BIAS LINEARIZATION AND DECOUPLING FOR ACTIVE MAGNETIC BEARINGS: A GENERALIZED THEORY

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ABSTRACT

The mathematical basis for bias linearization of active magnetic bearings is developed. The approach generalizes prior ad–hoc methods of linearizing the relationship between actuator force and electromagnet current. Growing from the properties of a fundamental representation for the current–force relationships, conditions are determined under which linearization may be possible. A numerical optimization problem is posed whose solution provides a linearization scheme which maximizes the available force capacity of the bearing. A corollary result is a method for obtaining coil–fault tolerance without adding coils to existing actuators. Several paper examples are presented to illustrate linearization of asymmetric bearings and those with failed coils.

INTRODUCTION

Conventional radial magnetic bearings employ a magnetic stator with radial legs surrounding a magnetic rotor. Electromagnet coils are wound on some or all of the stator legs and forces are exerted on the rotor by passing currents through these coils. With suitable coil currents, a radial force of a prescribed magnitude and orientation can be applied to the rotor. The desired force is determined in response to measured rotor motion in order to achieve stable rotor support with appropriate dynamic properties [5]. Thus, the bearing is a feedback device consisting of a rotor motion sensor, a magnetic force actuator, and a controller which regulates the coil currents in response to the sensed motion and other inputs.

Most commercial radial magnetic bearings have at least eight legs in the stator and at least four independent coils. (In many cases, collections of neighboring coils are wired in series. In this work, a collection of coils wound in series would be considered to be dependent.) This means that the number of independent coils in the stator substantially exceeds the number of force components which are to be generated – usually two.

As will be developed in the present work, the relationship between these currents and the resulting force components is fairly easily determined by analysis:

$$I_1, I_2, \ldots, I_n \Rightarrow F_1, F_2, \ldots, F_p \quad : \quad p < n$$

but the inverse relationship:

$$F_1, F_2, \ldots, F_p \Rightarrow I_1, I_2, \ldots, I_n$$

is not only difficult to find but is not unique. For the purposes of designing the feedback control, this latter relationship is crucial since the general dynamic problem relates the bearing forces to the rotor motion; ideally, the character of the magnetic device should not enter directly into the design of the controller.

The problem of determining a suitable set of control currents cannot be resolved simply by using pseudo–inverse methods because the relationship between coil currents and force components is quadratic. Instead, the coil currents are selected either by a nonlinear optimal rule or by a linear, suboptimal rule which permits a simpler interaction with the controller. While not all stator/coil configurations will permit a linear rule for selecting coil currents, many stator configurations can be linearized.

Linearization is accomplished by imposing a biasing current in each coil which produces a magnetic stress but no net force at an equilibrium position. Although the current to force relationship is quadratic, bias and control currents are chosen so that all second-order control current terms are identically zero at the equilibrium point. When the bias current magnitudes are held constant, the relationship between control current and bearing force is linear.

Bias linearization is widely used in a number of magnetic devices from radial magnetic bearings to six degree-of-freedom actuators (for example, [1, 2, 3, 8]). These applications employ symmetric geometries where the determination of a permissible set of linearizing currents proceeds by inspection. However, the previous work has not been extended to the general problem, particularly where coil failures produce substantial asymmetry in the stator.

The present work explores the manner in which n coil currents should be selected to provide the $x-$ and $y-$ force components in a radial magnetic actuator of arbitrary geometry. A more general representation for actuators producing m independent forces is developed in [9]. In stators which permit linearization, an optimal coil current map can be determined which relates the n coil currents to the desired force components in terms of a bias vector and two control vectors which determine the two force components. Finally, a method for determining an optimal set of currents is considered.

MODEL

Assuming negligible eddy current effects and a linear flux density to field intensity relationship with negligible hysteresis effects, a magnetostatic analysis can be employed. If losses from flux leakage and fringing are also assumed negligible, the applicable magnetostatic field equations become one dimensional. Flux density at any point in the bearing can then be computed using simple circuit theory [7].

An n pole magnetic bearing (as exemplified by Fig. 1) is characterized by Ni_j, B_j, A_j, g_j, and θ_j for $j = 1 \ldots n$, the impressed magnetomotive force, flux density, pole face area, air gap length, and centerline angle respectively for each pole. The reluctances of the permeable parts of the structure are neglected on the assumption that the relative permeability is well over 1000; virtually all of the circuit reluctance is due to the air gap associated with each pole. The sign convention adopted here assumes that positive fluxes are directed out of the stator poles into the rotor while positive coil currents pass counterclockwise around the stator poles when viewing the pole end from the gap. It is assumed that the only

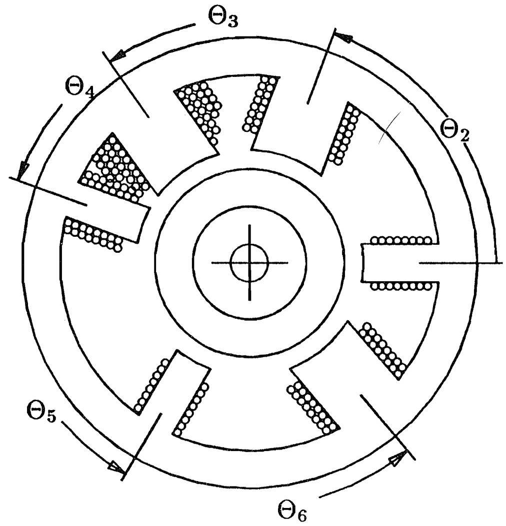

Figure 1: Generic bearing arrangement

sources of magnetic excitation are the coils, which specifically excludes bearings employing permanent magnets from this analysis.

Ampére's loop law for the magnetic circuit results in $n-1$ independent equations:

$$\frac{g_j}{\mu_o}B_j - \frac{g_{j+1}}{\mu_o}B_{j+1} = N_j i_j - N_{j+1} i_{j+1} \quad (1)$$

One additional independent equation results from conservation of flux:

$$\sum_{j=1}^{n} A_j B_j = 0 \quad (2)$$

Arranging these equations in matrix form produces

$$\frac{1}{\mu_o}\begin{bmatrix} g_1 & -g_2 & 0 & \cdots & 0 \\ 0 & g_2 & -g_3 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & g_{n-1} & -g_n \\ A_1 & A_2 & \cdots & A_{n-1} & A_n \end{bmatrix} \underline{B} = \begin{bmatrix} N_1 & -N_2 & 0 & \cdots & 0 \\ 0 & N_2 & -N_3 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & N_{n-1} & -N_n \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \underline{I} \quad (3)$$

This matrix relationship is represented more succinctly by

$$\mathbf{G}\underline{B} = \mathbf{N}\underline{I} \quad (4)$$

where $\mathbf{G}$ can easily be shown to be nonsingular which permits

$$\underline{B} = \mathbf{G}^{-1}\mathbf{N}\underline{I} \quad (5)$$

Note from (3) that the matrix **N** has a nullity of 1. Consequently, one of the currents in $\underline{I}$ is redundant if each leg has an independent coil. In this case, the redundancy is used to minimize the coil energy by defining

$$\underline{I} = \mathbf{K}\underline{\hat{I}} \qquad (6)$$

where the columns of **K** are selected to be orthogonal to the null vector of **N**. Further, (3) assumes that the coil currents on the n legs are all independent. In many cases, combinations of the coils are wired in series so that, for instance, $i_1 = -i_2$. This case can, again, be represented by (6) where the rows of **K** define the interdependencies. Finally, one or more of the n coils may have failed in which case the corresponding current is zero. This condition can also be represented by (6) where **K** is simply the identity matrix with columns corresponding to the nonfunctioning coils removed. It is significant that, in all cases, the dimension of $\underline{\hat{I}}$ is at most $n-1$.

Forces produced by the bearing can be most readily computed using Maxwell's stress tensor[6]. With some reasonable simplifications, this reduces to the form

$$\underline{F} = \frac{1}{2\mu_o}\int_A B^2 \underline{da} \qquad (7)$$

where the integral is taken over any closed surface which does not cut through magnetic material. For the geometry considered here, (7) becomes

$$F_x = \underline{B}'\mathbf{C}\underline{B} \quad : \quad \mathbf{C} \doteq \frac{1}{2\mu_o}\mathrm{diag}[A_j \cos\theta_j]$$

$$F_y = \underline{B}'\mathbf{S}\underline{B} \quad : \quad \mathbf{S} \doteq \frac{1}{2\mu_o}\mathrm{diag}[A_j \sin\theta_j]$$

Finally, by defining $\mathbf{V} \doteq \mathbf{G}^{-1}\mathbf{NK}$, these equations can all be combined to produce a very succinct statement of the force–to–current relationship:

$$F_x = \underline{\hat{I}}'\mathbf{X}\underline{\hat{I}} \quad : \quad \mathbf{X} \doteq \mathbf{V}'\mathbf{C}\mathbf{V} \qquad (8)$$

and

$$F_y = \underline{\hat{I}}'\mathbf{Y}\underline{\hat{I}} \quad : \quad \mathbf{Y} \doteq \mathbf{V}'\mathbf{S}\mathbf{V} \qquad (9)$$

LINEARIZATION

Equations (8) and (9) show that the relationship between the reduced order current vector $\underline{\hat{I}}$ and the forces produced is *quadratic*. In general, the bearing must be able to generate forces in an arbitrary direction. F_x and F_y must therefore be independent of one another, and each force should be able to be generated with an arbitrary sign. The latter condition is relatively easy to establish by simply examining the definiteness of the symmetric matrices **X** and **Y**. If any of these matrices is either semi-definite, the corresponding quadratic product will always be either non-negative or non-positive. An arbitrary force cannot then be realized.

If both **X** and **Y** are indefinite, then there may exist matrices **W** so that

$$\underline{\hat{I}} = \mathbf{W}\begin{Bmatrix} c_b \\ c_x \\ c_y \end{Bmatrix} = \mathbf{W}\underline{c} \qquad (10)$$

and

$$F_x = \underline{c}'\mathbf{W}'\mathbf{X}\mathbf{W}\underline{c} = c_b\, c_x \qquad (11)$$

$$F_y = \underline{c}'\mathbf{W}'\mathbf{Y}\mathbf{W}\underline{c} = c_b\, c_y \qquad (12)$$

In matrix form, the desired separation in (11) and (12) can be obtained if

$$\mathbf{W}'\mathbf{X}\mathbf{W} = 0.5\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{W}'\mathbf{Y}\mathbf{W} = 0.5\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad (13)$$

This form implies that the first column of **W** is the biasing current vector, the second column is the control current vector for the F_x, and the last column of **W** controls the F_y. Clearly, (13) represents twelve coupled quadratic equations in $3 \times m$ equations where m is the number of elements in $\underline{\hat{I}}$. Thus, $\underline{\hat{I}}$ must have at least four elements in it in order to permit solutions of (13): the stator must have at least five poles and at least four independent coils. Assuming that a matrix **W** can be found which satisfies (13) then the inverse current–to–force relationship which permits linear bearing force control is

$$\underline{I} = \frac{1}{c_b}\mathbf{KW}\begin{Bmatrix} c_b^2 \\ F_x \\ F_y \end{Bmatrix} \qquad (14)$$

Since the derivation of matrices **X** and **Y** imposes no restrictions on stator geometry, (13) is a general statement of the bias–linearization problem for radial magnetic bearings. Any matrix **W** satisfying (13) will permit independent linear control over the orthogonal force components produced by a given bearing via the currents specified by (14).

A closed–form solution of (13) has not been found. At present, linearizing matrices may only be obtained numerically. Generally, the approach is to pose the problem in vector form and solve the resulting nonlinear equation using a gradient descent algorithm. Details of the solution method can be found in [9].

CHOICE OF OPTIMAL W

For most magnetic bearings, the problem defined by (13) has many solutions. Therefore, a criterion must be established for selecting the ***best*** solution. One such measure is the maximum load which the bearing can generate before magnetic saturation occurs at some point on the stator or rotor.

To determine saturation in the stator, the fluxes in the legs, back-iron, and journal iron must all be computed. If the pole areas are equal to the air gap areas, then the pole flux densities are simply equal to the gap densities:

$$\underline{B}_p = \underline{B} \tag{15}$$

Most of the back iron flux densities can be found from the $n-1$ independent conservation of flux conditions:

$$A_{b,j}B_{b,j} - A_{p,j}B_{p,j} - A_{b,j+1}B_{b,j+1} = 0 \tag{16}$$

The one remaining equation required is most properly obtained by applying Ampére's loop law to the back iron:

$$\sum_{j=1}^{n} B_{b,j}L_j = 0 \tag{17}$$

However, as the circuit begins to saturate, the permeabilities of the back iron sections with higher flux density will begin to decrease. This will produce a redistribution of flux density which tends to minimize the peak flux density in the back iron, subject to conservation of flux.[1] On the basis of this heuristic argument, it may be best to solve these equations in such a manner as to minimize the peak flux density. The simplest approximation to this kind of solution is provided by the Moore–Penrose pseudoinverse. Summarize (16) as

$$\mathbf{A}_b\underline{B}_b = \mathbf{A}_p\underline{B}_p \tag{18}$$

Using the Moore–Penrose pseudoinverse results in

$$\underline{B}_b = \mathbf{A}_b^\dagger \mathbf{A}_p \underline{B}_p \;:\; \mathbf{A}_b^\dagger \doteq \mathbf{A}_b'(\mathbf{A}_b\mathbf{A}_b')^{-1} \tag{19}$$

The journal flux densities can be computed in a similar manner, leading to

$$\underline{B}_s = \begin{Bmatrix} \underline{B}_p \\ \underline{B}_b \\ \underline{B}_j \end{Bmatrix} = \begin{bmatrix} I \\ \mathbf{A}_b^\dagger\mathbf{A}_p \\ \mathbf{A}_j^\dagger\mathbf{A}_p \end{bmatrix} \underline{B} = \begin{bmatrix} I \\ \mathbf{A}_b^\dagger\mathbf{A}_p \\ \mathbf{A}_j^\dagger\mathbf{A}_p \end{bmatrix} \mathbf{V}\hat{\underline{I}} \tag{20}$$

[1] Of course, as the iron starts to saturate, flux leakage will also increase, reducing the validity of the simple conservation of flux conditions used here.

The transformation from the reduced order current vector to the distribution of flux densities throughout the stator can then be defined as:

$$\mathbf{V}_s \doteq \begin{bmatrix} I \\ \mathbf{A}_b^\dagger\mathbf{A}_p \\ \mathbf{A}_j^\dagger\mathbf{A}_p \end{bmatrix} \mathbf{V} \tag{21}$$

Rather than computing the saturation load directly, compute the flux density distribution for a force of magnitude 1.0 and arbitrary orientation Θ:

$$F_x = \cos\Theta \;,\quad F_y = \sin\Theta \tag{22}$$

If the parameters c_x, and c_y are chosen according to

$$c_x = \frac{\cos\Theta}{c_b} \;,\quad c_y = \frac{\sin\Theta}{c_b} \tag{23}$$

then the desired force of magnitude 1.0 and direction Θ will result. The flux distribution throughout the stator resulting from any selection of c_b and Θ is given by

$$\underline{B}_s(c_b,\Theta,\mathbf{W}) = \mathbf{V}_s\hat{\underline{I}} = \frac{1}{c_b}\mathbf{V}_s\mathbf{W}\begin{Bmatrix} c_b^2 \\ \cos\Theta \\ \sin\Theta \end{Bmatrix} \tag{24}$$

The maximum magnitude of the resulting flux density distribution is

$$B_{\max}(c_b,\Theta,\mathbf{W}) = |\underline{B}_s(c_b,\Theta,\mathbf{W})|_\infty \tag{25}$$

The achievable load capacity is then

$$F_{\max}(c_b,\Theta,\mathbf{W}) = \left(\frac{B_{\text{sat}}}{B_{\max}(c_b,\Theta,\mathbf{W})}\right)^2 \tag{26}$$

where B_{sat} is the saturation flux density of the magnet iron.

The achievable load capacity is dependent upon the choice of c_b and Θ. Typically, it is conservative to base the load capacity upon the worst case orientation:

$$B_{\max}(c_b,\mathbf{W}) = \max_{\Theta}|\underline{B}_s(c_b,\Theta,\mathbf{W})|_\infty \tag{27}$$

This choice might be modified for systems where a gravity load or some other load with fixed orientation is significant. Further, the choice of c_b is essentially free; it should be chosen in such a manner as to minimize the peak flux density (and thereby maximize the load capacity):

$$B_{\max}(\mathbf{W}) = \min_{c_b}\max_{\Theta}|\underline{B}_s(c_b,\Theta,\mathbf{W})|_\infty \tag{28}$$

In this manner, the best solution $\mathbf{W}^*$ is that which minimizes $B_{\max}$ (or maximizes $F_{\max}$):

$$B_{\max} = \min_{\mathbf{W}\to\mathbf{W}^*}\;\min_{c_b}\max_{\Theta}|\mathbf{B}_s(c_b,\Theta,\mathbf{W})|_\infty \tag{29}$$

The minimax problem defined by (29) along with the constraint equation (13) forms a nonlinear optimization problem for selecting **W**. At present, the only computational approach is to find many examples which satisfy (13) using the method mentioned previously from random seeds and then choose the best solution on the basis of (29). While this procedure yields usable solutions, there is no guarantee that these solutions are optimal or even represent local optima. However, it is unlikely that a single gradient descent optimization will yield a global optimum because the solutions for **W** are not necessarily connected.

EXAMPLES

Two examples will be explored in order to illustrate the concepts developed above. The first example considers a highly asymmetric stator while the second considers coil failure in an otherwise symmetric stator.

Stator Asymmetry

In the past, the problem of determining bias and control currents was only considered for symmetric cases. Under these conditions, the proper linearizing currents are obtained by inspection. However, when symmetry is lost, the determination of the proper currents is no longer a trivial problem. Take for example the six pole bearing pictured in Figure 1. The geometry of this bearing is described in Table 1, where $A = 1\,cm^2$, $g_o = 1\,mm$ and $N = 200$. Substituting the geometric parameters into (3) gives:

$$\frac{g_o}{\mu_o}\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 1 & 2 & 2 & 1 & 1 & 2 \end{bmatrix} \underline{B} = N \begin{bmatrix} 1 & -2 & 0 & 0 & 0 & 0 \\ 0 & 2 & -3 & 0 & 0 & 0 \\ 0 & 0 & 3 & -2 & 0 & 0 \\ 0 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \underline{I}$$

By the geometry of the stator, the matrices **C** and **S** are

$$\mathbf{C} = \frac{A}{2\mu_o}\text{diag}\,[1.00, 0.684, -1.147, -0.940, -0.50, 1.286]$$

$$\mathbf{S} = \frac{A}{2\mu_o}\text{diag}\,[0.00, 1.879, 1.638, 0.342, -0.866, -1.532]$$

Because this stator has an independent coil on each leg, one coil will be redundant: matrix **N** is singular. This singularity can be removed with a suitable

Table 1: Asymmetric bearing properties

Leg	θ	Area	Turns	Gap
1	0^o	A	N	g_o
2	70^o	2 A	2 N	g_o
3	125^o	2 A	3 N	g_o
4	160^o	A	2 N	g_o
5	240^o	A	N	g_o
6	310^o	2 A	2 N	g_o

K matrix. The **K** matrix should have columns orthogonal to the null space of **N** so that the power dissipation required to achieve a given set of flux densities is minimized. One such matrix is

$$\mathbf{K} = \begin{bmatrix} 0.447 & 0.256 & 0.338 & 0.488 & 0.1829 \\ -0.894 & 0.1278 & 0.1688 & 0.244 & 0.0915 \\ 0.0 & -0.958 & 0.1125 & 0.1625 & 0.0610 \\ 0.0 & 0.0 & -0.919 & 0.244 & 0.0915 \\ 0.0 & 0.0 & 0.0 & -0.786 & 0.1829 \\ 0.0 & 0.0 & 0.0 & 0.0 & -0.955 \end{bmatrix}$$

The force-current relationships are specified by (8) and (9) as:

$$\mathbf{X} = \begin{bmatrix} 4.76 & 1.865 & 0.983 & -0.395 & -3.11 \\ 1.865 & -12.88 & 5.99 & 2.49 & 0.862 \\ 0.983 & 5.99 & -7.67 & 1.203 & 2.13 \\ -0.395 & 2.49 & 1.203 & -1.196 & 1.731 \\ -3.11 & 0.862 & 2.13 & 1.73 & 8.08 \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} 9.68 & -9.83 & -2.48 & -0.1747 & -0.533 \\ -9.83 & 23.7 & -2.95 & 0.426 & 0.464 \\ -2.48 & -2.95 & 3.96 & 0.444 & 0.817 \\ -0.1747 & 0.426 & 0.444 & -1.891 & 0.673 \\ -0.533 & 0.464 & 0.817 & 0.673 & -8.58 \end{bmatrix}$$

This bearing can be linearized if a 5×3 matrix **W** can be found which satisfies (13). Such a matrix can be found through a numerical search. For example,

$$\mathbf{W} = \begin{bmatrix} 0.1363 & -0.314 & 0.611 \\ 0.205 & -0.0433 & 0.233 \\ 0.462 & -0.409 & 0.674 \\ 0.697 & 0.261 & -0.475 \\ -0.0278 & 0.227 & -0.1587 \end{bmatrix}$$

is one linearizing solution satisfying (13). The physical coil currents are then specified by (14) as

$$\begin{aligned} I_1 &= 0.604c_b - 0.1210F_x/c_b + 0.300F_y/c_b \\ I_2 &= 0.150c_b + 0.291F_x/c_b - 0.533F_y/c_b \\ I_3 &= -0.0329c_b + 0.0517F_x/c_b - 0.235F_y/c_b \\ I_4 &= -0.257c_b + 0.461F_x/c_b - 0.750F_y/c_b \\ I_5 &= -0.553c_b - 0.1636F_x/c_b + 0.344F_y/c_b \\ I_6 &= 0.0266c_b - 0.217F_x/c_b + 0.152F_y/c_b \end{aligned}$$

The best value of c_b depends upon the physical parameters of the stator including A, N, and the saturation flux density.

Coil Failure

Aside from linearizing unusual stator geometries, the procedures are more practically useful in developing fault-tolerant controllers for bearings with a large number of coils. Consider a conventional 8-pole symmetric bearing with each pole face having an area of $4.91\,10^{-4}\,m^2$, a nominal gap of $0.001\,m$, and 200 turn coils. In this example, the flux path areas in the back iron and in the journal are the same as the pole face area.

In the normal operating mode, coils on all 8 legs would be operational. This configuration allows for the linearizing current set

$$\mathbf{W} = \frac{g_o}{4N\sqrt{\mu_o A}}\begin{bmatrix} 2 & 2 & 0 \\ -2 & -\sqrt{2} & -\sqrt{2} \\ 2 & 0 & 2 \\ -2 & \sqrt{2} & -\sqrt{2} \\ 2 & -2 & 0 \\ -2 & \sqrt{2} & \sqrt{2} \\ 2 & 0 & -2 \\ -2 & -\sqrt{2} & \sqrt{2} \end{bmatrix}$$

which yields a load capacity of 562 N at a saturation density of 1.2 Tesla.

This stator can be linearized (solutions to (13) can be found) for coil failure configurations involving any one, two, or three coils. In addition, the stator can be linearized with four coils failed as long as the four coils are not all adjacent. As an example, consider the case where coils 1, 2, and 3 have failed. In this case,

$$\mathbf{W} = \frac{g_o}{4N\sqrt{\mu_o A}}\begin{bmatrix} 0. & 0. & 0. \\ 0. & 0. & 0. \\ 0. & 0. & 0. \\ 5.09 & 0.359 & 3.32 \\ 0. & 3.68 & 3.68 \\ 0. & 3.68 & 3.68 \\ 0. & 3.68 & 3.68 \\ 5.09 & 3.32 & 0.359 \end{bmatrix}$$

solves (13), but the load capacity is reduced to 256 N which is forty six percent of the capacity when all coils are functional.

CONCLUSIONS

An analysis has been described for linearizing and decoupling the force axes in complicated magnetic actuators. Using this analysis, a simple linear relationship can be found which relates the desired force components and an additional fixed biasing term to the best set of coil currents. Since the analysis is not limited to a specific geometry or number of actuator force components, it can be applied to asymmetric stators, stators with failed coils, and stators which generate more than the usual two orthogonal force components.

In addition, a clear mechanism has been demonstrated for achieving fault tolerance to coil failures. If one or more coils fail, a new coil current control scheme can usually be constructed which preserves the linear relationship between required forces and coil currents. This fault tolerance comes at some expense in load capacity because the necessary redistribution of magnetic flux in the stator in order to achieve high forces along vectors passing through the poles of the failed coils leads to premature saturation in the stator or journal.

REFERENCES

[1] A. P. Allan and C. Knospe, "A Six-Degree of Freedom Magnetic Bearing for Microgravity Isolation," International Symposium on Magnetic Suspension Technology, Hampton, VA, August 19-23, 1991.

[2] K. R. Bornstein, "Dynamic Load Capabilities of Active Electromagnetic Bearings," *Journal of Tribology*, 113:598-603, July 1991.

[3] C. R. Burrows *et. al.*, "Design and Application of a Magnetic Bearing for Vibration Control and Stabilization of a Flexible Rotor," Proceedings of the First International Symposium on Magnetic Bearings, Zurich, June 6-8, 1988.

[4] A. Chiba and M. Rahman, "Radial Force in a Bearingless Reluctance Motor," *IEEE Transactions on Magnetics*, MAG-27:786-790, March 1991.

[5] R. R. Humphris, R. D. Kelm, D. W. Lewis, and P. E. Allaire, "Effect of Control Algorithms on Magnetic Journal Bearing Properties," *Journal of Engineering for Gas Turbines and Power*, vol. 108, October 1986.

[6] O. D. Jefimenko, "Correct Use of Maxwell Stress Equations for Electric and Magnetic Fields," *American Journal of Physics*, 51(11):988-996, November 1983.

[7] A. F. Kip, *Fundamentals of Electricity and Magnetism*, New York: McGraw-Hill, 1962.

[8] C. Lee and J. Kim, "Modal Testing and Suboptimal Vibration Control of Flexible Rotor Bearing System by Using a Magnetic Bearing," *Journal of Dynamic Systems, Measurement, and Control*, 114:244-252, June 1992.

[9] E. H. Maslen and D. C. Meeker, "Bias Linearization of Magnetic Bearings," submitted for review to the IEEE *Transactions on Magnetics*.