STABILITY OF CYLINDRICAL AND CONICAL MOTIONS OF A RIGID ROTOR IN RETAINER BEARINGS

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ABSTRACT

We examine cylindrical and conical motions of a horizontal, symmetric, rigid rotor in rigid retainer bearings. The model also assumes a small ratio between air gap and rotor length, and a high rotor spin rate. The analyses are for small (linear) deviations about the cylindrical and conical configurations. Conical motions may be locally stable and cylindrical oscillations are locally unstable.

INTRODUCTION

The dynamics of rotors in retainer bearings is very important to the active magnetic-bearing (AMB) industry. In event of AMB failure, there must be confidence regarding safety, protection of the AMB assembly, and the lifetime of retainer bearings.

Related studies include resonance analysis of elastic rotors with bearing clearances [1], a nonlinear study of synchronous whirl with bearing clearances [2], numerical simulations of rotor drop [3], experiments on large retainer bearings [4], an experimental and numerical investigation on elastic rotors, with suggested remedies for problems associated with whirl [5], and an analysis on rigid rotors in rigid retainer bearings and the associated dynamic bearing loads [6]. Results in the latter study were based on cylindrical and conical modes of motion (Figure 1).

This paper compliments [6] with a stability analysis of cylindrical and conical motions of a rigid rotor in rigid retainer bearings with a small air gap. Motions which are nearly conical exist and turn out to be locally stable for a large scope of parameter values. Cylindrical motions exist but are locally unstable during small cylindrical oscillations.

MODEL

We examine a horizontal, perfectly balanced rigid rotor in a rigid bearing with dry-friction contacts. We assume that the air gap ρ is small, the rotor length L is large, the spin $\dot{\phi}$ of the rotor is large and constant, and the rotor stays in sliding contact with the bearings. (The transition from rotor drop to sliding may be complicated, as implied in [7]). The friction is the source of excitation, and is modeled with the Coulomb law.

The coordinate system is shown in Figure 2, in which y denotes the distance of the center of gravity G of the rotor from the center of the bearing housing, β is the angle of G from the bottom of the bearing housing, and ψ represents the pitch angle of the rotor. During contact, a constraint relates y and ψ by $y^2 = \rho^2 - L^2 \tan^2 \psi$. Since L is large and ρ is small, the constraint can be approximated as $y^2 = \rho^2 - L^2\psi^2$.

The constraint approximates an ellipse in y and ψ. When relating the velocities, $\dot{y}$ is singular near the $\psi = 0$ position, and $\dot{\psi}$ is singular near the $y = 0$ position. For this reason, we look at two models: one for "nearly cylindrical" motions in the neighborhood of $\psi = 0$; and another for "nearly conical" motions in the neighborhood of $y = 0$. The cylindrical model (for nearly cylindrical motion) is described by θ and ψ, with ψ assumed to be small. The conical model is described by θ and y, with y assumed to be small. The small-coordinate assumptions result in a linear stability analysis.

The coordinates described above are Lagrangian coordinates [8]. Together with the constraint, the

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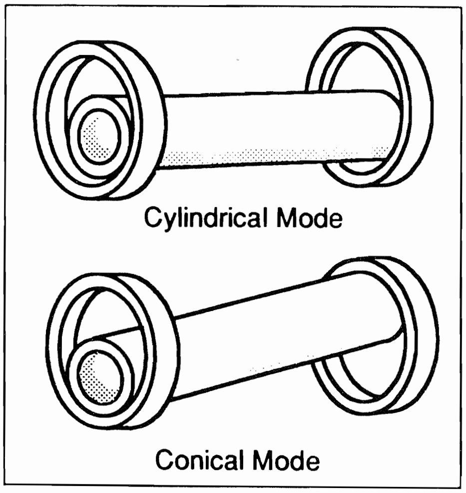

Figure 1: Hypothetical motions for a rigid rotor in rigid retainer bearings

kinetic and potential energies can be described:

$$T = \frac{1}{2}J\Omega_1^2 + \frac{1}{2}I(\Omega_2^2 + \Omega_3^2) + \frac{1}{2}m(v_1^2 + v_2^2 + v_3^2),$$

$$V = -mgy\cos\theta,$$

where $\Omega_1 = \dot\phi + \dot\theta\cos\psi, \Omega_2 = \dot\psi, \Omega_3 = \dot\theta\sin\psi, v_1 = -\dot\theta y\sin\psi, v_2 = \dot y$, and $v_1 = \dot\theta y\cos\psi$. Lagrange's equations can then be applied. This yields equations of motion for the frictionless system. To find the normal loads, we use $\dot{\mathbf{H}}_G = \sum \mathbf{M}_G$ and $\dot{\mathbf{L}}_G = \sum \mathbf{F}_G$, where $\mathbf{H}_G$ is the angular momentum, and $\mathbf{L}_G$ is the linear momentum. $\mathbf{M}_G$ and $\mathbf{F}_G$ are the moment about G and the resultant force, respectively, applied to the body by normal constraint forces N_1 and N_2 at each contact. We compute $\dot{\mathbf{H}}_G$ and $\dot{\mathbf{L}}_G$, and then solve the constraint forces. The friction is then applied as μN_i, where μ is the coefficient of friction, and added to the equations of motion in the appropriate way.

For a symmetric, horizontal rotor, the equations of nearly conical motions are [6]

$$m\ddot y - (m + \frac{(J-I)}{L^2})y\dot\theta^2 - \frac{J\Omega}{L^2}y\dot\theta - mg\cos\theta$$

$$+\mu m y\ddot\theta + 2\mu m\dot y\dot\theta = 0 \qquad (1)$$

$$I\ddot\theta + \frac{mgL^2}{\rho^2}y\sin\theta = -\mu(I-J)\dot\theta^2 + \mu\Omega J\dot\theta. \qquad (2)$$

and the equations for nearly cylindrical motions are

$$I\ddot\psi + J\dot\theta\dot\phi\psi - (I-J)\dot\theta^2\psi + mL^2\dot\theta^2\psi + mgL^2\psi\cos\theta$$

$$= -\mu(I\ddot\theta\psi + (2I-J)\dot\theta\dot\psi - J\dot\phi\dot\psi), \qquad (3)$$

$$\ddot\theta + \frac{g}{\rho}(\sin\theta + \mu\cos\theta) + \mu\dot\theta^2 = 0. \qquad (4)$$

Purely cylindrical motion ($\psi \equiv 0$) is a solution for equation (3). Purely conical motion ($y \equiv 0$) does not satisfy equation (1). However, if we consider the case of high whirling velocity ($\dot\theta$ is very large), and neglect the $\cos\theta$ term in equation (1), $y \equiv 0$, is a solution.

CONICAL MOTION

We can carry out a more thoughtful analysis of the conical equations of motion. This will lead to a better understanding of this mode of motion, and a conclusion that it is often stable.

To clean up the appearance of the equations, we let $a = J/I$, and $b = J/mL^2$. Suppose $\dot\phi = \Omega$ is fixed and large. Letting $\tau = \Omega t$, the equations of conical motion become

$$\Omega^2 y'' + c\Omega^2 y\theta'^2 - d\Omega y\theta' + 2\Omega^2 y'\theta' - g\cos\theta = 0$$

$$\Omega^2\theta'' + \mu(1-a)\Omega^2\theta'^2 - \mu a\Omega^2\theta' + \Gamma y\sin\theta = 0,$$

where $c = (b(1-a)/a + \mu^2(1-a) - 1), d = b - \mu^2 a$, $\Gamma = ag/b\rho^2$, and the prime refers to differentiation with respect to τ. Because the air gap ρ is taken to be small, $\Gamma = \xi\Omega$ may be considered to be large.

We introduce $\epsilon = 1/\Omega$ as a small bookkeeping parameter. The equations of motion are then

$$y'' + cy\theta'^2 - d\epsilon y\theta' + 2y'\theta' - \epsilon^2 g\cos\theta = 0 \qquad (5)$$

$$\theta'' + \mu(1-a)\theta'^2 - \mu a\theta' + \epsilon\xi y\sin\theta = 0. \qquad (6)$$

In the method of multiple scales [9], the dependent variables are expanded as $y = y_0 + \epsilon y_1 + \epsilon^2 y_2 + \ldots$, $\theta = \theta_0 + \epsilon\theta_1 + \epsilon\theta_2 + \ldots$, and the time variable is considered to have several time scales, i.e. $\tau = \tau(T_0, T_1, T_2, \ldots)$, where $T_i = \epsilon^i\tau$. Thus, $d/d\tau = D_0 + 2\epsilon D_0 D_1 + \epsilon^2 D_1 + 2\epsilon^2 D_0 D_2 + \ldots$, where $D_i = \partial/\partial T_i$. We plug these ideas into equations (5) and (6), and look at the coefficients of like powers of ϵ. The coefficient of ϵ^0 leads to

$$D_0^2 y_0 + 2D_0 y_0 D_0\theta_0 + cy_0(D_0\theta_0)^2 = 0 \qquad (7)$$

$$D_0^2\theta_0 + \mu(1-a)(D_0\theta_0)^2 - \mu a D_0\theta_0 = 0, \qquad (8)$$

The latter equation is invariant of y. Letting $\omega_0 = D_0\theta_0$, its steady-state solution is $\bar\omega_0 = a/(1-a)$. The complete solution for ω_0 is

$$\omega_0 = \frac{\mu a}{\mu(1-a) - e^{-\mu a(T_0 + A(T_1,T_2))}},$$

where $A(T_1, T_2)$ is a constant of integration with respect to T_0, and is dependent on the initial conditions. $\bar\omega_0 = 0$ is also a solution, but it is unstable.

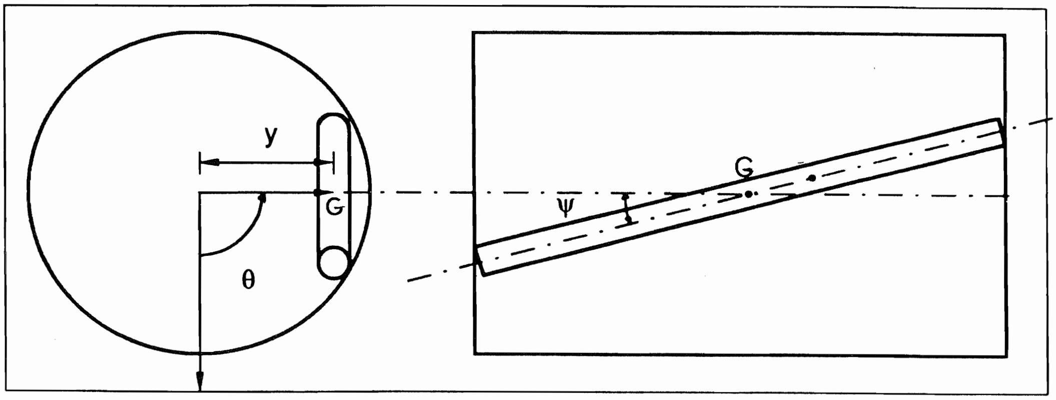

Figure 2: The coordinate system depicts θ as the circumferential whirling coordinate, ψ as the pitch angle, and y as the radial displacement of the center of the rotor.

There is also an unbounded solution for ω_0 in the opposite sense as $a/(1-a)$. The initial conditions determine whether the solution will be unbounded or approach the stable fixed point. For the stable fixed point, $D_0\omega_0, D_1\omega_0$, and $D_2\omega_0$ approach zero as $\tau \to \infty$.

Equations (7) and (8) together have an unstable equilibrium at $(\bar{y}_0, \bar{\omega}_0) = (\bar{y}_0, 0)$, and an equilibrium at $(\bar{y}_0, \bar{\omega}_0) = (0, a/(1-a))$, which is stable if $a < 1$ and $b > a/(1-a) - \mu^2$, and unstable if $a > 1$.

Continuing with the multiple-scales analysis for the case of the steady state solution for ω_0, the coefficients of ϵ^1 lead to

$$D_0^2 y_1 + 2\omega_0 D_0 y_1 + c\omega_0^2 y_1 = 0$$

$$D_0^2\theta_1 + 2D_0D_1\theta_1 + 2\mu(1-a)D_0\theta_0 D_0\theta_1$$
$$+2\mu(1-a)D_0\theta_0 D_1\theta_0 - \mu a D_0\theta_1 - \mu a D_1\theta_0 = 0.$$

Since the derivatives of $D_0\theta_0 = \omega_0$ approach zero in the steady state, the θ equation reduces to

$$D_0^2\theta_1 + (2\mu(1-a)\omega_0 - \mu a)D_0\theta_1 = 0.$$

Thus, the steady-state solution to these equations is $y_1 = 0$, again stable, and $\omega_1 = 0$, stable if $2(1-a)\omega_0 > a$, which is true for $a \neq 1$ $(J \neq I)$.

Similary, the coefficients of ϵ^2 lead to

$$D_0^2 y_2 + 2\omega_0 D_0 y_2 + c\omega_0^2 y_2 = g\cos(\omega_0 T_0)$$

$$D_0^2\theta_2 + (2\mu(1-a)\omega_0 - \mu a)D_0\theta_2 = 0.$$

whence

$$y_2 = Y_2\cos(\omega_0 T_0),$$

with $Y_2 = cg/2\omega_0^2\sqrt{(c-1)^2+4}$ and $\tan\phi = 2/(c-1)$. Again, $D_0\theta_2 = \omega_2 = 0$, stable if $J \neq I$.

Putting the results back into the original time variable t, we have $y = \epsilon^2 Y_2\cos(\omega_0\Omega t + \phi)$, and $\dot{\theta} = \omega_0\Omega$. This is stable.

Let us address the unbounded solution for θ' by studying equations (5) and (6) directly. The question is whether y approaches zero while the whirling θ' grows. For large $\theta' = \hat{\omega}$, equation (6) reduces to $\hat{\omega}' = -\mu(1-a)\hat{\omega}^2$. If $a < 1$, the unbounded whirl is in the negative θ direction, while if $a > 1$ it is in the positive θ direction. The y equation is then approximated by

$$y'' + 2\hat{\omega}y' + c\hat{\omega}^2 y = 0.$$

Since the coefficients are functions of time, we should be careful in accessing the stability of $(y, y') = (0,0)$. We consider, à la Lyapunov, a function V in the (y, y') plane such that $2V = cy^2 + {y'}^2/\hat{\omega}^2$. A level set of V is an ellipse for which, as $\hat{\omega}$ increases, the y axis of the ellipse remains fixed, and the y' axis of the ellipse grows. Differentiating V, and using equations (5) and (6), shows that $dV/d\tau = (\mu(1-a)-2){y'}^2/\hat{\omega}$. For $\hat{\omega} > 0$ $(a > 1)$ $dV/d\tau < 0$. Thus the y axis of the ellipse shrinks in time. Although this says nothing about the y' axis, visually, we see the rotor near the conical position during failure. For $\hat{\omega} > 0$ $(a < 1)$ the y axis grows, indicating instability.

CYLINDRICAL MOTION

Equations (3) and (4) have equilibrium solutions at $(\bar{\psi}, \bar{\theta}) = (0, -\tan^{-1}\mu)$, which is locally unstable. However, other types of motions are possible.

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Equation (4) is invariant of ψ. Under the change of coordinates $\theta = \eta - \pi/2 + \gamma$, where $\gamma = -\tan^{-1} 1/\mu$, this equation becomes

$$\ddot{\eta} + \alpha \sin \eta + \mu \dot{\eta}^2 = 0,$$

where $\alpha = g\sqrt{1+\mu^2}/\rho$. The ψ equation can be viewed as parametrically excited by the output of the η equation (θ equation).

The η equation can be solved to obtain a relationship between η and $\dot{\eta}$,

$$\dot{\eta}^2 = -\frac{2\alpha}{1+4\mu^2}(2\mu \sin \eta - \cos \eta) + Ce^{-2\mu\eta},$$

where C is a constant of integration. Depending on C, this solution may describe either closed or open curves in $(\eta, \dot{\eta})$ [6]. The closed curves correspond to periodic oscillations, and the open curves depict motions which become unbounded in finite time. Further manipulation leads to the following integral which relates η and time t:

$$t = \int_{\eta_0}^{\eta} \frac{dx}{\sqrt{-\frac{2\alpha}{1+4\mu^2}(2\mu \sin x - \cos x) + Ce^{2\mu x}}}.$$

In theory, when $dt/d\eta \neq 0$, we can invert this expression to obtain $\eta(t)$. Furthermore, for oscillatory motions, we can determine endpoints for the intergral equation corresponding to a half period of motion. Thus, we can obtain the period T by performing this integral. This, of course, would be done numerically. T depends upon the amplitude of oscillation. From T, we obtain the circular frequency $v = 2\pi/T$.

Given the frequency v of a periodic oscillation, the periodic solution can be expressed as a Fourier series:

$$\eta(t) = \eta_0 + \Sigma_{n=1}^{\infty} \eta_n \sin nvt.$$

For small oscillations, η can be expressed with a small bookkeeping parameter ϵ, such that

$$\eta(t) = \epsilon\beta(t) = \epsilon\beta_0 + \epsilon\Sigma_{n=1}^{\infty} \beta_n \sin nvt.$$

Letting $\theta = \epsilon\beta - \pi/2 + \gamma$, using $\sin(\epsilon\beta + \gamma) \approx \epsilon\beta \cos\gamma + \sin\gamma$, and assuming $J = \epsilon\hat{J}$, $\dot{\phi} = \Omega/\epsilon$), and I, mL^2, and μ are order one, equation (3) has the form

$$\ddot{\psi} - a\dot{\psi} + 2\mu\dot{\beta}(t)\dot{\psi} + [d + \epsilon(c\beta(t) + f\dot{\beta}(t) + \mu\ddot{\beta}(t))]\psi = 0, \quad (9)$$

where $a = \hat{J}\Omega\mu/I, c = mgl^2(\cos\gamma)/I + \mu\alpha, d = mgl^2(\sin\gamma)/I$, and $f = \hat{J}\Omega/I$.

The negative coefficient on the order-one damping term probably indicates instability. An analysis will show that other effects, such as geometry, forcing, and gyroscopic terms, do not overrule this negative linear damping. We can analyze equation (9) for small ψ, which will correspond to analyzing both equation (3) and equation (4) for small motions.

Perturbation Analysis

Since $\beta(t)$ is periodic, it can be viewed as a parametric excitation, similar to the Mathieu equation. For order-one damping, we follow the treatment of [10] to transform the system into an equation with small damping. We let $\psi = ze^{at/2}$. Substituting, the order-one damping term cancels, and we obtain

$$\ddot{z} - \epsilon b\dot{\beta}(t)\dot{z}$$

$$+[d - \frac{a^2}{4} + \epsilon(c\beta(t) - 2\mu a\dot{\beta}(t) + f\dot{\beta}(t) + \mu\ddot{\beta}(t))]z = 0.$$

Using $\beta(t) = \Sigma_{n=1}^{\infty}\beta_n \sin n\Omega t$, rescaling time such that $\tau = \Omega t$, and renaming τ as t, we have

$$\ddot{z} + w^2 z - \epsilon u\dot{z}\Sigma_{n=1}^{\infty} n\beta_n \cos nt$$

$$+\epsilon(p\Sigma_{n=1}^{\infty}\beta_n \sin nt + q\Sigma_{n=1}^{\infty} n\beta_n \cos nt)z = 0, \quad (10)$$

where $w^2 = 1(d - \frac{a^2}{4})/v^2, u = 2\mu/v, p = c/v^2$, and $q = (f - a\mu)/v$. Equation (10) can be analyzed using the method of multiple scales. Stability is determined by comparing the growth of z with $e^{at/2}$.

Using the method of multiple scales, we again replace t by $t(T_0, T_1, \ldots)$. Thus, $d/dt = D_0 + \epsilon D_1 + \cdots$, where $D_0 = \partial/\partial T_0$, and $D_1 = \partial/\partial T_1$. We also expand the frequency parameter such that $w^2 = w_0^2 + \epsilon w_1 + \cdots$. We seek solutions expansions of the form $z(t) = z_0(T_0, T_1, \ldots) + \epsilon z_1(T_0, T_1, \ldots) + \cdots$. Substituting these expansions into equation (10), and separating like powers of ϵ, we obtain an equations at order one and at order ϵ. The order-one equation is

$$D_0^2 z_0 + w_0^2 z_0 = 0,$$

and has solutions of the form

$$z_0 = A(T_1)\cos(w_0 T_0) + B(T_1)\sin(w_0 T_0).$$

Plugging z_0 into the order-ϵ equation, and using trigonometric identities, yields

$$D_0^2 z_1 + w_0^2 z_1 = 2A' w_0 \sin w_0 T_0 - 2B' w_0 \cos w_0 T_0$$

$$-w_1 A \cos w_0 T_0 - w_1 B \sin w_0 T_0$$

$$+\frac{uw_0 B - qA}{2}\Sigma_{n=1}^{\infty} n\beta_n(\cos(n - w_0)T_0 + \cos(n + w_0)T_0)$$

$$+\frac{uw_0 A + qB}{2}\Sigma_{n=1}^{\infty} n\beta_n(\sin(n - w_0)T_0 - \sin(n + w_0)T_0)$$

$$-\frac{A}{2}\Sigma_{n=1}^{\infty}(p - \mu n^2)\beta_n(\sin(n - w_0)T_0 + \sin(n + w_0)T_0)$$

$$-\frac{B}{2}\Sigma_{n=1}^{\infty}(p - \mu n^2)\beta_n(\cos(n - w_0)T_0 - \cos(n + w_0)T_0),$$

where the prime indicates partial differentiation with respect to T_1.

The particular solution is unbounded when there are resonances, i.e. when the frequencies in the nonhomogeneous terms are equal to the frequency of the homogeneous solution. Such terms are called secular terms. The resonances occur when $n - w_0 = w_0$, or $w_0 = n/2$. During resonance, the secular terms can be eliminated by setting their coefficients to zero. This leads to a linear system of first-order differential equations in the amplitudes of response $A(T_1)$ and $B(T_1)$:

$$A' = \frac{(2p - un^2)\beta_n}{4n} A + \frac{2w_1 - qn\beta_n}{2n} B$$

$$B' = -\frac{2w_1 + qn\beta_n}{2n} A - \frac{(2p - un^2)\beta_n}{4n} B.$$

The eigenvalues of this system of equations provide the exponential growth rates of the amplitudes of $z_0(t)$. Therefore, we can determine the stability of $\psi(t)$ by examining the growth of $z_0 e^{at/2}$. In this case, the eigenvalues are either imaginary, or else real with opposite signs. Thus, $ze^{at/2}$ has exponential growth, and the system is unstable. Similarly, the nonresonant case is also unstable.

Physical insight arises from a simplified system. Suppose a balanced rotor rests symmetrically on two rails such that the normal support load is the the same at each contact, and translational motions are constrained as it spins. If a slight rotational disturbance is imposed in the plane of the rails, the gyroscopic effect changes the normal contact loads, and thus the frictional moment, in such a way as to encourage more rotation in the direction of the disturbance.

Cylindrical Whirl

We noted that the η equation (and hence θ) has a solution which becomes unbounded in finite time. We address whether this motion may be stable in the cylindrical configuration. Suppose $\omega = \dot{\theta}$. For purely cylindrical motion, rolling occurs when $\omega = r\Omega/\rho$. If the air gap ρ is indeed very small, there is a range in which $\omega >> \Omega$.

Equations (3) and (4) can be approximated large ω. If we further assume $\omega >> \Omega$, the gyroscopic effect is overshadowed, and the approximate equations are

$$\ddot{\psi} + \mu(2-a)\omega\dot{\psi} + k\omega^2\psi = 0$$

$$\dot{\omega} = -\mu\omega,$$

where $k = a - 1 + b + \mu^2$, and a and b are defined as in the section on conical motion. Note that $\mu\omega < 0$ for the unbounded whirling solution. Writing $2V = \psi^2 + \dot{\psi}^2/k\omega^2$, a level set of V again describes an ellipsoid whose ψ axis is fixed when ω^2 increases. We find that $\dot{V} = \mu(a-1)\dot{\psi}^2$. Since $\mu/\omega < 0$, $\dot{V} < 0$ if $(a-1)/k > 0$. This is satisfied if $a > 1$ or if $a < 1 - b - \mu^2$. The interpretation of $\dot{V} < 0$ is that ψ remains close to zero and hence cylindrical motions are observed.

Thus, for a range of parameters, if the whirl speed is so large that it overshadows the gyroscopic effect, the cylindrical configuration may be observed at this stage of the unbounded motion. Somewhere between low-speed whirl and high-speed whirl there can be a transition of local stability.

CONCLUSIONS

The equations of motion for a perfectly balanced rotor in contact with retainer bearings have been written for small, nearly cylindrical and nearly conical motions, and a local analysis of these configurations has been performed.

The analysis also shows that a nearly conical motion exists in the form of a steady conical whirl, which is stable. There is also an exploding whirl which remains conical if $J > I$. Indeed, engineers have observed conical motions during rotor drop.

A purely cylindrical motion exists but is locally unstable during small cylindrical oscillations (in the θ coordinate). An unbounded cylindrical whirl also exists. As the whirling velocities increase, there is a local stabilizing effect for this configuration for a range of parameter values. This takes place for regimes in which the whirl rate is so large that the gyroscopic effects are neglegible. This type of motion would lead to quick failures in the system [6].

Future studies may deal with the global dynamics. This calls for a new model which encompasses all sliding motions from the cylindrical extreme to the conical extreme. It would address whether the locally unstable cylindrical oscillations can be globally stabilized, for example as limit-cycle oscillations of the pitch coordinate ψ.

The effects of elasticity, imbalance, and impacts would be other areas to broaden the scope of the model.

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