INTERACTION OF PERMANENT CYLINDRICAL MAGNETS WITH AXIAL MAGNETIZATION

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ABSTRACT

Known equivalence of the field of a permanent magnet having the form of a circular cylinder and axial magnetization and that of the field of a circular tape with constant surface current is used. Exact calculation of potential energy of interaction between two circular currents makes it possible:

1) to express in accurate formulae force and moment characteristics of interaction between magnets;
2) retaining the second-order memebers with respect to small displacements of two initially coaxial rings to build "the stiffness matrix" and analyse quantitatively the known static unstability of the system. One of the possible rotor suspension configurations with permanent magnets is given to illustrate why the main quantitative characteristic of unstability - negative relationship of eigenvalues of the stiffness matrix - remains valid for all axially symmetric systems.

INTRODUCTION

Axial and radial rotor stabilization in machines and devices being provided by permanent magnets have, as a rule, axial symmetry and more or less complicated boundary composed by cylindrical (in rare cases, by conical) surfaces [1]. Modern metal-ceramic materials and magnetization technology make it possible to regard magnetization as permanent uniform and either radial or axial depending on configuration.

Analysis of force and rigidity characteristics of a magnetic suspension system under complicated configuration of magnets themselves and magnetic circuits is a complex problem dealt with in numerous special works and computer program packages. There is a natural tendency to obtain simple, visible, though approximate estimation formulae representing the effect caused by main parameters (such as dimensions relationship). One can mention analysis of Baermann's bearing made by Bekkers [1], work [2], and others. Frequently, simplification of physical model or geometry of magnets is done to make the analysis easier. Alternative approach is possible in simple situations. In such a case an exact field problem solution which is further simplified by using not physical but mathematical ways of approximation is obtained.

There is a special stability problem requiring exact relationships. It is a very urgent problem since numerous attempts to provide complete body levitation in permanent magnetic field do not cease. Fruitlessness of such attempts except the case of diamagnetic is theoretically proven by Earnshow [3] and Braunbeck [4]. Using approximate formulae for stability analysis can qualitatively break the result.

The present paper deals with interaction between two axis-symmetric permanent magnetic systems with axial magnetization under small displacement from coaxiality. Analysis of such systems is based on the coupling energy between two circular rings powered by direct current. Analytical solution of the problem up to forming matrix of quadratic form for potential energy is made without approximations which permits to analyze stability exactly.

To provide rotor suspension circular permanent magnets with axial magnetization are often used as radial and axial bearings. It is known [5] that fields of such magnets are equivalent to those of permanent ring currents distributed along cylindrical surfaces. One can regard them as superposition of circular currents and use energy additivity which makes it possible, firstly, to find force and moment characteristics of interaction between actual magnets by integrating with respect to the axial coordinate and, secondly, to consider conclusions obtained during sta-

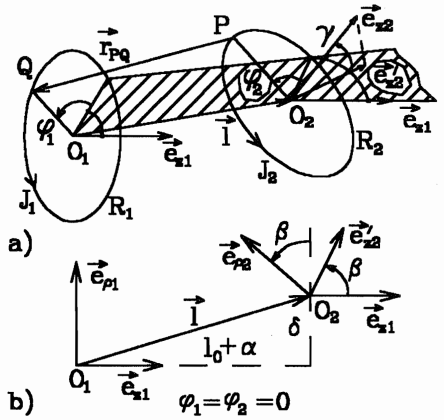

FIGURE 1: System unit vectors and coordinates for rings arrangement

bility analysis of a simple ring pair of current to be true for any system that can be reduced to interaction between such pairs.

POTENTIAL ENERGY OF INTERACTION BETWEEN CIRCULAR CURRENTS

Let us consider two circular arbitrarily located rings with currents (Fig. 1a). Subscript i=1, 2 indicates that characteristics belongs to the i-th rings:

R_i are the ring radii; J_i are the ring currents; O_i are the ring centers.

Vector $\vec{l}=\overrightarrow{O_1O_2}$ connects centers. Let each ring be related to a cylindrical coordinate system $\{\rho_i, \varphi_i, z_i\}$ with unit vectors $\{\vec{e}_{\rho i}, \vec{e}_{\varphi i}, \vec{e}_{zi}\}$, dependent on polar angle:

$$\begin{cases} \vec{e}_{\rho i}(\varphi_i)=\vec{e}^{\,0}_{\rho i}\cos\varphi_i+\vec{e}^{\,0}_{\varphi i}\sin\varphi_i \\ \vec{e}_{\varphi i}(\varphi_i)=-\vec{e}^{\,0}_{\rho i}\sin\varphi_i+\vec{e}^{\,0}_{\varphi i}\cos\varphi_i \end{cases} \tag{1}$$

Here superscript $(^\circ)$ corresponds to angle $\varphi_i=0$. Axis z_i passes through the center of the i-th ring perpendicularly to its plane, current J_i is considered to be positive if it flows in the ring counterclockwise relative to the unit vector $\vec{e}_{zi}$.

To link cooridinate systems let us draw a plane **H** containing $\vec{l}$ and $\vec{e}_{z1}$ (shaded in Fig. 1a) and count polar angles φ_i from it. This breaks the symmetry of the coordinate systems used but turns out to be convinient enough for our purposes.

Let $\vec{e}\,'_{z2}$ denote the projection of unit vector $\vec{e}_{z2}$ to the plane **H**. Let us introduce angles:

$\beta=\widehat{\vec{e}_{z1},\vec{e}\,'_{z2}}$ - in the plane **H**;

$\gamma=\widehat{\vec{e}\,'_{z2},\vec{e}_{z2}}$ - in perpendicular plane.

Mutual location of unit vectors in the plane **H** is shown in Fig.1b. The same figure shows α-axial and δ-radial displacement of the center O_2 of the "movable" ring relative to its "nominal" position on axis O_1z_1 at distance l_0 from the center O_1 of the "fixed" ring.

Vector potential $\vec{A}_1$ of the field arising from ring 1 at the point P of ring 2 is determined by the formula [6]:

$$\vec{A}_1(\vec{r}_p)=\frac{1}{c}\oint_{l_1}\frac{\vec{J}_1(Q)}{r_{PQ}}\,dl_1 \tag{2}$$

where dl_1- is the linear element of the ring 1; Q - is the current point of ring 1; $\vec{r}_p=\overrightarrow{O_1P}$ - is the radius vector of point P relative to the center O_1; $r_{PQ}=|\overrightarrow{r_{PQ}}|$ - is the distance between points P and Q ; c - is the speed of the light. (Here and further Gauss system of units is used.)

Potential energy U_{12} of interaction between rings is determined by integrating with respect to ring 2 [6]:

$$U_{12}=-\frac{1}{c}\oint_{l_2}\vec{A}_1(P)\cdot\vec{J}_2(P)\,dl_2. \tag{3}$$

($\vec{a}\cdot\vec{b}$ - is the scalar product of vectors.)

Vector-currents and linear elements are determmined by formulae

$$\vec{J}_i=J_i\vec{e}_{\varphi i},\quad dl_i=R_i d\varphi_i,\quad (i=1,2).$$

Substituting $\vec{A}_1$ (2) into U_{12} (3) we shall need quantities $\vec{J}_1\cdot\vec{J}_2=J_1J_2\vec{e}_{\varphi 1}\cdot\vec{e}_{\varphi 2}$,

$$\begin{aligned} r_{PQ}^2&=\left|R_1\vec{e}_{\rho 1}-R_2\vec{e}_{\rho 2}-\vec{l}\right|^2= \\ &=R_1^2+R_2^2+l^2-2R_1R_2\vec{e}_{\rho 1}\cdot\vec{e}_{\rho 2}-2R_1\vec{l}\cdot\vec{e}_{\rho 1}+2R_2\vec{l}\cdot\vec{e}_{\rho 2} \end{aligned} \tag{4}$$

for whose calculation let us express the unit vectors of the system 2 in term of unit vectors of the system 1 at $\varphi_1=0$. In this basis, obviously, $\vec{e}^{\,0}_{r1}=(1,0,0)$, $\vec{e}^{\,0}_{\varphi 1}=(0,1,0)$, $\vec{e}_{z1}=(0,0,1)$. From Fig. 1b and determination of angle γ we obtain

$$\left.\begin{aligned} \vec{e}^{\,0}_{\rho 2}&=\vec{e}^{\,0}_{\rho 1}\cos\beta-\vec{e}_{z1}\sin\beta, \\ \vec{e}_{z2}&=\vec{e}^{\,0}_{\varphi 1}\sin\gamma+(\vec{e}^{\,0}_{\rho 1}\sin\beta+\vec{e}_{z1}\cos\beta)\cos\gamma, \end{aligned}\right\} \tag{5}$$

then $\vec{e}^{\,0}_{\varphi 2}$ is found by vector product:

$$\vec{e}^{\,0}_{\varphi 2}=\vec{e}_{z2}\times\vec{e}^{\,0}_{\rho 2}=\vec{e}^{\,0}_{\varphi 1}\cos\gamma-(\vec{e}^{\,0}_{\rho 1}\sin\beta+\vec{e}_{z1}\cos\beta)\sin\gamma$$

Using these representations and formulae (1) of rotation of unit vectors , we obtain

$$\left.\begin{aligned}&\vec{e}_{\rho 1}(\varphi_1)\cdot\vec{e}_{\rho 2}(\varphi_2)=\cos\beta\cos\varphi_1\cos\varphi_2-\\&\quad-\sin\beta\sin\gamma\cos\varphi_1\sin\varphi_2+\cos\gamma\sin\varphi_1\sin\varphi_2\\&\vec{e}_{\varphi 1}(\varphi_1)\cdot\vec{e}_{\varphi 2}(\varphi_2)=\cos\beta\sin\varphi_1\sin\varphi_2+\\&\quad+\sin\beta\sin\gamma\sin\varphi_1\sin\varphi_2+\cos\gamma\cos\varphi_1\cos\varphi_2\end{aligned}\right\}\quad(6)$$

which enter in (4).

Expansion of vector $\vec{l}$ is illustrated by Fig. 1b:

$$\vec{l}=(l_0+\alpha)\,\vec{e}_{z1}+\delta\vec{e}^{\,0}_{\rho 1}\quad(7)$$

from where

$$l^2=\left|\vec{l}\right|^2=(l_0+\alpha)^2+\delta^2\quad(8)$$

$$\left.\begin{aligned}&\vec{l}\cdot\vec{e}_{\rho 1}(\varphi_1)=\delta\cos\varphi_1\\&\vec{l}\cdot\vec{e}_{\rho 2}(\varphi_2)=\delta\cos\beta\cos\varphi_2-\delta\sin\beta\sin\gamma\sin\varphi_2-\\&-(l_0+\alpha)\sin\beta\cos\varphi_2-(l_0+\alpha)\cos\beta\sin\gamma\sin\varphi_2\end{aligned}\right\}\quad(9)$$

Substitution of the expression obtained into (3) will result for potential energy in terms of double integral with respect to $\varphi_{1,2}$: $0\le\varphi_1,\ \varphi_2\le 2\pi$ depended on four parameters:

$$U_{12}=U_{12}(\alpha,\beta,\gamma,\delta)\quad(10)$$

and dimensions of the ring system: R_1, R_2, l_0.

Currents J_i enter into multiplicate constant

$$g=\frac{J_1J_2}{c^2}2\pi\quad(11)$$

Parameters $\alpha\ldots\delta$ are generalized coordinates of a mechanical two-ring system with four degrees of freedom: rotation about axes z_1 and z_2, obviously, do not change energy. Complete expression for energy is not given because of awkwardness and also because it will not be used further (small values of parameters $\alpha\ldots\delta$ will be considered), however the expressions given above can be useful for studying non-linear effects of interaction.

QUADRATIC APPROXIMATION: FORCE FACTORS AND STIFFNESS MATRIX

Let generalized coordinates $\alpha\ldots\delta$ characterize displacement of "movable" ring 2 relative to nominal coaxial position ($|\overrightarrow{O_1O_2}|=l_0,\alpha=\delta=0,\beta=\gamma=0$) and be small. Potential energy in quadratic approximation has the form

$$U_{12}\approx U^0_{12}+\sum_{j=1}^{4}f_{qj}q_j+\frac{1}{2}\sum_{j=1}^{4}\sum_{k=1}^{4}C_{jk}q_jq_k$$

where $q_1\ldots q_4$ substitute $\alpha\ldots\delta$. Constant component U^0_{12} is of no importance. Determining force factors f_{qj} and elements of stiffness matrix C_{jk} requires that the 2-nd order members in subintegral expression (3) be retained. From the 2-nd formula (6) we obtain

$$\vec{e}_{\varphi 1}\cdot\vec{e}_{\varphi 2}\approx\beta\gamma\sin\varphi_1\cos\varphi_2+$$

$$+(1-\frac{1}{2}\beta^2)\sin\varphi_1\sin\varphi_2+(1-\frac{1}{2}\gamma^2)\cos\varphi_1\cos\varphi_2\quad(12)$$

Similar approximation in (6), (9) and usage of (4), (8) makes it possible after obvious but awkward calculations to find

$$\begin{aligned}r^{-1}_{PQ}\approx{}&r_0^{-1}-r_0^{-3}[l_0\alpha+(R_2\cos\varphi_2-R_1\cos\varphi_1)\delta-\\&-R_2l_0(\beta\cos\varphi_2+\gamma\sin\varphi_2)]-\\&-\frac{1}{2}r_0^{-3}[\delta^2+\alpha^2-2R_2\alpha(\beta\cos\varphi_2+\gamma\sin\varphi_2)+\\&+R_1R_2(\beta^2\cos\varphi_1\cos\varphi_2+2\beta\gamma\cos\varphi_1\sin\varphi_2+\gamma^2\sin\varphi_1\sin\varphi_2)]+\\&+\frac{3}{2}r_0^{-5}\left[l_0^2\alpha^2+(R_2\cos\varphi_2-R_1\cos\varphi_1)^2\delta^2+\right.\\&+2l_0(R_2\cos\varphi_2-R_1\cos\varphi_1)\alpha\delta-2l_0^2R_2\alpha(\beta\cos\varphi_2+\gamma\sin\varphi_2)-\\&-2l_0R_2(R_2\cos\varphi_2-R_1\cos\varphi_1)(\beta\cos\varphi_2+\gamma\sin\varphi_2)\delta+\\&\left.+l_0^2R_2^2(\beta^2\cos^2\varphi_2+2\beta\gamma\cos\varphi_2\sin\varphi_2+\gamma^2\sin^2\varphi_2)\right]\quad(13)\end{aligned}$$

Here

$$r_0\equiv r_{PQ}\big|_{q=0}=\left(R_1^{\,2}+R_2^{\,2}+l_0^{\,2}-2R_1R_2\cos\varphi\right)^{1/2},\quad(14)$$

$$\varphi=\varphi_1-\varphi_2\quad(15)$$

By substituting (12), (13) into (3) and calculating partial derivatives we find force factors:

axial force

$$f_\alpha=\left(\frac{\partial U_{12}}{\partial\alpha}\right)_{q=0}=\frac{gl_0}{2\pi}\int_0^{2\pi}\!\!\int_0^{2\pi}r_0^{-3}\cos(\varphi_1-\varphi_2)d\varphi_1d\varphi_2;$$

passing to integrating with respect to φ (15) instead of φ_1, using periodicity and evenness of functions we can find easily

$$f_\alpha=2gl_0\int_0^{\pi}r_0^{-3}\cos\varphi d\varphi;\quad(16)$$

radial force f_δ vanishes as so **moments** f_β, f_γ. Thus,

$$f_\delta=0,\ f_\beta=f_\gamma=0,\quad(17)$$

which was easy to foresee due to axial symmetry of nominal position.

As to axial force f_α, according to (16) it vanishes only under coplanar ($l_0=0$) position of rings.

Let us pass to calculation of the stiffness matrix.

Axial stiffness is determined as

$$C_{\alpha\alpha}=\left(\frac{\partial^2 U_{12}}{\partial \alpha^2}\right)_{q=0}=-\frac{g}{2\pi}\int_0^{2\pi}\int_0^{2\pi} r_0^{-3}\cos(\varphi_1-\varphi_2)\left(1-3\left(\frac{l_0}{r_0}\right)^2\right)d\varphi_1 d\varphi_2$$

and is easily reeduced to the form

$$C_{\alpha\alpha}=2g\int_0^{\pi} r_0^{-3}\cos\varphi\left(1-3\left(\frac{l_0}{r_0}\right)^2\right)d\varphi \qquad (18)$$

For **radial stiffness** we have

$$C_{\delta\delta}=\left(\frac{\partial^2 U_{12}}{\partial \delta^2}\right)_{q=0}=-\frac{1}{2}C_{\alpha\alpha} \qquad (19)$$

Further, for solving stability problem this result is of crucial importance.
Similarly, we find

$$C_{\beta\beta}=C_{\gamma\gamma}=g\int_0^{\pi} r_0^{-1}\cos\varphi\left(1+\frac{R_1R_2\cos\varphi}{r_0^2}-\frac{3l_0^2R_2^2}{r_0^4}\right)d\varphi \qquad (20)$$

$$C_{\beta\delta}=C_{\delta\beta}=3gl_0R_2\int_0^{\pi} r_0^{-5}\cos\varphi(R_2-R_1\cos\varphi)d\varphi \qquad (21)$$

The rest elements of the stiffness matrix are equal to zero, which makes it possible to give its representation determined by three quantities only:

$$\begin{pmatrix} C_{\alpha\alpha} & 0 & 0 & 0 \\ 0 & C_{\beta\beta} & 0 & C_{\delta\beta} \\ 0 & 0 & C_{\beta\beta} & 0 \\ 0 & C_{\delta\beta} & 0 & -\frac{1}{2}C_{\alpha\alpha} \end{pmatrix} \qquad (22)$$

CALCULATION OF PARAMETERS OF INTERACTION BETWEEN CYLINDRICAL MAGNETS

The formulae obtained above for circular current rings allow us to find force and stiffnes characteristics of interaction between coaxial cylindrical magnets. For this purpose it is enough to substitute circular currents J_i by differentials of surface current [5]

$$dJ_i=M_i c dz_i\ ,\ (i=1,2), \qquad (23)$$

(here M_i - is the magnetization of the i-th cylindrical magnet which is considered to be axial and constant in magnets body) and integrate with respect to axial dimensions of interacting magnets. For cylinders this integration can always be made in explicit form.
As an illustration consider calculation of interaction force and axial stiffness of two equal magnets having height h and radius R with the distance between the centers l_0 (Fig. 2a). Such pair of magnets can serve as axial bearing of rotor suspension.
Reverse magnetization generates repulsive force act-

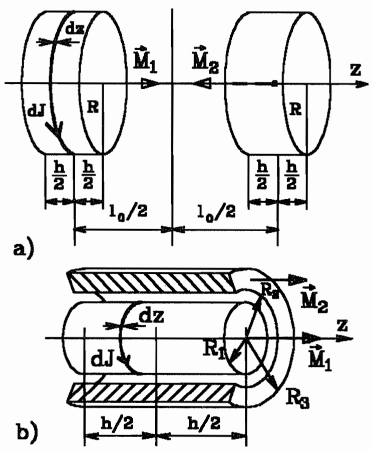

FIGURE 2: To computation of axial (a) and radial (b) interaction

ing along axis and determined according to (16), (23)

$$f_\alpha=2G\int_0^{\pi}\cos\varphi d\varphi\int_{-\frac{(l_0+h)}{2}}^{-\frac{(l_0-h)}{2}} dz_1\int_{\frac{l_0-h}{2}}^{\frac{l_0+h}{2}} r_0^{-3} z dz_2 \qquad (24)$$

Here $z=z_2-z_1$ takes in place of l_0 in (16),

$$G=2\pi M_1 M_2 \qquad (25)$$

$$r_0^2=a^2+z^2 \qquad (26)$$

$$a^2=2R^2(1-\cos\varphi) \qquad (27)$$

Integrating with respect to z instead of z_2 in (24), we have

$$f_\alpha=2G\int_0^{\pi}(2\Phi(0,\varphi)-\Phi(h,\varphi)-\Phi(-h,\varphi))\cos\varphi d\varphi, \qquad (28)$$

where function is introduced

$$\Phi(\eta,\varphi)=\ln\left(l_0+\eta+\sqrt{a^2+(l_0+\eta)^2}\right), \qquad (29)$$

and value $a=a(\varphi)$ is determined (27).
Similarly, axial stiffness is calculated.

$$C_{\alpha\alpha}=2G\int_0^{\pi}(2\psi(0,\varphi)-\psi(h,\varphi)-\psi(-h,\varphi))\cos\varphi d\varphi \qquad (30)$$

where

$$\psi(\eta,\varphi)=\left(a^2+(l_0+\eta)^2\right)^{-1/2} \qquad (31)$$

Calculation of radial stiffness of coaxial magnets (Fig. 2b) forming a radial bearing unit can be regarded as another illustration of the foregoing. At first, take into accont interaction between cylindrical surfaces of radii R_1 and R_2.
Formulae (18), (19) give

$$C_{\delta\delta}^{12}=G\int_0^{\pi}\cos\varphi d\varphi\int_{-h/2}^{h/2} dz_1\int_{-h/2}^{h/2}(r_0^{-3}-3z^2r_0^{-5})dz_2,$$

where as before $z=z_2-z_1$ but substitution of l_0 by z in (14) gives

$$r_0^2=b^2+z^2 \tag{32}$$

$$b^2=R_1^2+R_2^2-2R_1R_2\cos\varphi. \tag{33}$$

Calculation result in the formula of the form (30) but without multiplier 2, and in (31) should be replaced a^2 by b^2 according to (33).

Interaction between surfaces radii R_1 and R_3 contributes to the opposite sign, i. e.

$$C_{\delta\delta}=C_{\delta\delta}^{12}-C_{\delta\delta}^{13}, \tag{34}$$

$C_{\delta\delta}^{13}$ being calculated according to the same formulae with replacing $R_2 \rightarrow R_3$ in (33).

For practical calculation, in (28), (30) most frequently gap smallness between magnets can be used: $l_0=h+\alpha_0$ where $\alpha_0<<h$. Similarly, for a radial bearing $R_2=R_1+\delta_0$, $(\delta_0<<R_1)$.

Note that (30) permits representation in complete elliptic integrals which is natural for interaction between coaxial solenoids actually replacing cylindrical permanet magnets.

SYMMETRIC ROTOR SUSPENSION IN PERMANENT CYLINDRICAL MAGNETS

Fig. 3 shows symmetric rotor suspension provided by two radial magnetic bearings(stator ring are not shown) and two axial magnetic bearings with stiffness matrices C^r and C^α respectively. Such a configuration is known to be statically unstable, which is corroborated by the following analysis. Dimensions L and l are considered to be large enough to take into account only interaction inside the pair of magnetic rings ("movable-immovable").

A rotor position is determined by quantities (α, β, γ, δ) already shown in Fig. 1. and connect to the plane passing through axis z_1, and centers O_1 and O_2.

The problem arises from misalignment between the plane of central eccentricity and the eccentricity planes in pairs of interacting rings.

Consider the pair forming the right radial bearing with centers O_{r1}',O_{r2}' through which a plane containing axis z_1 is drawn. As in Fig. 1b let us introduce parameters (α_{r1}, β_{r1}, γ_{r1}, δ_{r1}) for these centers and the plane and express them in teerms of coordinates α . . . δ. For this purpose we introduce unit vectors of cylindrical systems ($\vec{e}_{\rho r1}, \vec{e}_{\varphi r1}, \vec{e}_{Z1}$) and $(\vec{e}_{\rho r2}, \vec{e}_{\varphi r2}, \vec{e}_{Z2})$ related to immovable and movable rings. They differ from the central unit vectors not indicated by r due to the existing misalignment between the planes of eccentricities.

Let us present the explicit vector equality

$$\vec{O}_{r2}'\vec{O}_{r2}'=\vec{O}_{r2}'\vec{O}_1'+\vec{O}_1\vec{O}_2+\vec{O}_2\vec{O}_{r2}'$$

as an expansion in unit vectors

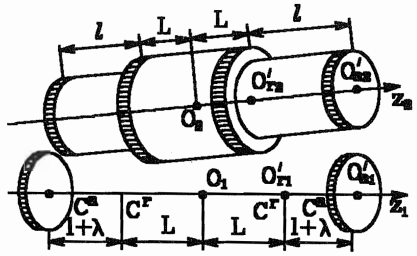

FIGURE 3: Symmetric rotor on permanent cylindrical magnet suspension

$$\alpha_{r1}e_{z1}+\delta_{r1}e_{\rho r1}=\alpha e_{z1}+\delta e_{\rho 1}^0-Le_{z1}+Le_{z2} \tag{35}$$

(Index (^{0}) corresponds to $\varphi_k=0;\ k=\rho 1,\ \rho r1$)

Equality (35) allows us to find the required relation.

1. Multiply it scalarly by $\vec{e}_{z1}$ and use (5). With accuracy up to the second order inclusive we have

$$\alpha_{r1}\approx\alpha-\frac{1}{2}L(\beta^2+\gamma^2) \tag{36}$$

2. Take squares of modules in the left and right sides of (32) using (5) again. This gives subject to (36)

$$\delta_{r1}^2\approx\delta^2+2L\delta\beta+L^2(\beta^2+\gamma^2) \tag{37}$$

3. Mulitiply (35) subsequently by $\delta\vec{e}_{\rho 1}$ and $\delta\vec{e}_{\rho r1}$ and sum the result, which gives

$$\delta_{r1}\beta_{r1}\approx\delta\beta+L(\beta^2+\gamma^2) \tag{38}$$

4. Present $\vec{e}_{z2}$ as an expansion similar to (5) but in unit vectors $(\vec{e}_{\rho r1}, \vec{e}_{\varphi r1}, \vec{e}_{z1})$. Multiply the result scalarly by $\vec{e}_{z1}$, which allows us to state that

$$\beta_{r1}^2+\gamma_{r1}^2\approx\beta^2+\gamma^2 \tag{39}$$

The relationships obtained are sufficient for the potential energy U_r' of the right radial bearing to be determined.

$$U_r'\approx U_r^0+\frac{1}{2}C_{\delta\delta}^r\cdot(\delta^2+2L\delta\beta+L^2(\beta^2+\gamma^2)-2\alpha^2)+$$

$$+\frac{1}{2}C_{\beta\beta}^r(\beta^2+\gamma^2) \tag{40}$$

For the left radial bearing we shall obtain the same relationship replacing L by ($-L$).

For the right radial bearing (the centers of rings are O_{a1}' and O_{a2}') the same relationships (36-39) for (α_{al}, β_{al}, γ_{al}, δ_{al}) can be used if is replaced L by $-(L+l)$.

In the formulae for calculating force and stiffness characteristics regular displacement λ of the centers of rings should be taken into account which makes nonzero as force $f_\alpha^a(\lambda)$ so nondiagonal stiffness $C_{\delta\beta}^a(\lambda)$ according to (16) and (21) (there l_0 corresponds to λ). For potential energy we shall obtain

$$U_a'\approx U_a^0+f_\alpha^a(\lambda)\cdot\left(\alpha+\frac{1}{2}(L+l)(\beta^2+\gamma^2)\right)+$$

$$+\frac{1}{2}C^{a}_{\delta\delta}(\delta^2-2(L+l)\delta\beta+(L+l)^2(\beta^2+\gamma^2)-2\alpha^2)+$$

$$+\frac{1}{2}C^{a}_{\beta\beta}(\beta^2+\gamma^2)+C^{a}_{\delta\beta}(\lambda)(\delta\beta-(L+l)(\beta^2+\gamma^2)) \quad (41)$$

For the left axial bearing $(L+l)$ and λ should be replaced by $-(L+l)$ and $(-\lambda)$ respectively. The latter results in changing the sign

$$f_\alpha(-\lambda)=-f_\alpha(\lambda),\ C_{\delta\beta}(-\lambda)=-C_{\delta\beta}(\lambda),$$

whereas $C_{\delta\delta}$ and $C_{\beta\beta}$ do not change.

Summary expression for potential energy is simple diagonal quadratic form

$$U=U^0+\frac{1}{2}C^{\Sigma}_{\delta\delta}(\delta^2-2\alpha^2)+\frac{1}{2}C^{\Sigma}_{\beta\beta}(\beta^2+\gamma^2), \quad (42)$$

where summary stiffnesses are given by

$$\frac{1}{2}C^{\Sigma}_{\delta\delta}=C^{r}_{\delta\delta}+C^{a}_{\delta\delta} \quad (43)$$

$$\frac{1}{2}C^{\Sigma}_{\beta\beta}=C^{r}_{\beta\beta}+C^{a}_{\beta\beta}-2C^{a}_{\delta\beta}(L+l)+C^{r}_{\delta\delta}L^2+$$

$$+C^{a}_{\delta\delta}(L+l)^2+f^{a}_{\alpha}(L+l) \quad (44)$$

Note that for summary stiffnesses the relationship obtained for simple rings $C_{\delta\delta}=-\frac{1}{2}C_{\alpha\alpha}$, remaines valid. It is not difficult to see what is the effect of principle premises: coaxiality of nominal magnetic systems and additivity of potential energy.

CONCLUSION

The result obtained make it possible to calculate mechanical characteristics - force and stiffness parameters - of interacting axially symmetric permanent magnetic fields under small rotor displacements.

Classical results on principle unstability of rotor suspension provided by permanent magnets in conformity to the field geometry considered have been confirmed. However, quantitative expressions obtained for the elements of the stiffness matrix make it possible to estimate additional stiffness to be provided for combined suspension. Such problem arises when, for instance, along with permanent magnets active electromagnetic bearings are used by some degrees-of-freedom.

The form of obtained stiffness matrix allows us to offer unusual rotor configuration in which achive ment of complete levitation in permanent magnetic field do not contradict the conclusion on unstability. Namely, two current coplanar circular rings with opposite directions of currents exert repulsive force and

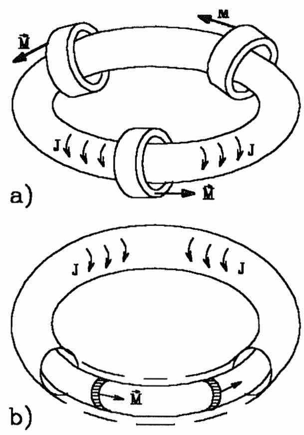

FIGURE 4: Toroidal rotors on permanent magnets suspension

have positive radial and angular stiffnesses. Moreover, axial stiffness is negative. In the structure of toroidal rotor shown in Fig. 4a this unstable axial degree-of-freedom corresponds to the rotation coordinate of a rotor and is neutral for stabilization requirements. From the viewpoint of field energy efficiency the inverse configuration (Fig. 4b) is more preferable.

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