NON LINEAR CONTROL OF ACTIVE MAGNETIC BEARINGS

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ABSTRACT

That paper presents a non linear control scheme for active magnetic bearings involved in magnetic suspension of high speed rotor. It is based on exact linearization techniques. After introducing the non linear model of the plant, a lower dimension model is used to present the control law design.
It utilizes a fictitious model of both electromagnets involved in the production of electromagnetic force along one direction. Although this model contains non analytical relations, it is possible to derive a static state feedback that linearizes this system. Extension to the global model doesn't introduce more difficulties since the sum of relative degrees equals the dimension of the model. Simulation results show the good behaviour of the controlled system, even in case of commutation between electromagnets.

I - INTRODUCTION

This paper presents a non linear control scheme of active magnetic bearings involved in the magnetic suspension of an horizontal rotating shaft. The process under consideration presented in section II is composed of an horizontal rotating shaft, suspended by means of two active magnetic bearings located in parallel planes orthogonal with rotation axis. The problem is to maintain the rotation axis in a specified position despite of various disturbances (working efforts or unbalance) by acting on electromagnets feeding. Among the six degrees of freedom of a body in three dimensional space, four of them are controlled by means of electromagnetic forces. The two others are rotation and translation along the main axis of the rotor. The former is controlled by angular velocity regulation, while the latter is passively controlled by annular permanent magnets located on the rotor and on the stator in such a way that the equilibrium position is stable when considering displacement along the main axis. Of course, this device introduces destabilizing effects in radial directions, which must be compensated by control of active magnetic bearings.
The problem is to build a non linear multivariable control law, without assuming linearization of the plant model around a working point. Furthermore, in order to reduce thermal losses, there is no polarization current, which roughly means that only one electromagnet is working at one time in each direction. The only assumption which is made, in order to simplify the presentation, is that both coils involved in the production of the force along one direction have identical characteristics (resistances and inductances).
Describing physical phenomena by laws issued from mechanical engineering, or electromagnetics, it is possible to derive an initial model with eight inputs, four outputs and sixteen state variables. One drawback of such a model is the number of control inputs which is twice the number of outputs. In fact, there is some redundancy since two input variables are involved in the production of the force variable in one direction (one for positive values, the other for negative values). In order to suppress this redundancy, and in the same time to reduce the model complexity, an equivalent actuator is designed which replace the two coils located on the same axis : section III. This actuator is characterized by a resistance and an inductance equal to those of each coil, and the convention on current variable is such that the produced

force has the same sign as the current. Every time this current variable cross the zero value, there is exchange of the roles played by coils involved in the plant (one is active while the other is neutral). It is then possible to reduce the number of inputs to four and the number of states to twelve. This new model is non linear, due to the relation linking current and produced force, which also depends on air gap, that is the position of the rotating shaft. It is shown that the non linear relations are non analytical, but sufficiently derivable to determine a state feedback and the diffeomorphisms that linearizes the closed loop with decoupling. It is then easy to build a second loop which fix the dynamics of the whole system. This last step is made easy since the linearizing and decoupling state feedback doesn't introduce unobservable modes.
Tests have been carried out on the model of an experimental process of the laboratory : section IV. Simulation concerns the use of a computer to implement the non linear control scheme. The discrete version is obtained by blocking the value of the continuous control during the sampling interval. Various tests have been carried out in order to determine the maximum sampling period without altering too much the dynamical behaviour. These tests were proceeded in the case of rising the shaft from the lower position, and then in case of constant disturbances acting at the end of the shaft.

II - MODELLING

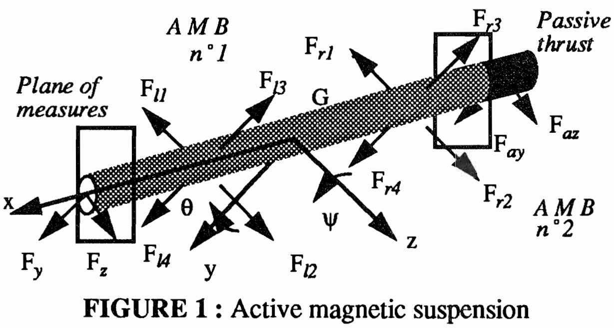

FIGURE 1 : Active magnetic suspension

Let us denote (figure 1) :
(y_0, z_0) : the displacements of G (translations along y and z axes respectively),
(θ, ψ) : the rotations of the body around y and z axes respectively,
F_{li} and F_{ri} : forces created by electromagnets,
P_y and P_z : components of weight,
F_y and F_z : disturbing forces acting at one end of the shaft,
F_{ay} and F_{az} : reaction of passive thrust,
p : rotation speed of the shaft around X axis.

Under the assumption of small displacements, it is possible to describe the mechanical model by the following state space representation :

$$\begin{bmatrix} \dot{y}_0 \\ \dot{z}_0 \\ \dot{\theta} \\ \dot{\psi} \\ \ddot{y}_0 \\ \ddot{z}_0 \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-k_r}{m} & 0 & 0 & \frac{k_r \cdot l_a}{m} & 0 & 0 & 0 & 0 \\ 0 & \frac{-k_r}{m} & -\frac{k_r \cdot l_a}{m} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-k_r \cdot l_a}{J_y} & \frac{-k_r \cdot l_a^2}{J_y} & 0 & 0 & 0 & 0 & -p \cdot \frac{J_x}{J_y} \\ \frac{k_r \cdot l_a}{J_y} & 0 & 0 & \frac{-k_r \cdot l_a^2}{J_y} & 0 & 0 & p \cdot \frac{J_x}{J_y} & 0 \end{bmatrix} \cdot \begin{bmatrix} y_0 \\ z_0 \\ \theta \\ \psi \\ \dot{y}_0 \\ \dot{z}_0 \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{m} & \frac{1}{m} & \frac{1}{m} & 0 \\ \frac{1}{m} & \frac{1}{m} & 0 & 0 & 0 & \frac{1}{m} \\ \frac{-l_1}{J_y} & \frac{l_2}{J_y} & 0 & 0 & 0 & \frac{-l_b}{J_y} \\ 0 & 0 & \frac{l_1}{J_y} & \frac{-l_2}{J_y} & \frac{l_b}{J_y} & 0 \end{bmatrix} \cdot \begin{bmatrix} F_{l2} - F_{l1} + \left(\frac{\sqrt{2}}{2} \cdot \frac{l_2}{l_1 + l_2} \cdot m \cdot g \right) \\ F_{r2} - F_{r1} + \left(\frac{\sqrt{2}}{2} \cdot \frac{l_1}{l_1 + l_2} \cdot m \cdot g \right) \\ F_{l4} - F_{l3} + \left(\frac{\sqrt{2}}{2} \cdot \frac{l_2}{l_1 + l_2} \cdot m \cdot g \right) \\ F_{r4} - F_{r3} + \left(\frac{\sqrt{2}}{2} \cdot \frac{l_1}{l_1 + l_2} \cdot m \cdot g \right) \\ F_y \\ F_z \end{bmatrix}$$

It is assumed that along radial directions, passive thrust behaves like a spring with negative stiffness k_r. l_1, l_2, l_a, l_b, m, J_x, J_y are constants parameters describing geometrical or mechanical characteristics of the shaft. Outputs are taken as displacement of rotor axis measured in sensors planes which are parallel to magnetic bearings.

The second step concerns modelling of electromagnets : electrical behaviour is really simple and can be resumed in the following equation : $u_j(t) = R \cdot i_j(t) + L \cdot \frac{d}{dt} i_j(t)$.
It means that variations of inductance L are negliged. In fact, taking into account these variations wouldn't increase the dimension of the problem and so could be broated following the same reasoning. It is only a matter of report simplicity.

Last step deals with force production. A model which is often presented in the literature consists in the following static equation : $F_j(t) = \frac{k \cdot i_j^2(t)}{(w_0 - w(t))^2}$, where k is a constant with resumes physical characteristics of the coil and kernel, while w_0 is another constant which describes the nominal equivalent magnetic circuit length, including iron and air gap weighted by relative permeability.

III - CONTROL

III-1 - Objectives

There are different ways to tackle the problem of control of such a system. The two main difficulties are due to the multivariable feature of the process on one hand, and to non linearities that exist in the electromagnetical aspects modelling on the other hand.

At the first glance, it appears that four variables have to be controlled (four degrees of freedom), with eight control inputs (voltage input of each electromagnet) : that let think that there is some redundancy in the system. In fact, it comes from the positiveness of the force produced by electromagnet, and the necessity to dispose of two devices. One way to overcome this redundancy problem, which provides at the same time with some kind of linearization, is to use polarization currents in electromagnets. The main drawback of this solution is to create useless thermal losses in the coil on the one hand, but also in the rotor on the other hand.

It is why it is to be hoped that only one electromagnet is working at one time along one direction according to the sign of the required force.

In order to simplify the presentation of the linearization method, we focus our attention on a reduced system, which consists in a pair of electromagnets that sustain a ball. This system presents the same non linear features as the preceding, so it will be easy to extend the control scheme to the whole system.

III-2 - Reduced subsystem

Let us consider one axis of an active magnetic bearing as shown in figure 2.

The model of such a system is described by the following equations :

$$m \cdot \ddot{x} = F_1 - F_2 - F_0$$

$$F_1 = \frac{k_1 \cdot i_1^2}{(x_0 - x)^2}$$

$$F_2 = \frac{k_2 \cdot i_2^2}{(x_0 + x)^2}$$

$$F_0 = m \cdot g$$

$$u_1(t) = R_1 \cdot i_1(t) + L_1 \cdot \frac{d}{dt} i_1(t)$$

$$u_2(t) = R_2 \cdot i_2(t) + L_2 \cdot \frac{d}{dt} i_2(t)$$

where m is the mass of the ball, R_i, L_i are resistance and inductance of coils.

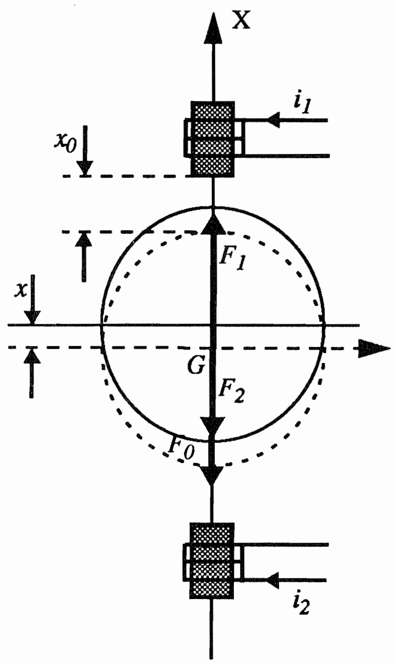

FIGURE 2 : Reduced system

In order to simplify the writing, we assume that both electromagnets have the same characteristics $((R_1 = R_2 = R), (L_1 = L_2 = L), (k_1 = k_2 = k))$.

When only one electromagnet is working at one time, one of both current i_1 and i_2, and one of both forces F_1 and F_2 are null. Since the sign of i_1 or i_2 doesn't matter, let us define

$$i(t) = |i_1(t)| - |i_2(t)|$$

$$F(t) = F_1(t) - F_2(t)$$

It is then possible to handle a fictitious device with resistance R, inductance L, fed with input voltage $u(t)$, which produces a force $F(t)$ according to the relation

$$F(t) = \frac{k \cdot i(t) \cdot |i(t)|}{[x_0 - x \cdot sgn\,(i(t))]^2}$$

The *sgn* function in the denominator takes into account the commutation between gaps, according to the electromagnet which is used.

The control laws will determine the control variable $u(t)$ that drives the evolution of $x(t)$ in the good direction.

Since $u(t)$ is a fictitious variable, actual control inputs $u_1(t)$ and $u_2(t)$ are determined by means of following relations

as long as $i(t)$ is > 0 $\quad u_1(t) = u(t) \quad u_2(t) = 0$

as long as $i(t)$ is < 0 $\quad u_1(t) = 0 \quad u_2(t) = u(t)$

Let us remark that commutation occurs when current is null, so there is no difficulty to implement such a switching law.
Of course, that system presents one singular point corresponding to $i(t) = 0$. We will come back further to that problem.

III-3 - Control scheme

The problem is to derive a control law for the system described by the following state space representation :

$$\begin{cases} \dot{x}_1 = x_2(t) \\ \dot{x}_2 = \dfrac{k \cdot x_3(t) \cdot |x_3(t)|}{m \cdot [x_0 - x_1 \cdot sgn(x_3(t))]^2} - \dfrac{F_0}{m} \\ \dot{x}_3 = \dfrac{1}{L} \cdot (-R \cdot x_3 + u(t)) \end{cases}$$

where $x_1(t)$, $x_2(t)$ and $x_3(t)$ represent position x, velocity $\dot{x}$ and fictitious current $i(t)$ respectively.

It is possible to linearize such a model and then to build a second loop that stabilizes the plant with the appropriate dynamics? This question arises because of the non analytical feature of the second equation.
In fact, this equation is twice derivable with respect to x_3, and so sufficiently derivable to allow the computation of the state feedback that linearizes the system.
The relative degree is equal to 3, except for $x_3 = 0$.
It is easily shown that the static state feedback

$$u(t) = -\frac{L \cdot x_2 \cdot x_3 \cdot sgn(x_3)}{x_0 - x_1 \cdot sgn(x_3)} + R \cdot x_3 + \frac{L \cdot m \cdot (x_0 - x_1 \cdot sgn(x_3))^2}{2 \cdot k \cdot |x_3|} \cdot v(t)$$

transforms the non linear system in the following one :

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_4 \\ \dot{x}_4 = v \end{cases}$$

It is then easy to build a second static state feedback

$$v = \alpha_1 \cdot x_1 + \alpha_2 \cdot x_2 + \alpha_3 \cdot x_4 + e$$

where e is the reference signal, that fixes the right dynamics of the system.
The extension to the initial complete model doesn't comprise other difficulties, if it weren't the writing of equations : they are found in [5].
The next section is devoted to the presentation of simulation results concerning the whole system.

IV - TESTS

IV-1 - Implementation

The control scheme that was derived in previous section is essentially continuous. However, it is interesting to get a numerical version of it in order to use a computer to control the system. Assuming that the sampling period is sufficiently small, the following approximation is used :

$$u_d(t) = u_c(j \cdot T) \quad \text{for} \quad t \in [j \cdot T, (j+1) \cdot T]$$

when $u_d(t)$ and $u_c(t)$ are the discrete and continuous control laws respectively.
The second problem that must be overcome is the crossing of singularities of the control law, namely every time that a commutation between electromagnets occurs.
The solution that was adopted consists in :

- choosing a lower bound of $|i(t)|$, that is

$$i_c(j \cdot T) = sgn(i_m(j \cdot T)) \cdot max(i_b, |i_m(j \cdot T)|)$$

where i_m is the measured current, i_c its value used in computation of control law, and i_b the lower bound, chosen such that there is no numerical difficulties in the computation of the control variables.

- fixing an upper bound to $u(t)$, i.e. :

$$u(j \cdot T) = sgn(u_d(j \cdot T)) \cdot min(u_s, |u_d(j \cdot T)|)$$

where u_d is the computed value of the control variable and u_s is chosen such that $u(t)$ evolves in an acceptable range.
It means that during crossing of singularities, the behaviour of the whole system is not entirely mastered. Furthermore, it is not possible to maintain the system around a singular point with this control scheme : one has to check that it doesn't coincide with the equilibrium.

IV-2 - Simulation results

Several situations have been considered for simulation, namely rising of shaft and influence of constant disturbances acting at one end of shaft. These two cases lead to crossing of singularities, and thus, display the non linear features of the process.

IV-2-1 - Shaft rising : initial conditions correspond to the shaft lying down on ball bearings. Upper electromagnets drive the shaft to the nominal position, but according to acceleration given to the body, which depends on the dynamics of the global control scheme, it may be necessary to brake its evolution by creating negative forces produced by lower electromagnets.

Figures (3) show the evolution of output measure (a), of the current (b) and the voltage input (c) of the fictitious coil corresponding to a pair of electromagnets. It is clear that commutation has no effect on output behaviour, even in case of saturation of the voltage input.

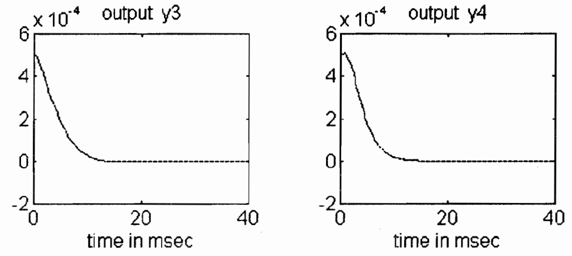

FIGURE 3-a : output measure

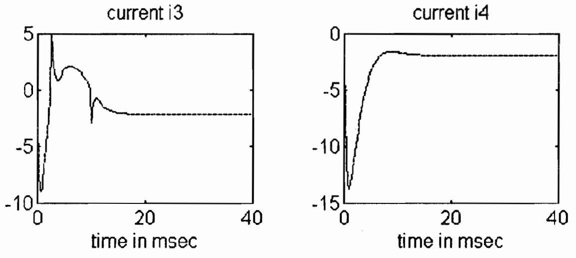

FIGURE 3-b : current in electromagnet

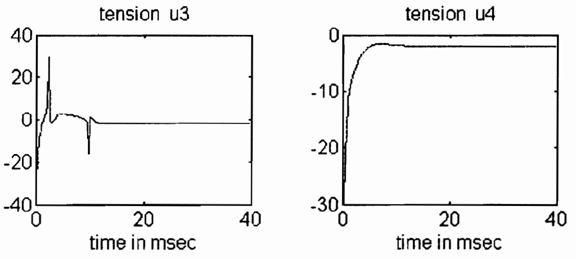

FIGURE 3-c : voltage input

Convention for current and force signs is the same as for y and z axes. Since the shaft is not rotating, displacements are identical in *(G,x,y)* and *(G,x,z)* planes. It is why only two among the four outputs, currents or voltage inputs are presented. Dissymmetry between variables indexed by 3 (opposite to passive thrust) and 4 (near passive thrust) is due to the influence of this device during transient behaviour and different lengths between centre of mass and active bearings in case of equilibrium. It is to be noticed that during transient behaviour, current i_3 crosses twice the singular point. Let us also note the saturation of voltage input just after $t = 0$, due to the start from singular point for each electromagnet.

IV-2-2 - Constant force disturbances : the aim of this test is to show the good behaviour of the control law in case of strong disturbances that couldn't be properly compensated by means of control law build upon a linearized model around a working point. In that example, the shaft is rotating at 120 000 rpm, around its nominal position and then after 50 sampling periods, a constant disturbance in applied at the end of the shaft with $(F_y,F_z) = (10N,20N)$.

In this case, due to gyroscopic effects induced by shaft rotation, and due to different values of F_y and F_z, there is not exact symmetry between evolutions in *(G,x,y)* and *(G,x,z)* planes. Since there is no integral action, a new equilibrium point is reached. The influence of disturbances leads to the tilting of the shaft, and causes new crossing of singularities by electromagnets 2 and 4. Of course, this depends on relative amplitude of disturbances and weight.

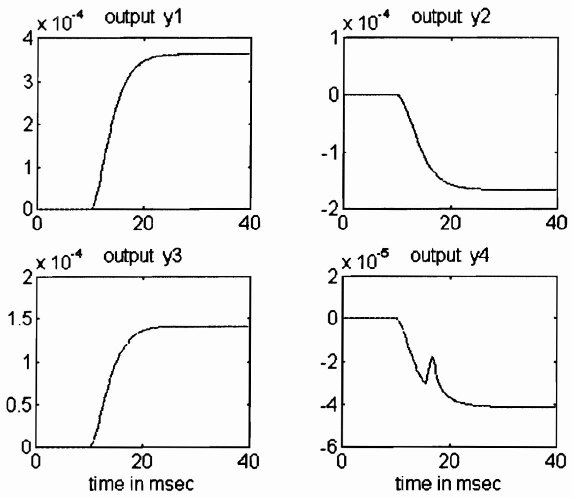

FIGURE 4-a : output measure

This is illustrated on figure (4-a) to (4-c) which show the outputs, currents and voltage input evolutions.

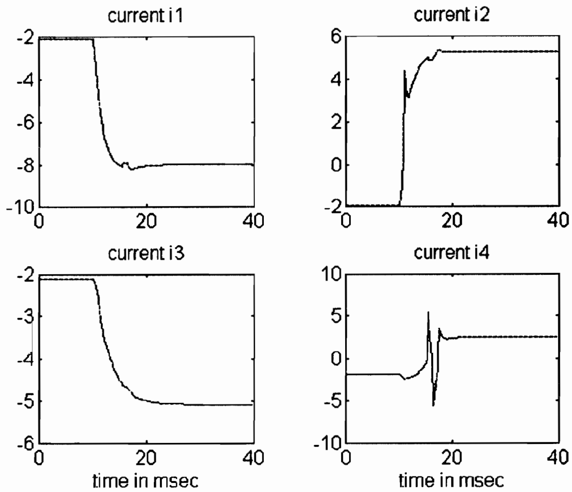

FIGURE 4-b : current in electromagnet

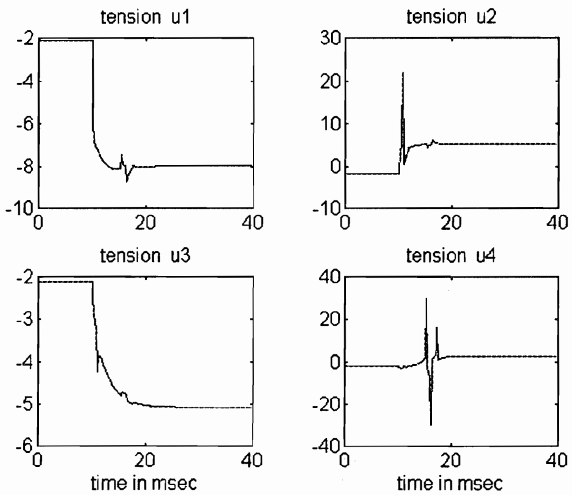

FIGURE 4-c : voltage input

V - CONCLUSION

A non linear control scheme has been presented, which apply to active magnetic bearings suspension. In order to treat the apparent redundancy of actuators, a fictitious model including both electromagnets involved in the production of force along one direction is introduced. It permits the synthesis of a linearizing control scheme and simulation results show a good behaviour even when non linear features are activated. However, it is not properly a state model, since the actual dimension of the system is reduced; others control schemes based on flatness properties are under study.

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