INVERSE PROBLEMS OF MAGNETIC BEARING DYNAMICS

Yuri N. Zhuravlyov
Pskov Branch of St.Petersburg State Technical University,
Pskov, Russia
Mikhail V. Afanasiev
Active Magnetic Engineering Ltd, Tampere, Finland,
Pskov office center, Pskov, Russia
Erkki Lantto
Helsinki University of Technology, Espoo, Finland

ABSTRACT
This paper describes the design procedure of the optimal active magnetic bearing control for a rigid gyroscopic rotor from the position of inverse problems of dynamics. The procedure is based on the analytical solution of the four-dimensional Linear-Quadratic optimization problem and results in the coupled speed-dependent optimal control. The simulation results are compared with those obtained for a conventional PID control.

INTRODUCTION
The conventional approach to the optimal Active Magnetic Bearing (AMB) control may be formulated in the following way. Given the control forces (and/or the coil currents and voltages) as functions of the state variables, one should determine the control law by finding such a system motion trajectory on which an extreme value of the performance index takes place. From this point of view such AMB control methods as the "classical" root locus and state space and, as well as the "modern" H^{∞} [1], Q-factorisation [2], sliding mode [3] and the others may be considered as methods based on the solution of direct problems of dynamics (i.e. to find the motion trajectory if the forces are known).
On the other hand, the optimal AMB control may rightfully be formulated as the problem of finding the control forces, currents and voltages which provide the desirable motion trajectory or the programmed trajectory for the AMB system. The programmed trajectory may be given by both the explicit function of time or the set of differential equations having the solution coinciding with the desirable trajectory [4]. Methodologically, the control forces, currents and voltages may be found by solving the inverse problem of dynamics (i.e. to find the forces if the motion trajectory is known). Such approach is known to be used in robot arm and flight control and leads to simple and effective control algorithms.
In this paper, the inverse problem of dynamics method is applied to the radial AMB control system for a rigid gyroscopic rotor. Attention is focused on the methodology for synthesis of the fourth (full)-order and second-order optimal linear controllers ensuring the minimization of the AMB force reactions. In order to minimize forces the Linear-Quadratic optimization theory is applied. This theory is known to be based on the solution to the nonlinear Riccati equation. Until now, only numerical iterative methods are used in such problems. But the problem under consideration is actually unique because the analytical solution to the Riccati equation exists and it has been obtained in [5]. The inverse problem of dynamics method leads to the coupled speed-dependent optimal control law.
The controllers obtained are compared with a conventional PID controller for the flywheel energy storage system by numerical simulation.

MODELLING
Modelling of Rotor
As shown in Fig.1, a rigid gyroscopic rotor of mass M, equatorial J_1 and axial J_3 principal moments of inertia spins at the constant rotational speed ω in two radial active magnetic bearings AMB1 and AMB2.The rotor displacement sensors S1 and S2 are not collocated with the AMB actuators, i.e. $z_1 \neq z_1{}^*$ and $z_2 \neq z_2{}^*$.

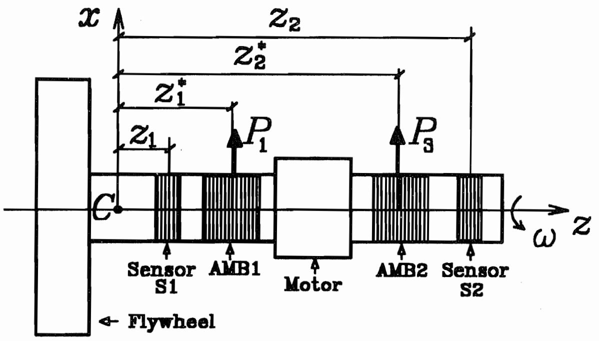

FIGURE 1: Schematic of a rotor

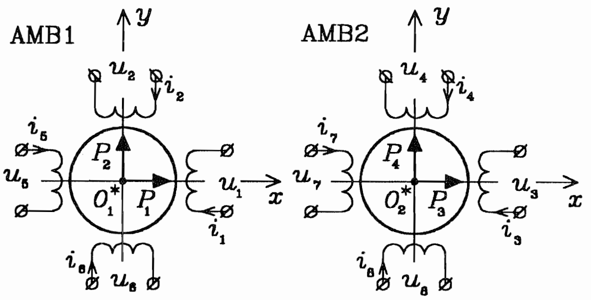

FIGURE 2: Schematic of AMB actuators

We introduce the vector $\eta=(x_c,y_c,\varphi_x,\varphi_y)^T$ of the coordinates x_c and y_c of the center of mass C and the angles of rotation φ_x and φ_y around x and y axes. Then the vectors of rotor displacements at the sensor locations $q=(x_1,y_1,x_2,y_2)^T$ and at the actuator centers $q_*=(x_1{}^*,y_1{}^*,x_2{}^*,y_2{}^*)^T$ are given by the linear transformations

$$q = Z\eta, \qquad q_* = Z_*\eta \tag{1}$$

with 4x4 matrices Z and Z_*.
The equations of motion of the rotor are given by

$$M\ddot{x}_c = F_1, \quad M\ddot{y}_c = F_2 \tag{2}$$
$$J_1\ddot{\varphi}_x + \omega J_3\dot{\varphi}_y = F_3, \quad J_1\ddot{\varphi}_y - \omega J_3\dot{\varphi}_x = F_4 \tag{3}$$

or in the matrix form

$$J\ddot{\eta} + G\dot{\eta} = F \tag{4}$$

Here J and G are, respectively, the inertia and gyroscopic matrices, and F is the vector of the generalized forces consisting of the magnetic forces F_m and the external loads $F_e = F_e(\eta, \dot{\eta})$ and the disturbance forces $F_d = F_d(t)$, i.e. $F = F_m + F_e(\eta, \dot{\eta}) + F_d(t)$. The generalized magnetic forces F_m are related to the actuator reacting forces $P=(P_1,P_2,P_3,P_4)^T$ by

$$F_m = Z_*^T P \tag{5}$$

The equations of motion of the rotor in terms of coordinates q may be written in the form

$$\ddot{q} + H\dot{q} = WP + Z_F F_e(q, \dot{q}) + Z_F F_d(t) \tag{6}$$

where $H = ZJ^{-1}GZ^{-1}$, $W = ZJ^{-1}Z_*^T$, $Z_F = ZJ^{-1}$.

Modelling of AMB actuators

As shown in Fig.2, the AMB actuators incorporate eight electromagnets with the coil currents i_k, input voltages u_k, resistances r_k and self and mutual inductances L_{ks}, k,s=1,...,8. To obtain the linear model we represent the currents and voltages by the sums $i_k(t) = i_{kc} + i_{kv}(t)$, $u_k(t) = u_{kc} + u_{kv}(t)$, where i_{kc} and u_{kc} are the bias values, and $i_{kv}(t)$ and $u_{kv}(t)$ are the control variables.
Introducing the vectors of the difference control currents $I=(I_1,I_2,I_3,I_4)^T$ and voltages $U=(U_1,U_2,U_3,U_4)^T$ with elements $I_1 = i_{1v} - i_{5v}$, $U_1 = u_{1v} - u_{5v}$, $I_2 = i_{2v} - i_{6v}, \ldots$, $U_4 = u_{4v} - u_{8v}$, we can write the expression for the control forces

$$P = C_q q + C_i I \tag{7}$$

and the dynamic equation for the AMB actuators

$$L\dot{I} + B_E\dot{q} + RI = U \tag{8}$$

where C_q, C_i, L, B_E and R are, respectively, the 4x4 matrices of negative stiffnesses, current stiffnesses, inductances, electromotive forces of motion and resistances [6].

OPTIMAL AMB CONTROL

Conceptions of Control

Decomposing the control system (6)-(8) into the mechanical subsystem (6) and the electromagnetic subsystem (7)-(8), we shall solve the control problem in two steps. We first consider the plant (6) and find the optimal programmed control forces $P = P^0$ which cause the desirable motion of the rotor $q(t) = q^0(t)$. Next, considering the plant (7)-(8) we determine the optimal control currents $I = I^0$ and voltages $U = U^0$ which generate the optimal forces P^0.

Applying the conception of the inverse problems of dynamics to the plant (6) we assume that the desirable trajectory of motion $q(t)=q^0(t)$ is known and it coincides with the solution to the differential equation

$$\ddot{q} = f^0(q, \dot{q}, \sigma, \omega) + Z_F F_d^0(t) + Z_F F_d(t) \qquad (9)$$

where f^0 is the given vector-function of q, $\dot{q}$, σ and ω and where σ is the integral variable defined as $\dot{\sigma} = q$, and $F_d^0(t)$ is the vector of the desirable disturbance forces. Equation (9) can be readily constructed, for example, from common engineering positions by introducing the restoring, damping, correcting and integral terms which may be both linear and nonlinear and, dependent on the rotational speed ω. The force $F_d^0(t)$ may be introduced to suppress vibrations of the rotor. As it will be shown further, the optimization approach can be used to construct Eq. (9), as well.
Substituting the acceleration $\ddot{q}$ from Eq. (9) into Eq. (6) we obtain the programmed forces

$$P^0 = W^{-1}\left(f^0(q, \dot{q}, \sigma, \omega) + H\dot{q} - Z_F F_e(q, \dot{q}) + Z_F F_d^0(t)\right) =$$
$$= -\left(C_0 q + B_0 \dot{q} + D_0 \sigma\right) + W^{-1} Z_F F_d^0(t) \qquad (10)$$

where C_0, B_0 and D_0 are, respectively, the proportional, derivative and integral feedback gain matrices with elements which may be dependent on q, $\dot{q}$ and ω.
From relation (7) we find the programmed currents

$$I^0 = -C_i^{-1}\left(\left(C_0 + C_q\right)q + B_0\dot{q} + D_0 s\right) + C_i^{-1} W^{-1} Z_F F_d^0(t) \qquad (11)$$

To reduce the difference $\Delta I(t)=I^0(t)-I(t)$ between the programmed currents and actual ones to zero we must introduce a current tracking system. In accordance with the conception of the inverse problems of dynamics we are to give the desirable trajectory $\Delta I(t)$. There are many ways to do it. Let us first consider the following trajectory

$$\Delta I_j(t) = \Delta I_j(0)\, exp\left(\lambda_j t\right),\ \left(\lambda_j \succ 0\right) \qquad (12)$$

which is the solution to the differential equation

$$\dot{I} = \dot{I}^0 + \Lambda(I^0 - I) \qquad (13)$$

where $\Lambda = diag(\lambda_1, \ldots, \lambda_4)$. Substituting the derivative $\dot{I}$ from Eq.(13) into Eq.(8) yields the control voltages $U = U^0$. If matrices C_0, B_0 and D_0 are time- invariant and $F_d^0(t) \equiv 0$, the control law is given by

$$U^0 = -\left(K_q q + K_v \dot{q} + K_I I + K_\sigma \sigma + K_a \ddot{q}\right) \qquad (14)$$

where $K_q = L\Lambda C_i^{-1}\left(C_0 + C_q\right) + LC_i^{-1}D_0$, $K_v = L\Lambda C_i^{-1}B_0 + LC_i^{-1}\left(C_0 + C_q\right) - B_E$, $K_I = L\Lambda - R$, $K_\sigma = L\Lambda C_i^{-1}D_0$, $K_a = LC_i^{-1}B_0$ are, respectively, proportional, derivative, current, integral and acceleration feedback gain matrices. Next, using the tracking system

$$\dot{I} = \Lambda(I^0 - I) \qquad (15)$$

instead of system (13) results in the control law of the form (14) without the acceleration feedback. Finally, a relay tracking system leads to the time optimal control law given by

$$U_j^0 = +\hat{U} \ \text{ if } \Delta I_j(t) \geq 0$$
$$U_j^0 = -\hat{U} \ \text{ if } \Delta I_j(t) \prec 0 \qquad (16)$$

where $\hat{U}$ is the maximum value of U_j.

Full-Order Controller
The goal is to find the optimal control forces $P = P^0$ for the four-degrees-of-freedom (4-DOF) control system (4) which incorporates two 1-DOF systems (2) and one 2-DOF system (3). For each of these systems we write the state equations in the form

$$\dot{x} = Ax + Bf$$
$$y = Cx \qquad (17)$$

where x is the state vector; y is the output variable; f is the input variable; A, B and C are the constant matrices.
It is required to find the control $f = f^0$ (or $F = F^0$) which brings the system (17) from an arbitrary initial state to the zero state by minimizing the criterion

$$\int_0^\infty \left(f^T(t) f(t) + \rho\, y^T(t) y(t)\right) dt \qquad (18)$$

where ρ is the positive weighting scalar. This approach means to minimize the integrated square forces with the constrained integrated square displacements of the rotor.

The analytical solution of this optimization problem has been obtained in [5] and it is given by

$$\begin{aligned} F_1^0 &= -M\left(\omega_0^2 x_c + 2\xi\omega_0 \dot{x}_c\right) \\ F_2^0 &= -M\left(\omega_0^2 y_c + 2\xi\omega_0 \dot{y}_c\right) \\ F_3^0 &= -J_1\left(k_1\varphi_x + k_2\dot{\varphi}_x + k_3\varphi_y\right) \\ F_4^0 &= -J_1\left(k_1\varphi_y + k_2\dot{\varphi}_y - k_3\varphi_x\right) \end{aligned} \quad (19)$$

where $\omega_0 = \rho^{1/4}$ is the desirable natural frequency of translational motions of the rotor; $\xi = \sqrt{2}/2$ is the optimal relative damping; k_1, k_2 and k_3 are, respectively, the optimal stiffness, damping and radial correcting factors of rotational motions of the rotor given by

$$k_1 = \sqrt{h^4/16 + \Omega_0^4} - h^2/4,\ k_2 = \sqrt{2k_1},\quad k_3 = h\sqrt{k_1/2} \quad (20)$$

Here $h = \omega J_3/J_1$ is the gyroscopic parameter, and $\Omega_0 = \rho^{1/4}$ is the desirable natural frequency of rotational motions of the non-rotating rotor. From (20) follows that as $\omega \to \infty$ the stiffness and damping factors k_1 and k_2 approach to zero, and the radial correcting factor k_3 becomes equal to Ω_0^2.
The optimal forces P^0 are determined from Eq.(19) and (5). Adding the integral control we can easily arrive at the full (fourth)-order speed-dependent optimal control law (14). The problem is the relatively high order of the controller obtained which may limit the performance.

Second-Order Controller

Let us obtain the reduced-order optimal controller without the cross couplings between AMB1 and AMB2. In this case, we have two the 2-DOF control systems obtained from Eq.(6) for AMB1 and AMB2, respectively. Consider, for example, the control system for AMB1

$$\begin{aligned} \ddot{x}_1 - h_1\dot{y}_1 &= w_{11}P_1 \\ \ddot{y}_1 + h_1\dot{x}_1 &= w_{22}P_2 \end{aligned} \quad (21)$$

where
$h_1 = \omega J_3 z_1 / J_1(z_2 - z_1),\ w_{11} = w_{22} = M^{-1} + z_1 z_1^* J_1^{-1}$.
Applying the procedure (17)-(20) to Eq.(21) yields the optimal forces

$$\begin{aligned} P_1^0 &= -w_{11}^{-1}\left(c_1 x_1 + b_1\dot{x}_1 - g_1 y_1\right) \\ P_2^0 &= -w_{22}^{-1}\left(c_1 y_1 + b_1\dot{y}_1 + g_1 x_1\right) \end{aligned} \quad (22)$$

where c_1, b_1 and g_1 are, respectively, the optimal stiffness, damping and radial correcting factors of the AMB1 given by

$$c_1 = \sqrt{h_1^4/16 + \omega_0^4} - h_1^2/4,\ b_1 = \sqrt{2c_1},\quad g_1 = h_1\sqrt{c_1/2} \quad (23)$$

The optimal forces P_3^0 and P_4^0 can be determined in a similar manner. Introducing the integral control we obtain two second-order speed-dependent optimal control laws (14), one of them is for AMB1 and the other for AMB2.

SIMULATION

The simulation model shown in Fig.1, is the flywheel energy storage system prototype [6]. Table 1 shows the main parameters of the system. The simulations are done for the closed-loop system having the 4th, 2nd and 1st-order optimal PIDA (with the acceleration feedback in (14)) and PID (without the acceleration feedback in (14)) controllers. The 1st-order PID controller is the conventional PID controller. In all the cases the Butterworth pole distribution of the radius v_0 at $\omega=0$ is used [6].

TABLE 1: Specification of the simulation model

Parameter	Symbol	Value	Unit
Mass	M	67.2	kg
Moment of inertia equat.	J_1	3.3	kgm^2
axial	J_3	2.12	kgm^2
Location of sensors	z_1	-0.1	m
	z_2	0.35	m
of electromagnets	z_1*	-0.045	m
	z_2*	0.29	m
Air gap length	δ	0.5	mm
Coil inductance	L	0.1	H
Coil resistance	r	1.5	Ω
Bias current	i_c	0.8	A
Nominal rotational speed	ω_n	12000	r.p.m

Due to the integral control and gyroscopic effect the closed-loop system becomes unstable at the bounding rotational speed $\omega=\omega_*$. Fig. 3 shows ω_* as function of v_0. The value of ω_* increases with v_0 and the controller order. For the 4th-order PIDA controller the value of ω_* is greater than 40000 r.p.m.

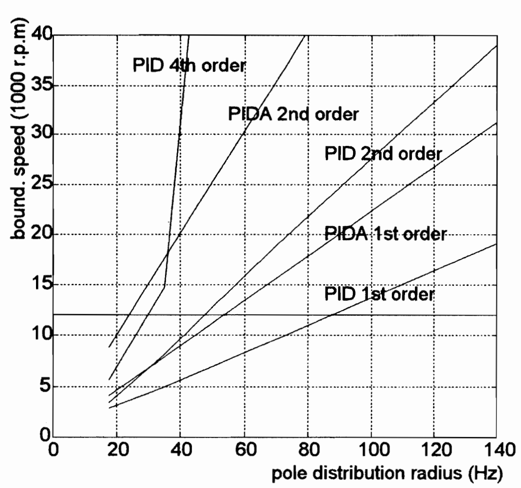

FIGURE 3: The bounding rotational speed as a function of pole distribution radius.

Fig. 4 shows the responses of the most loaded AMB1 on the impulse 1 Ns at the nominal rotational speed 12000 r.p.m.

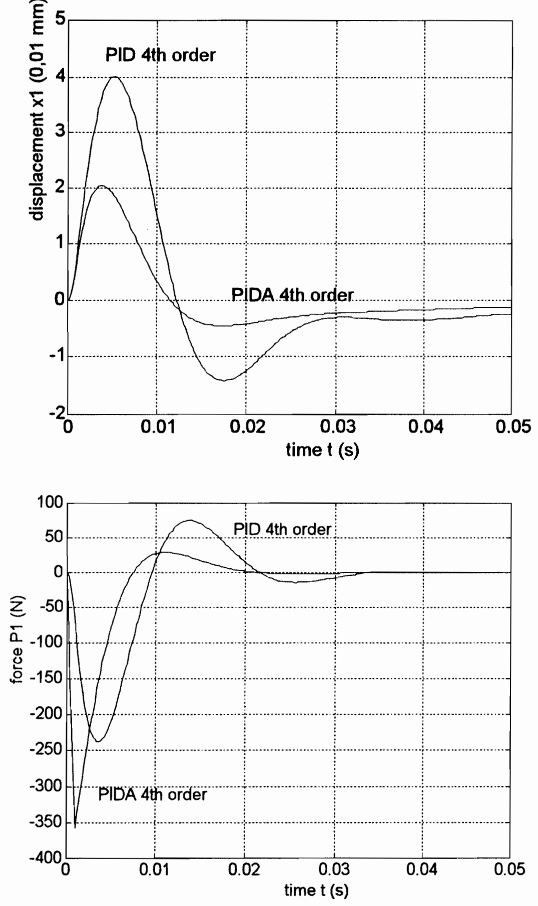

FIGURE 4: Impulse responses of the 4th order PID and PIDA controllers for v_0=50 Hz.

Fig.5 shows the unbalance responses of AMB1 on the 1 μm eccentricity of the center of mass of the rotor. The pole distribution radius v_0 is chozen in such a way that ω_*=1.3ω_n=15600 r.p.m is the same for each system.

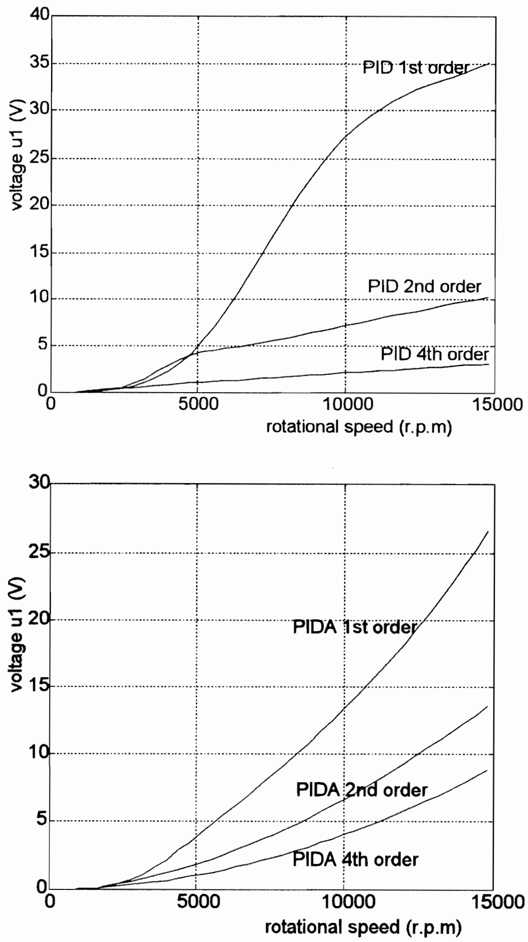

FIGURE 5: The unbalance responses.

It is easily seen that the multi-order controllers are characterizied by significantly smaller control voltages than the 1st-order controller (control forces and currents are approximately proportional to voltages).
Table 2 shows the robustness concerning stability with parameter deviations. We vary the air gap length δ and the rotor mass M (together with the moments of inertia J_1 and J_3) and find the ratio of the changed values $\tilde{\delta}$ and $\tilde{M}$ to the nominal values δ and M at which the system becomes unstable. So, table 2 shows the lower and upper values of the parameters at which the system is stable. We can see that the multi-order controllers have powerful robustness to the parameter variations with the exeption of decreasing the air gap length. That occurs because of increasing the negative stiffness and the relatively small values of v_0. Robustness becomes better with increasing v_0 but the voltages increases, as

well. So, it is necessary to find a compromise.

TABLE 2: Robustness with parameter variations

Controller	ν_0 (Hz)	$\tilde{\delta}/\delta$	$\tilde{M}/M$
PID 1st order	110	0.25 - 1.11	0 - 1.2
PID 2nd order	70	0.073 - 1.18	0 - 1.26
PID 4th order	65	0.32 - 1.48	0 - 1.78
PIDA 1st order	35	0.58 - 1.85	0 - 1.97
PIDA 2nd order	35	0.85 - 3.7	0 - 5
PIDA 4th order	20	0.94 - 6.5	0 - 5
	35	0.59 - 3.1	0 - 5

CONCLUSIONS

The inverse problem of dynamics method is applied to AMB control. This method leads to the simple and physically clear control algorithms and may be applied not only to a linear AMB control but to a nonlinear and time-variant control, too. Using the analytical solution to the LQ optimization problem results in the multi-order speed-dependent control. Such control, as it is seen from the simulations, has a strong robustness for parameter deviations and an unbalance cancellation effect and it is superior to the conventional PID control.

REFERENCES

1. Fujita M., Matsumura F., and Shimizu M., H^∞ Robust Control Design for a Magnetic Suspension System,Proc. of the 2nd Int. Symp. on Magnetic Bearings, University of Tokyo, Japan, 1990.
2. Siegwart R., Larsonneur R., Herzog R., and Traxler A., A Versatile Controller Layout CAD Tool for AMBs, Proc. of the 3rd Int. Symp. on Magnetic Bearings, Technotomic Publishing Co., Virginia, 1992.
3. Nonami K., and Yamaguchi H., Robust Control of Magnetic Bearing Systems by Means of Sliding Mode Control, Proc. of the 3rd Int. Symp. on Magnetic Bearings, Technotomic Publishing Co., Virginia, 1992.
4. Petrov B.N., and Krutko P.D., Inverse Problems of Control System Dynamics. NonLinear Models, Izvestia AN USSR, Tech. Cybernetics, no. 5, 1980, (in Russian).
5. Zhuravlyov Y.N., Optimization of the Force Responses of Controlled Suspension For Gyroscopic Rotor. Rakenteiden Mekaniikka, vol. 24, no. 4, Helsinki Univ. of Tech., 1991.
6. Zhuravlyov Y.N., Active Magnetic Bearings, Report 37, Helsinki Univ. of Tech., 1992.