Hybrid Nonlinear Robust Control for Magnetically Supported Flexible Beam

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1 ABSTRACT

This paper describes a design and simulation study for nolinear robust control to a magnetic levitation system on a flexible beam. The beam investigated here is supposed so flexible that the gravitation causes noticeable static deflection. An unified dynamic model including static modes is induced an d then an exactly linearized state equation system is derived by utilizing the feedback linearization technique. H^2/H^∞ robust control system is applied to the linearized systems to compensate the modelling error. A simulatioon result proves the effectiveness of the proposed design method.

2 INTRODUCTION

The use of magnetic suspension is increasing in its various forms with its proven high level of reliability, negligible maintenance and low energy consumption. The benefits come from the noncontact support which results in negligibly small friction and low vibration level of the supported body. Its use spreads from a high speed railway vehicle to rotational machineries. One of the most attractive advantages of the magnetic suspension is applicability to the vibration control of the flexible structure. The control systems concerned with maintaining a floating gap is also applied to the vibration suppression. Modern machinery tends to decrease its weight and accordingly its stiffness. On the other hand the motion speed of the machinery consistently increases. These trends often result in increase of the control band and decrease of the natural frequency of the structures. Incidentally we are forced to cope with the problem of resonance between the structures and the control systems. Another difficulty of the magnetic suspension is due to non-linearity in the magnetic force . The non-linearity makes designing of the control system difficult especially in the case of wide range of the controlled gap.

This paper presents a design and simulation study for non-linear robust control to a magnetic levitation system on a flexible beam. The beam is supported horizontally by two electro-magnets. The vertical position of the beam is detected by two position sensors colocated with electro-magnet actuators. The beam is supported so flexible that the gravitational force may cause noticeable static deflection. We should treat with "static mode", to which the static mode is refered, in addition to dynamic modes on designing the control systems. This means that hybrid system of state equations involving static mode variables must be derived. The controlling force for stable suspension must include the static force for compensating the gravitational force along with the stabilizing force for vibration suppression. The static force is conventionally compensated separately from the dynamic force. But, as a non-linear control is introduced in the systems, unified control algorythm should be derived, to which the hybrid control is refered. The electro-magnets generate attractive force represented by non-linear function of the electric current in electro-magnet coil, and the gap between the end surface of the iron yoke attached to the beam and pole surface of the electro-magnet. Then hybrid system of the state equations involves the non-linear terms representing the control force. This systems of the equation is transformed to the exact linear equation systems by using the feed back linearization technique, by which the linearized terms are collected in the input of the control systems. The effectiveness of the proposed designing method is proven in the simulation of the step response of the magnetic levitation.

3 A MODEL OF THE SYSTEM

A model of magnetically suspended flexible beam / control system is shown in Fig.1. The suspended flexible beam of Length $L(=600\text{mm})$ and thickness $t(=1\text{mm})$ has its basic bending frequency at about Hz. The beam is supported contactless and horizontally by two electro-magnets (**magnet1 and magnet2**). The gravitational force is compensated by the magnetic attractive force generated by electro-magnet coils. The vertical position of the beam is detected through air gap by a eddy current displacement sensors, which are colocated with the electro-magnet in the horizontal position (along x-axis). The sensoor signals are fed to the currents ($\mathbf{i_1}$ and $\mathbf{i_2}$) in the poewr amplifiers, and finally they are supplied to the electro-magnet coils. While thevertical position of the beam is actively controlled, the horizontal position is passively stabilized by the attractive force tween the magnetic poles.

4 MODELLING

We assume that bending deflection of the beam w is governed by the following equation,

$$\frac{\partial^2 w}{\partial t^2} + a^2\frac{\partial^4 w}{\partial x^4} = g - \frac{1}{\rho A}[f_1(t)\delta(x+l_1) + f_2(t)\delta(x-l_1)] \tag{1}$$

where $a^2 = EI/\rho A$, EI is bending stiffness of the beam, ρ is the mass density of the beam and A is the cross sectional area. g means the acceleration of gravitation. $f_1(t)$ and $f_2(t)$ mean the magnetic attractive force generated by **magnet1** and **magnet2**, which are located at $x=-l_1$ and $x=l_1$, respectively. The magnetic attractive force $f_1(t)$ and $f_2(t)$ are related nonlinearly with the current i_1 and i_2 and the gap y_1 and y_2 as follows,

$$f_1 = \frac{k_{l1}i_1^2}{2y_1^2}, \qquad f_2 = \frac{k_{l2}i_2^2}{2y_2^2} \tag{2}$$

where k_{l1} and k_{l2} are the magnet constant related with electro-magnet 1 and electro-magnet 2 respectively. The current i_1 and i_2 are consisted of three elements; equilibrium current i_{s1}, i_{s2}, direct feedback current i_{k1}, i_{k2} and linearization component a_1, a_2.

$$i_1^2 = i_{s1}^2 + i_{k1}^2 + a_1, \qquad i_2^2 = i_{s2}^2 + i_{k2}^2 + a_2 \tag{3}$$

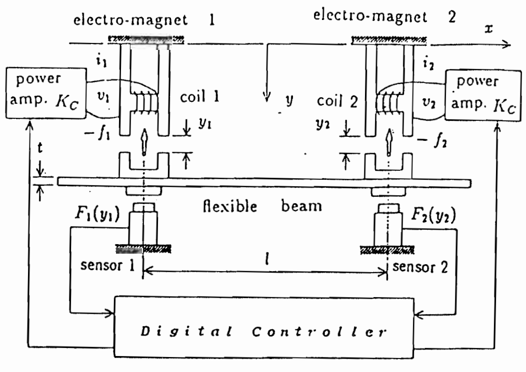

Fig.1 Schematical view of a model

Then i_{s1} and i_{s2} are supplied to the electro-magnets to compensate the gravitation of the beam as follows,

$$\frac{k_{l1}i_{s1}^2}{2y_1^2} = \frac{Mg}{2} = \frac{k_{l2}i_{s2}^2}{2y_2^2} = \frac{Mg}{2} \tag{4}$$

where M means mass of the beam. As mentioned above, Eq.(1) represents not only dynamic behavior but also static behavior of the beam. The static mode results from the following equation,

$$EI\frac{\partial^4 w}{\partial x^4} = -\rho Ag + f_1\delta(x+l_1) + f_2\delta(x-l_1) \tag{5}$$

One of solutions of Eq.(5) is represented by the fourth order algebraic equation of x as $X_0(x)$ as well known, and it satisfies the current component represented by Eq.(4). A mode shape of $X_0(x)$ that satisfies the boundary condition $X_0(x=-l_1) = X_0(x=l_1) = 0$ is shown in **Fig.2**. But any static deflection at the supporting points,

$$w(x=-l_1) = w_{s1} \quad \text{and} \quad w(x=l_1) = w_{s2} \tag{6}$$

can satisfy Eqs.(4) and (5). Then two static mode variables are defined as follows,

$$\eta_{01} = \frac{w_{s1}+w_{s2}}{2l_1}, \qquad \eta_{02} = \theta = \frac{w_{s1}-w_{s2}}{2l_1} \tag{7}$$

where η_{01} represents translational deflection and η_{02} does rotational deflection. Henceforth two static mode shape functions are defined as follows,

$$\begin{cases} X_{s1}(x) = X_0(x) + l_1\eta_{01} \\ X_{s2}(x) = X_0(x) - x\eta_{02} \end{cases} \tag{8}$$

These two static mode variables can be introduced into the dynamic equations without modifying above mode shape functions, because the dynamic magnetic forces $f_1(t)$ and $f_2(t)$ work only on the supporting point $x=-l_1$ and $x=l_1$. Then dynamic equations of the static variables are described as follows by introducing new variables $\eta_{s1} = 2Ll_1^2\eta_{01}, \eta_{s2} = 2k^2L\eta_{02}$,

$$\begin{cases} \ddot{\eta}_{s1} = -\frac{g}{l_1} + \frac{1}{Ml_1}[f_1(t)+f_2(t)] \\ \ddot{\eta}_{s2} = \frac{l_1}{Mk^2}[f_1(t)-f_2(t)] \end{cases} \tag{9}$$

where $2L$ means the whole length of the beam and k means the radius of the gyration of the moment of inertia around the center. After introducing equilibrium current i_{s1} and i_{s2}, Eq.(9) is rearranged as follows,

$$\begin{cases} \ddot{\eta}_{s1} = \frac{1}{Ml_1}\left[\left(\frac{k_{l1}}{2y_1^2}\right)(i_{k1}^2+a_1) + \left(\frac{k_{l2}}{2y_2^2}\right)(i_{k2}^2+a_2)\right] \\ \ddot{\eta}_{s2} = \frac{l_1}{Mk^2}\left[\left(\frac{k_{l1}}{2y_1^2}\right)(i_{k1}^2+a_1) - \left(\frac{k_{l2}}{2y_2^2}\right)(i_{k2}^2+a_2)\right] \end{cases} \tag{10}$$

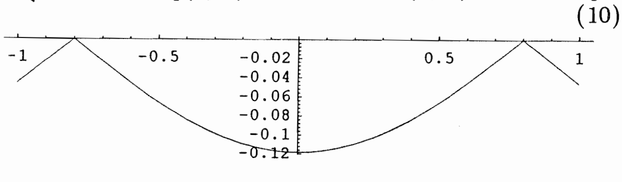

Fig.2 Mode shape of static bending deflection

Other dynamic eigen mode variables $\eta_i(t)$ and mode shape function $X_i(x)$ which satisfies Eq.(1) are related with bending deflection $w(x,t)$ by

$$w(x,t) = \sum_{i=1}^{N} X_i(x)\eta_i(t) \tag{11}$$

Mode shape functions satisfying the boundary conditions,

$$\frac{d^2X_i}{dx^2} = \frac{d^3X_i}{dx^3} = 0; \qquad x = \pm L \tag{12}$$

are expressed by the following equation,

$$X_i(x) = C_i\Big\{\cosh b_i(x+L) + \cos b_i(x+L) - \frac{\cosh 2b_iL - \cos 2b_iL}{\sinh 2b_iL - \sin 2b_iL}[\sinh b_i(x+L) + \sin b_i(x+L)]\Big\} \tag{13}$$

where b_i is determined by the following condition

$$\cos(2b_iL)\cosh(2b_iL) = 1 \tag{14}$$

$X_i(x)$ satisfies orthogonal condition and normalization condition from which C_i is induced, then also the integration of $X_i(x)$ always reduces to null.

$$\int_{-L}^{L} X_i(x)dx = 0 \tag{15}$$

Incidentally equations of mode variables are represented after introducing Eq.(4) into Eq.(2) as follows,

$$\ddot{\eta}_i + \omega_i^2\eta_i = \frac{L}{M}\Big[X_i(-l_1)\left(\frac{k_{l1}}{2y_1^2}\right)(i_{k1}^2 + a_1) + X_i(l_1)\left(\frac{k_{l2}}{2y_2^2}\right)(i_{k2}^2 + a_2)\Big] \tag{16}$$

where i_{s1}^2 and i_{s2}^2 are dropped, as they are indifferent with dynamic behavior of $w(x,t)$. The second current components i_{k1}^2 and i_{k2}^2 are related with direct displacement feedback[1] at the supporting points as follows,

$$i_{k1}^2 = -k_0\frac{My_1^2}{k_{l1}}w_1, \qquad i_{k2}^2 = -k_0\frac{My_2^2}{k_{l2}}w_2 \tag{17}$$

where k_0 means the feedback gain, and w_1, w_2 means the bending deflection at $x = -l_1, l_1$ respectively. Then they are expressed as follows,

$$\begin{cases} w_1 = \frac{1}{Ll_1}\eta_{s1} + \frac{l_1}{k^2L}\eta_{s2} + X_1(-l_1)\eta_1 + \cdots + X_N(-l_1)\eta_N \\ w_2 = \frac{1}{Ll_1}\eta_{s1} - \frac{l_1}{k^2L}\eta_{s2} + X_1(l_1)\eta_1 + \cdots + X_N(l_1)\eta_N \end{cases} \tag{18}$$

Third current components are supplied for the exact linearization as follows,

$$a_1 = \frac{2Mgy_1^2}{k_{l1}}\left(\frac{i_1'}{i_{s1}}\right), \quad a_2 = \frac{2Mgy_2^2}{k_{l2}}\left(\frac{i_2'}{i_{s2}}\right) \tag{19}$$

where i_1' and i_2' are linearized currents for the robust control systems. When introducing Eq.(17) and Eq.(19) into Eq.(10) and Eq.(16) we redefine the extended variables systems,

$$\eta = [\eta_{s1} \quad \eta_{s2} \quad \eta_1 \cdots \eta_N]^T \tag{20}$$

then deflection vector $w = [w_1, w_2]^T$ are expressed as follows,

$$w = B_1{}^T\eta$$

$$B_1{}^T = \begin{bmatrix} \frac{1}{Ll_1} & \frac{l_1}{k^2L} & X_1(-l_1) & \dots & X_N(-l_1) \\ \frac{1}{Ll_1} & -\frac{l_1}{k^2L} & X_1(l_1) & \dots & X_N(l_1) \end{bmatrix} \tag{21}$$

Finally we can derive the equation of dynamic bending as follows,

$$\ddot{\eta} + (\Omega_N{}^2 + k_0B_1B_1{}^T)\eta = gLB_1\begin{pmatrix} i_1'/i_{s1} \\ i_2'/i_{s2} \end{pmatrix} \tag{22}$$

where $\Omega_N{}^2 = \mathrm{diag}.[0, 0, w_1{}^2, \cdots, w_N{}^2]$ is defined.

5 ROBUST CONTROL SYSTEMS DESIGN

An exactly linearized equation as shown in Eq.(22) is obtained by feedback linearization technique [2]. We should introduce a standard state equations by modifying Eq.(22) with structure damping effect $\Lambda\dot{\eta}$ and redefining the state variable x by $[\eta^T, \dot{\eta}^T]^T$ and the input variables v_1', v_2' by R_1i_1', R_2i_2' respectively,

$$\dot{x} = Ax + Bu, \tag{23}$$

$$A = \begin{bmatrix} O & I \\ -\Omega_N^2 - k_0B_1B_1^T & -\Lambda \end{bmatrix}, \qquad u = \begin{bmatrix} v_1' \\ v_2' \end{bmatrix},$$

$$B = \begin{bmatrix} O \\ gLEB_1 \end{bmatrix}, \quad E = \begin{bmatrix} (R_1i_{s1})^{-1} & O \\ O & (R_2i_{s2})^{-1} \end{bmatrix}$$

As detected variables w_1 and w_2 represented by Eq.(21) should be redefined by y and it should be related with state variable x as follows,

$$y = Cx, \qquad C = [B_1^T \ O] \tag{24}$$

We have derived an exactly linearized equation systems Eqs.(23) and (24) for not only dynamic mode variables but also static mode variables by utilizing feedback linearization. Even this modelling includes inevitable errors resulted by truncation of eigen mode variables by Nth degree, linearized assumption between v_1', v_2' and i_1', i_2', magnetic attractive force expression by Eq.(2) and others. We apply robust control algorithms to cope with the modelling errors. In order to introduce H^2/H^∞ norm based contol systems to the magnetic levitation mechanisms an augmented plant systems expression[3] shown by Fig.3 is utilized, where w means a disturbance vector and z_1, z_2 are controlled variables. When z_1 and z_2 are evaluated with weighting function by $W_1(s)$ and $W_3(s)$ respectively, the transfer matrix $P(s)$ from the input vectors w, u to the output vectors $[z_1^T, z_2^T, y_1^T]^T$ is

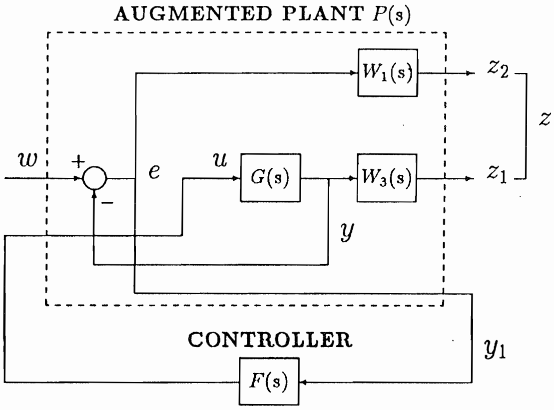

Fig.3 Block diagram of augmented plant

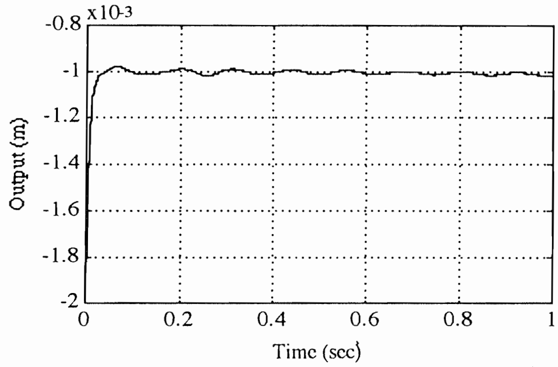

Fig.4 Simulation of step response of vertical position

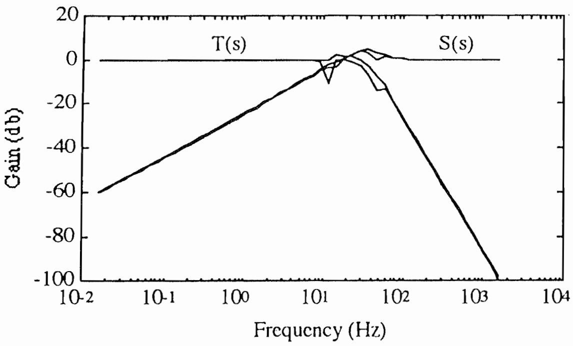

Fig.5 Bode diagram of $S(s)$ and $T(s)$

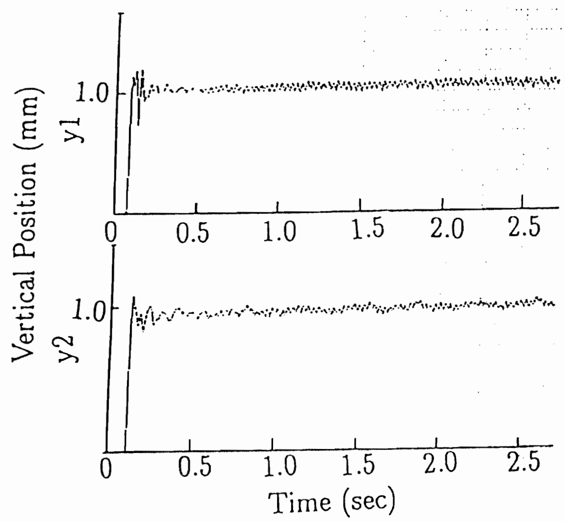

FIg.6 Experimental result of step response

expressed by the following equation,

$$\begin{bmatrix} z_1 \\ z_2 \\ y_1 \end{bmatrix} = P(s) \begin{bmatrix} w \\ u \end{bmatrix}, \qquad P(s) := \begin{bmatrix} W_1 & -W_1 G \\ O & W_3 G \\ I & -G \end{bmatrix} \tag{25}$$

where G means the transfer matrix from u to y, which is derived by Eqs.(23) and (24). Where a controller $F(s)$ is introduced between $y_1 = w - y$ and operation variable u and a closed loop is formed by the controller, transfer matrices, $\Psi_1(s)$ from w to z_1 and $\Psi_2(s)$ from w to z_2 are represented by the following equations.

$$\begin{cases} \Psi_1(s) := W_1(s)S(s), & S(s) = (I + GF)^{-1} \\ \Psi_2(s) := W_3(s)T(s), & T(s) = GF(I + GF)^{-1} \end{cases} \tag{26}$$

Then a joint transfer matrix $\Psi(s)$ is redefined from w to $z = [z_1^T, z_2^T]^T$ as follows,

$$\Psi(s) := \begin{pmatrix} \Psi_1(s) \\ \Psi_2(s) \end{pmatrix}, \qquad \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \Psi(s) w \tag{27}$$

The designing of H^2/H^∞ robust controller is reduced to derive a controller which optimize $||\Psi(s)||_2$ or minimizes $||\Psi(s)||_\infty$. The controller can be derived by utilizing robust controller design software[3].

6 SIMULATION

A simulation is conducted for a flexible beam/magnetic support systems shown in Fig.1. Proper eigen frequencies of the first and second mode are 12Hz and 35Hz respectively. On designing the controller first to second mode variables are reserved in the state equation in addition to the two static mode variables η_{s1} and η_{s2}. Diagonal entry functions, $w_1(s)$ of the weight matrix $W_1(s)$ and $w_3^{-1}(s)$ of $W_3^{-1}(s)$ are assigned respectively as follows,

$$w_1(s) = \frac{k_1(s+\rho)}{T_1 s + 1}, \qquad w_3^{-1}(s) = \frac{k_2(T_3 s + 1)}{T_2 s + 1} \tag{28}$$

Fig.4 shows a step response simulation of y_1 variable where H^∞ controller is designed under $k_1 = 50$, $T_1 = 2000$(sec), $\rho = 2000$(rad/s), $k_2 = 5 \times 10^{-3}$, $T_2 = (800)^{-1}$(sec), $T_3 = 1$(sec). Fig.5 shows Bode diagram of the $T(s)$ function and the $S(s)$ function. We can find the satisfactory performance for the step response of the controller. We should note that no limitatios are imposed on the controller designed for the linearized state equation systems, but $i_{s1}^2 + i_{k1}^2 + a_1 \geq 0$ and

$i_{s2}^2 + i_{k2}^2 + a_2 \geq 0$ should be satisfied anytime. Then an implicit limitation is imposed on the nonlinear controller. Fig.6 shows an examle of experimental result where a stiffer beam of which the first eigen mode frequency is 28Hz is levitated by H^2 robust controller, where a linear direct velocity feedback is supplemented. The beam is supported stable by the control systems.

7 CONCLUDING REMARKS

A design method for hybrid nonlinear control systems is derived for a magnetically levitated flexible beam. The simulation result proves the effectiveness of the design method. An experimet is planned for proving the simulation result.

References

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