COMPARISON OF CONTROLLER DESIGNS FOR ATTENUATION OF VIBRATION IN A ROTOR-BEARING SYSTEM UNDER SYNCHRONOUS AND TRANSIENT CONDITIONS

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ABSTRACT

In this paper two controllers are compared with regard to their performance in a computer simulation of loss of mass from a rotor. Both controllers are considered to use a magnetic actuator to apply the control forces and operate only on the displacement of the rotor at certain measurement locations. The first controller is designed using H_∞ optimisation. The second, called a synchronous feedback controller, is a closed-loop version of an open-loop controller used for the attenuation of synchronous vibration. The synchronous feedback controller can control vibration under both steady state and transient conditions. The performance of the synchronous feedback controller is slightly inferior to the H_∞ controller. However, this is to be offset against the fact that the synchronous feedback controller would be much simpler to implement than the H_∞ controller in practical applications.

NOTATION

B_f, B_u	force distribution matrices
$C(\Omega)$	system damping matrix
e	control state vector
f	disturbance force vector
$G(s)$	transfer function matrix
$H(s)$	controller transfer function matrix
$K(\Omega)$	system stiffness matrix
M	system mass matrix
q	generalised coordinate
r	rotor displacement
s	Laplace transform variable
t	time
T	sampling period
$T_{ef}(s)$	closed-loop transfer function matrix
u	control force vector
$W(s)$	weighting function matrix
y	measurement state vector
β	feedback parameter
$\Delta(\Omega)$	synchronous system matrix
ε	tolerable rotor deflection
ϕ	controller performance measure
ω	frequency
Ω	running speed frequency
A	general synchronous amplitude of a
$\hat{A}$	general Laplace transform of a

Subscripts and superscripts are defined in the text.

1 INTRODUCTION

The problem of attenuation of vibration in rotor-bearing systems is well established. The majority of work on the problem deals with steady synchronous vibration caused by unbalance of the rotor. Another aspect of the problem is the control of transient vibration that occurs after a component failure, such as blade loss in a turbine. The attenuation of such transient vibration is important because it allows the rotor-bearing system to be shut down without further damage.

There has been much interest in the attenuation of rotor-bearing vibration using forces generated by magnetic actuators. Both open and closed-loop strategies have been used for the control of synchronous vibration. In such a case the advantage of an open-loop strategy is that it can optimise performance without incurring instability problems. However, for transient vibration attenuation, a closed-loop controller must be used. Relatively little work

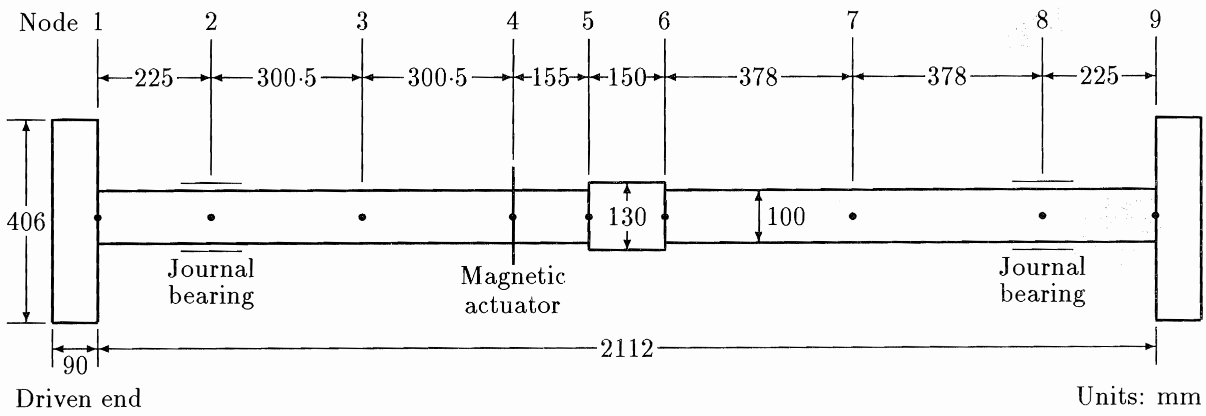

FIGURE 1 Rotor dimensions and discretisation for the finite-element model.

has been done on the attenuation of such vibration. Palazzolo *et al.* [1] have used piezoelectric actuators with feedback of position and velocity. Viggiano and Schweitzer [2] investigated the use of both decentralised and centralised controllers in the attenuation of transient vibration of a rigid rotor.

In this paper two controllers are designed for the attenuation of vibration in a rotor-bearing system under a mass loss situation. The system consists of a flexible rotor supported by two oil-lubricated journal bearings. The first controller is designed using H_∞ optimisation. The second is called a synchronous feedback controller as it is a closed-loop version of an open-loop synchronous vibration algorithm [3, 4]. The advantage of the synchronous feedback controller is that it can attenuate vibration under both steady and transient conditions. The controller was first introduced by Berry *et al.* [5]. Here it is shown that, with a relatively small increase in the complexity of the control law used in [5], enhanced performance can be achieved.

2 MODEL OF ROTOR-BEARING SYSTEM

The rotor-bearing system considered here is an experimental rig which has been described in detail in [4, 6]. The rotor is a steel shaft with end mounted rigid discs. The dimensions of the rotor are shown in figure 1. The static weight of the rotor is supported by two oil-lubricated journal bearings. A magnetic actuator can apply control forces in both horizontal (x) and vertical (y) directions. The vibration of the rotor is considered measured with horizontal and vertical position transducers at both of the discs and at the magnetic actuator location (nodes 1, 4 and 9).

The shaft of the rotor is modelled using finite-elements based on Timoshenko beam theory [7]. The journal bearings are modelled using linear short bearing theory [8] and exhibit asymmetric dynamic characteristics. The laminations of the magnetic actuator that are attached to the shaft are modelled as a 15 kg annulus with an outer diameter of 180 mm. The magnetic actuator incorporates a current amplifier and, hence, its dynamic properties can be ignored. When these models are combined an equation of motion can be derived in the form

$$M\ddot{q} + C(\Omega)\dot{q} + K(\Omega)q = B_f f + B_u u \qquad (1)$$

where q is the vector of generalised coordinates, f is the vector of disturbance forces, u is the vector of control forces from the magnetic actuator, and Ω is the running speed. Eight finite-elements (figure 1) are used in the model of the shaft in order to predict the first and second flexure modes with errors less than 6%. The vector $q = [{q_1}^T, {q_2}^T, \ldots, {q_9}^T]^T$, where each q_i $(i = 1, 2, \ldots, 9)$ is the vector of generalised coordinates at the ith node. Each q_i is defined by $q_i = [q_{1_i}, q_{2_i}, q_{3_i}, q_{4_i}]^T$ where q_{1_i} and q_{2_i} are the displacements along the x and y axes, and q_{3_i} and q_{4_i} are the rotations about the x and y axes respectively.

There are four imaginary part eigenvalue intersections with the running speed in the range 0–400 rad s^{-1} at 229, 258, 275 and 317 rad s^{-1}. The first two modes at 229 and 258 rad s^{-1} are highly damped, as can be seen from figure 2. The lightly damped modes at 275 and 317 rad s^{-1} correspond to the first flexural mode of the rotor where splitting has been caused by the asymmetric characteristics of the journal bearings.

3 TRANSIENT PERFORMANCE

In order to specify the performance that is required in the loss of mass situation, a measure of the vibration of the rotor is needed. At each node, in response

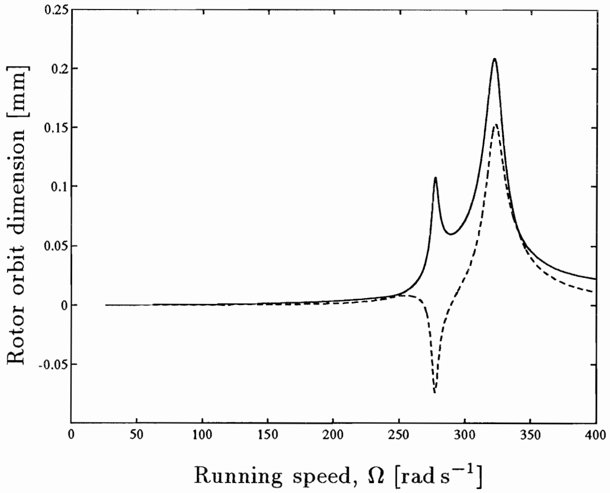

FIGURE 2 Major (solid) and minor (dashed) axes of the synchronous elliptical orbit at the magnetic actuator position in response to an unbalance of 0·01 kg on the edge of the non-driven disc.

to a disturbance f occurring at $t = 0$, the displacement of the centre of rotation of the rotor from its equilibrium position is given by

$$r_i(t, f) = \sqrt{q_{1_i}^2(t, f) + q_{2_i}^2(t, f)}, \quad t \geq 0 \qquad (2)$$

A natural measure of the vibration of the rotor at each node in response to a disturbance f is

$$\| r_i(f) \|_\infty = \sup\{|r_i(t, f)| : t \geq 0\} \qquad (3)$$

This is the maximum displacement at the ith node of the centre of rotor rotation. The disturbance f is the unbalance force caused by a loss of mass. A particular example of the disturbance is considered to be f^* caused by a loss of mass of 0·01 kg from the edge of the non-driven disc at time $t = 0$. The required performance can be defined by the set of inequalities

$$\{\| r_i(f^*) \|_\infty \leq \varepsilon_i, \quad i = 1, 2, \ldots, 9\} \qquad (4)$$

where each margin ε_i is the largest tolerable value of rotor deflection. Here the margins are all chosen as $\varepsilon_i = 0{\cdot}1$ mm $(i = 1, 2, \ldots, 9)$ as this corresponds to 10% of the air gap in the magnetic actuator. The performance of the two controllers designed in this paper can be compared on the basis of the measure of performance

$$\phi = \max\{\| r_i(f^*) \|_\infty : i = 1, 2, \ldots, 9\} \qquad (5)$$

i.e. the maximum vibration over all the nodes.

Here, for simplicity, it is required only to satisfy (4) at a single critical speed $\Omega = 322$ rad s^{-1}, which corresponds to the largest synchronous vibration frequency in the uncontrolled case (see figure 2). In general (4) could be extended to consider a range of running speeds Ω, and also a set of disturbances f instead of the particular one f^*.

The required performance (4) is consistent with the principle of matching [9, 10, 11]. The main objective of the principle of matching is to ensure that the set P of possible inputs is a subset of the set T of tolerable inputs (i.e. $P \subseteq T$). In this case, the tolerable set T contains the disturbances f that satisfy the set of inequalities (4), and the possible set P contains only a single disturbance f^*.

4 H_∞ CONTROLLER

One method of attempting to satisfy the set of inequalities (4) would be to use a closed-loop controller H and solve the minimisation problem of finding ϕ^* such that

$$\phi^* = \min_H \phi \qquad (6)$$

A minimisation problem is needed that can be solved using the H_∞ optimisation method to yield a value of ϕ close to ϕ^*. The definition of the H_∞ norm of the transfer matrix $G(s)$ is

$$\| G \|_\infty = \sup\{\bar{\sigma}(G(j\omega)) : \omega \in \mathbb{R}\} \qquad (7)$$

where $\bar{\sigma}(G(j\omega))$ is the maximum singular value of the matrix $G(j\omega)$. Using equation (1), it is possible to derive the open-loop relation in the Laplace transform domain as

$$\left[\begin{array}{c} \widehat{E} \\ \hline \widehat{Y} \end{array}\right] = \left[\begin{array}{c|c} G_{11}(s) & G_{12}(s) \\ \hline G_{21}(s) & G_{22}(s) \end{array}\right] \left[\begin{array}{c} \widehat{F} \\ \hline \widehat{U} \end{array}\right] \qquad (8)$$

where $\widehat{E}$ is a vector of control states and $\widehat{Y}$ is a vector of measurement states. The partitioned transfer matrix $G(s)$ is also speed dependent. The controller is derived from the measurement states as

$$\widehat{U} = H\widehat{Y} \qquad (9)$$

and enables the closed-loop system to be expressed in the form

$$\widehat{E} = T_{ef}\widehat{F} \qquad (10)$$

In the present paper, the minimisation problem considered is

$$\min_H \| T_{ef} W \|_\infty \qquad (11)$$

where W is a diagonal transfer matrix weighting. The control states $\widehat{E}$ are chosen to contain the x and y velocities, $s\widehat{Q}_{1_i}$ and $s\widehat{Q}_{2_i}$, respectively, at all the nine nodes (figure 1). The velocities were chosen in preference to the displacements in the controller design since they are more sensitive to changes in unbalance.

The weighting function matrix $W(s)$ (see figure 3) was chosen to approximate unbalance forces with very slowly decaying sinusoids (the decay is needed to ensure that the weight W is stable). The min-

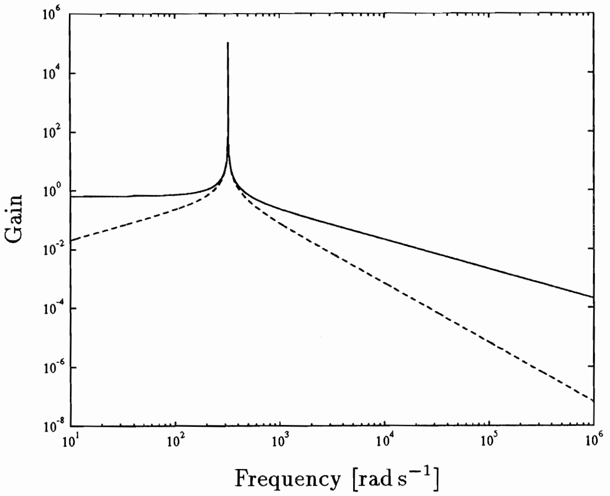

FIGURE 3 Maximum and minimum singular values of the weight W.

imisation problem was solved using MATLAB software [12]. Figure 4 shows the uncontrolled and controlled rotor response $r_4(f^*)$ at the actuator position. The value of $\phi = 0{\cdot}06$ mm implies that the required performance, as defined by (4), has been achieved. The corresponding controller is a constant coefficient state-space model of order 84. The high order together with the short sampling period needed for implementation in digital from would necessitate the use of powerful and expensive digital signal processors. In practice, some form of model reduction in the design would reduce the controller order, but the present example is suitable for simulation and comparison purposes.

5 SYNCHRONOUS FEEDBACK CONTROLLER

The synchronous feedback controller is a closed-loop version of a synchronous open-loop controller [3, 4] and was introduced by Berry *et al.* [5]. An enhanced design is considered in this paper. In order to reduce steady synchronous vibration at a running speed Ω the open-loop control force is of the form

$$u(t) = \mathrm{Re}\left\{U e^{j\Omega t}\right\} \tag{12}$$

where the complex amplitude U is to be selected. The steady state response of q to a control force

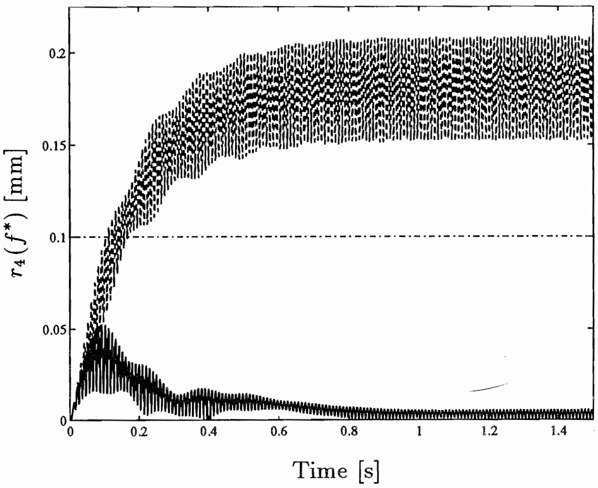

FIGURE 4 Uncontrolled (dashed) and H_∞ controlled (solid) responses $r_4(f^*)$, where f^* is the unbalance force due to a loss of mass of 0·01 kg from the edge of the non-driven disc. The margin ε_4 is shown as a dash-dotted line.

(12) is

$$q_u(t) = \mathrm{Re}\left\{Q_u e^{j\Omega t}\right\} \tag{13}$$

where

$$Q_u = \left(-\Omega^2 M + j\Omega C + K\right)^{-1} B_u U \tag{14}$$

Defining Q_u^m to contain the elements of Q_u corresponding to the measurement locations, implies that

$$Q_u^m = \Delta(\Omega) U \tag{15}$$

where $\Delta(\Omega)$ contains the appropriate rows of $(-\Omega^2 M + j\Omega C + K)^{-1} B_u$.

The steady state response at the defined measurement locations to a control force (12) and an unbalance force f is

$$q^m(t) = \mathrm{Re}\left\{(\Delta(\Omega)U + Q_f^m) e^{j\Omega t}\right\} \tag{16}$$

where Q_f^m is the synchronous amplitude due to the unbalance.

The open-loop control force is obtained by solving

$$\min_U \left\| \Delta(\Omega) U + Q_f^m \right\|_2 \tag{17}$$

where $\|\cdot\|_2$ is the Euclidean norm. This choice minimises the amplitude of q^m in a least squares sense. The solution of (17) is given by

$$U^* = -\Delta^\dagger Q_f^m \tag{18}$$

where $\Delta^\dagger = (\Delta^T \Delta)^{-1} \Delta^T$ is the pseudo-inverse of Δ.

In [5] the controller defined by (12) and (18) was converted into a feedback controller that operates on a synchronous cycle-by-cycle basis. The control force generated is of the form

$$u(t) = \mathrm{Re}\left\{U_k e^{j\Omega t}\right\}, \qquad t \in [(k-1)T, kT) \quad k = 1, 2, \ldots \qquad (19)$$

where $T = 2\pi/\Omega$. The amplitude U_k is defined by

$$U_{k+1} = U_k - \beta\Delta^{\dagger} Q_k \qquad (20)$$

where Q_k is the synchronous amplitude for the kth period, which can be evaluated digitally using fast Fourier transform techniques, and β is a real number.

The recursion relation (20) can be re-written in the form

$$U_{k+1} = -\beta\Delta^{\dagger} \sum_{i=1}^{k} Q_i \qquad (21)$$

The summation in (21) allows the controller defined by (19) and (20) to be interpreted as a form of integral control acting on the synchronous amplitude Q_k. In an attempt to imitate a generalised form of proportional+integral+derivative control the recursion (20) is extended to

$$\begin{aligned} U_{k+1} &= U_k - \beta_{\mathrm{P}}\Delta^{\dagger}(Q_k - Q_{k-1}) - \beta_{\mathrm{I}}\Delta^{\dagger} Q_k \\ &\quad -\beta_{\mathrm{D}}\Delta^{\dagger}(Q_k - 2Q_{k-1} + Q_{k-2}) \end{aligned} \qquad (22)$$

or, equivalently,

$$\begin{aligned} U_{k+1} &= -\beta_{\mathrm{P}}\Delta^{\dagger} Q_k - \beta_{\mathrm{I}}\Delta^{\dagger} \sum_{i=1}^{k} Q_i \\ &\quad -\beta_{\mathrm{D}}\Delta^{\dagger}(Q_k - Q_{k-1}) \end{aligned} \qquad (23)$$

where β_{P}, β_{I} and β_{D} are parameters to be selected.

The form of the required performance (4) is in accordance with the method of inequalities [13, 14, pp. 341–345] which requires that a design problem be stated as a set of inequalities in the form

$$\{\phi_i(p) \le \varepsilon_i, \quad i = 1, 2, \ldots, n\} \qquad (24)$$

where p is a parameter to be selected from a given set P, the ϕ_i are real functions of p, and ε_i is the largest tolerable or permissible value of $\phi_i(p)$. Any parameter p that satisfies (24) is a solution to the problem. Another aspect of the method of inequalities is the recognition that numerical search methods may, in general, be used to find solutions to a set of inequalities (24). One such numerical search method is the moving boundaries process [13, 14, pp. 344–345].

An implementation of the moving boundaries process in MATLAB was used to find solutions to the set of inequalities (4). Initially the synchronous feedback controller was defined by (19) and (22) with the parameters β_{P}, β_{I} and β_{D} real numbers, implying that the parameter $p = (\beta_{\mathrm{P}}, \beta_{\mathrm{I}}, \beta_{\mathrm{D}})^T \in \mathbb{R}^3$. An extensive search was unable to find a solution to (4). In order to allow for the asymmetric dynamic characteristics of the journal bearings, it was decided to provide independent proportional, integral and derivative parameters for the horizontal and vertical directions by making β_{P}, β_{I} and β_{D} diagonal matrices ($\beta_{\mathrm{P}} = \mathrm{diag}(\beta_{\mathrm{P}_1}, \beta_{\mathrm{P}_2})$, $\beta_{\mathrm{I}} = \mathrm{diag}(\beta_{\mathrm{I}_1}, \beta_{\mathrm{I}_2})$, $\beta_{\mathrm{D}} = \mathrm{diag}(\beta_{\mathrm{D}_1}, \beta_{\mathrm{D}_2})$). Now the parameter $p = (\beta_{\mathrm{P}_1}, \beta_{\mathrm{P}_2}, \beta_{\mathrm{I}_1}, \beta_{\mathrm{I}_2}, \beta_{\mathrm{D}_1}, \beta_{\mathrm{D}_2})^T \in \mathbb{R}^6$. In only a few iterations a solution $p = (2{\cdot}55 \times 10^{-3}, 0{\cdot}305, -0{\cdot}401, -3{\cdot}02 \times 10^{-2}, 0{\cdot}38, 0{\cdot}359)^T$ was found. The corresponding value of $\phi = 0{\cdot}08$ mm. The uncontrolled and controlled responses $r_4(f^*)$ are shown in figure 5.

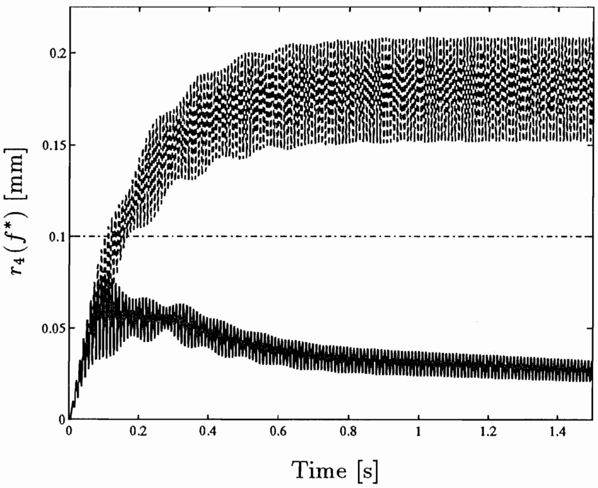

FIGURE 5 Uncontrolled (dashed) and synchronous feedback controlled (solid) responses $r_4(f^*)$, where f^* is the unbalance force due to a loss of mass of 0·01 kg from the edge of the non-driven disc. The margin ε_4 is shown as a dash-dotted line.

6 CONCLUSIONS

This paper has compared the performance of two controllers in a simulated loss of mass from a rotor-bearing system. One controller was based on H_∞ optimisation and the other was directed towards the control of synchronous vibration components in a generalised form of proportional+integral+derivative feedback. Both controllers were capable of attenuating steady and transient rotor vibration. The performance of the synchronous feedback controller was slightly inferior to the H_∞ controller although both were within tolerable bounds set on the rotor vibration amplitudes. The results demonstrated the potential of either controller for use with flexible rotor systems. The disadvantage of the H_∞ controller is the high order and for practical implementations this may be a problem. However, it is recognised that model reduction techniques could be used to overcome this. The synchronous feedback controller utilised the synchronous components in the vibration and was always of low order.

ACKNOWLEDGEMENT

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