A NONLINEAR FUZZY CONTROLLER FOR MAGNETIC BEARINGS WITHOUT PREMAGNETIZATION

B. WEIDEMANN and W. XIAO

Institut für Elektrische Energietechnik
Universität Kassel, Wilhelmshöher Allee 71, 34109 Kassel
Germany

ABSTRACT
This paper deals with the development of nonlinear fuzzy controllers. Because of their properties, such as robustness against external disturbing signals, nonlinear behaviour which is desired for a strongly nonlinear system, the possibilities to optimize the controller performances, etc., fuzzy controllers can be well used in such complex systems as magnetic bearings without premagnetization. Besides the discussions about adjusting the controllers, statements concerning the practical realization will be made.

INTRODUCTION
In principle, magnetic bearings can be divided in two groups, that is magnetic bearings with and without premagnetization. The controller for magnetic bearings which are linearized by means of premagnetization can be designed with a simple linear structure. However, this concept has the following disadvantages:

This type of magnetic bearing requires a structural measure of premagnetization which causes additional loss of energy. Moreover, it is confined to a linear controller and the possibilities to improve the system performance with regard to various disturbing signals are thus limited.

Compared with the linearized ones, magnetic bearings without premagnetization do not only have a more simple construction but also a higher efficiency. Owing to the strong nonlinear relation among the magnetic force, the displacement of the shaft, and the regulating current it is necessary to make use of a nonlinear controller. A unified method cannot be found for the synthesis of nonlinear controllers. The controller structure and the difficulty degree of the design are different from plant to plant. Two approaches are, for example, given by "exact linearization"[1] and "dynamic programming with the help of the Lyapunov-function"[2].

A general nonlinear model of controllers can be obtained by utilizing fuzzy logic. The fuzzy controller is rule-based and offers more possibilities than traditional ones for the realization of a well-performed closed control system. If the plant cannot be described mathematically, the experiences of experts, linguistically expressed, can be integrated in the fuzzy controller, which is almost impossible for conventional approaches. Using the experiences of experts it is possible to develop a Self-Organizing Controller (SOC). If enough information about the plant is available, the optimal fuzzy controller can be constructed by means of numerical methods, such as dynamic programming, since an arbitrary function can be formed approximately with the help of a set of fuzzy equations.

This paper describes a fuzzy controller with decentralized structure which is applied to magnetic bearings without premagnetization. In order to improve the performance of the controller, the possibilities of adjusting the controller are discussed.

THE DECENTRAL FUZZY CONTROLLER
It is assumed that a multi-variable system has n variables $x_i \in \chi_i$, $i=1,2...,n$, and m inputs $u_j \in \upsilon_j$, $j=1,2,...,m$ (they are outputs of the controller), where χ_i and υ_j denote the corresponding universes of discourse which are discretized with the dimensions $Dim(\chi_i) = l_i$ und $Dim(\upsilon_j) = o_j$, respec-

tively. Therefore, for each $x_i \in \chi_i$ ($u_j \in \upsilon_j$) we get a unique fuzzy vector

$$X_{fi} = \left[\mu_{\tilde{A}1}(x_i)\ \mu_{\tilde{A}2}(x_i)\ \dots\ \mu_{\tilde{A}l_i}(x_i)\right]$$

$$\left(U_{fj} = \left[\mu_{\tilde{B}1}(u_j)\ \mu_{\tilde{B}2}(u_j)\ \dots\ \mu_{\tilde{B}o_j}(u_j)\right] \right) .$$

With these vectors the rule base can be described as follows:

IF $X_{f1} = [\mu_{\tilde{A}1}(x_1)\ \mu_{\tilde{A}2}(x_1)\ \dots\ \mu_{\tilde{A}m1}(x_1)]^i$

AND IF $X_{f2} = [\mu_{\tilde{A}1}(x_2)\ \mu_{\tilde{A}2}(x_2)\ \dots\ \mu_{\tilde{A}m2}(x_2)]^i$

AND IF ...

AND IF $X_{fn} = [\mu_{\tilde{A}1}(x_n)\ \mu_{\tilde{A}2}(x_n)\ \dots\ \mu_{\tilde{A}mn}(x_n)]^i$

THEN $U_{fj} = [\mu_{\tilde{B}_1}(u_j)\ \mu_{\tilde{B}_2}(u_j)\ \dots\ \mu_{\tilde{B}_{o_j}}(u_j)]^i$

$$i = 1,2,\dots,n_r \qquad (1)$$

Including the rule base, the general model of fuzzy controllers is given by a system of fuzzy equations

$$B_{ij} = \Theta(R_j, A_i), \quad i=1,2,\dots,n_r,\ j=1,2,\dots,m \qquad (2)$$

where $A_i = (X_{f1} \times_{t_1} X_{f2} \times_{t_1} \dots \times_{t_1} X_{f_n})^{(i)}$ and

$B_{ij} = U_{fj}^{(i)}$. The system of fuzzy equations can take the form of $B = A \circ_t R$ as well as of $B = A \triangleright_t R$ which are defined as follows:

$$B = A \circ_t R = {}_{def}\{y \| \exists x (x \varepsilon A \wedge_t (x,y) \varepsilon R)\} \qquad (3)$$

$$B = A \triangleright_t R = {}_{def}\{y \| \forall x (x \varepsilon A \rightarrow_t (x,y) \varepsilon R)\} \qquad (4)$$

where $\wedge_t$ denotes the conjunction operator which is connected to the t-norm **t** and $\rightarrow_t$ the implication operator which is defined via a Φ_t-operator connected to a t-norm **t** [3].

For the general model of fuzzy controllers there are $l_1 \cdot l_2 \cdot \dots \cdot l_n$ to be set up. The fuzzy relation contains $l_1 \cdot l_2 \cdot \dots \cdot l_n \cdot o_j$ elements. Thus the computing time both for the establishment of the fuzzy relation R and for the fuzzy controller increases with the order of the system. On this point of view the fuzzy controller with the general structure is not of practical use for the magnetic bearing system with 9 variables (8 state variables and 1 rotational speed of the shaft).

It is assumed that the whole system is divided into subsystems; for each a local fuzzy controller is constructed which works only according to the information from the subsystem. A control system with decentral fuzzy controllers is thus obtained. It is pointed out that the coupling between the subsystems will be treated as disturbing signals. Similar to the general model, the decentral fuzzy controller can be expressed by the following set of equations:

$$B_{ij} = \Theta(R_{dj}, A_{di}), \quad i=1,2,\dots,n_{dr},\ j=1,2,\dots,m \qquad (5)$$

with $A_{di} \subset A_i$, $R_{dj} \subset R_j$ and $n_{dr} < n_r$.

This structure shows at first the simplicity which makes it possible to realize fuzzy control on multivariable systems. Furthermore, the boundary conditions which are constituted by the rule base in eq. (1) can also be satisfied by the system of fuzzy equations. The same is valid for the general model of fuzzy controllers. The decentral fuzzy controller formed in this way can be improved in several stages:

1. **Selection of the suitable system of fuzzy equations.** Generally a more continuous description is given by the fuzzy equation $B = A \circ_t R$ than that given by $B = A \triangleright_t R$. In order to achieve an accurately adjusted fuzzy controller, the type of fuzzy equation $B = A \circ_t R$ is suggested to be used.

2. **Choice of a t-norm t.**
For a certain rule base the output of a fuzzy controller can be affected by different **t**-operators. Of course, the choice of a **t**-operator is not based on the continuity of the controller function but rather on the performance of the fuzzy controller. Normally the computing time which is caused by the **t**-operator plays an important role in the determination. For this reason the t-norm $\mathbf{t}_G$ = min is often used.

3. **Adjustment of the membership functions.**
Due to the unique solution of a fuzzy controller, the convex fuzzy sets should be considered. A fuzzy set $\tilde{A} \in \mathcal{F}(\xi)$ is called convex fuzzy set, if for the real values $x<y<z$ the condition

$$\mu_{\tilde{A}}(y) \geq \min\left(\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(z)\right) \quad (6)$$

is fulfilled. A membership function can be described with various forms, such as triangle function, trapezium function, single-tone function, normal distribution, gauß-distribution, etc.. Because of its simplicity, the triangle function becomes the favourite one in describing membership functions. In addition, the position of membership functions can be adjusted to improve the performance of the fuzzy controller. It is of great importance to minimize the complexity of the controller structure. As shown in figure 1, the distribution of the membership functions can be represented by the discretizing vector $\left[x_{imin}\ x_{i1}\ x_{i2}\ \cdots\ x_{il_i-2}\ x_{imax}\right]$.

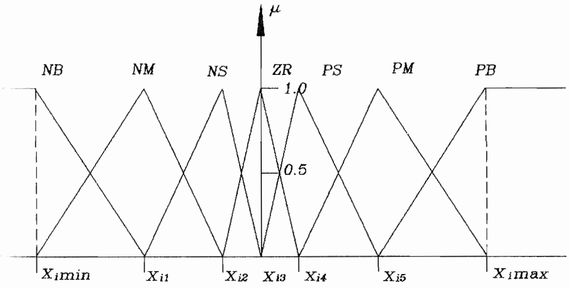


Fig. 1 The distribution of membership functions

Hence the purpose of adjusting the position of membership functions is to determine the discretizing vector $\left[x_{imin}\ x_{i1}\ x_{i2}\ \cdots\ x_{il_i-2}\ x_{imax}\right]$. In practice, the adjustment leads to a nonlinear transformation of the real variable x_i.

4. **Alternation of the fuzzy relation.** In the system of fuzzy equations the fuzzy relation plays a decisive role in determining the behaviour of the fuzzy controller. The modification of the fuzzy relation R leads to a self-organizing fuzzy controller. However, it requires an effective supervision of the control process as well as a self-organizing algorithm. For a principal application to magnetic bearing systems the approach will not be utilized in this paper.

APPLICATION TO MAGNETIC BEARING SYSTEMS

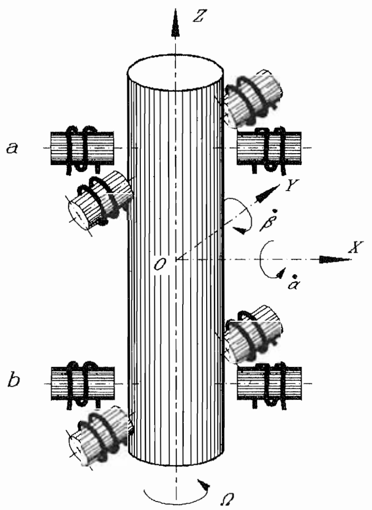


Fig. 2 Magnetic bearing without premagnetization

The magnetic bearing without premagnetization, whose rotor is modelled as a rigid body, was designed with the following structural data:

number of pole-pair p = 4;
air gap δ = 0.5 mm;
inner diameter of the stator D_i = 200 mm;
outer diameter D_a = 345 mm;
bearing length L = 60 mm;
width of pole-face arc H_p = 74.5 mm;
slot depth N_t = 35.2 mm;
maximal magnetomotive force Θ_s = 700 A.

One of the difficulties in the development of magnetic bearing systems is the nonlinearity of the plant, that is the nonlinear relation between magnetic force, regulating current, and displacement, $\vec{f} = \vec{f}(\vec{i},\vec{e})$. The following figures show the characteristic curves, which were obtained with the help of the finite-element method [4].

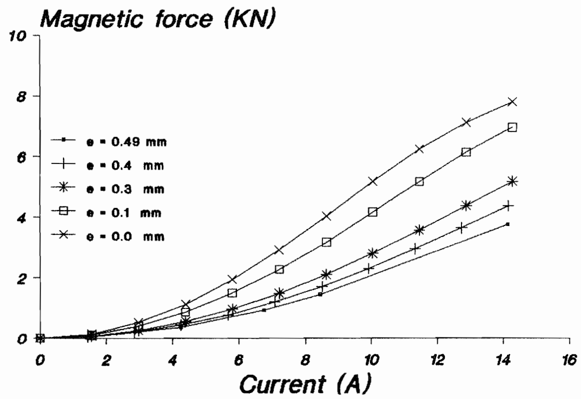


Fig. 3 Magnetic force as a function of current; e denotes the displacement, the angle of the current vector = 45° and that of the displacement vector = 225°.

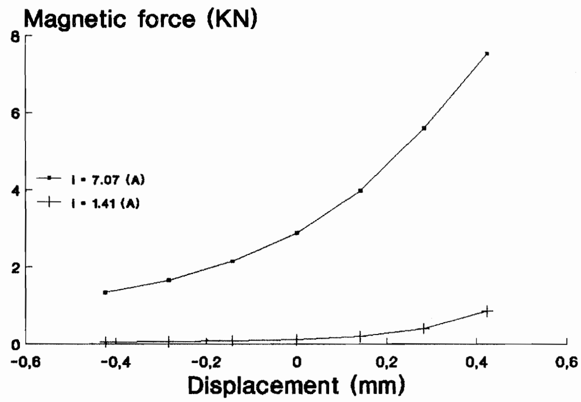


Fig. 4 Magnetic force as a function of displacement; **i** denotes the current.

The system of fuzzy equations

$$B_j = A_d \circ_t R_j, \; j=1,2,\ldots,n_{dr} \,, \tag{7}$$

where $A_d = X_{f1} \times_{t_1} X_{f2}$ and **t** = **t**$_1$ = min, describes the decentral fuzzy controller with two variables for the magnetic bearing.

The displacement and the velocity of the rotor shaft are taken as controller variables, which are discretized by 5 and 3 fuzzy sets, respectively. Thus,there are altogether 15 rules to be established. Due to its simplicity the triangle function was used for the fuzzification. Therefore, the rule base is expressed by the fuzzy vectors in the table 1.

X_{f1}	X_{f2}	U_{fj}
0 0 0 0 1	0 0 1	1 0 0
0 0 0 0 1	0 1 0	0.9 0.1 0
0 0 0 0 1	1 0 0	0 1 0
0 0 0 1 0	0 0 1	1 0 0
0 0 0 1 0	0 1 0	0.9 0.1 0
0 0 0 1 0	1 0 0	0 0.8 0.2
0 0 1 0 0	0 0 1	0.7 0.3 0
0 0 1 0 0	0 1 0	0 1 0
0 0 1 0 0	1 0 0	0 0.3 0.7
0 1 0 0 0	0 0 1	0.2 0.8 0
0 1 0 0 0	0 1 0	0 0.1 0.9
0 1 0 0 0	1 0 0	0 0 1
1 0 0 0 0	0 0 1	0 1 0
1 0 0 0 0	0 1 0	0 0.1 0.9
1 0 0 0 0	1 0 0	0 0 1

Table 1 Rule base for the decentral fuzzy controller with two variables

The discretizing vectors for the fuzzy controller of he magnetic bearing are given as follows:

Inputs
for the displacement
$x \Rightarrow [-8.0\times10^{-5} \;\; -4.0\times10^{-5} \;\; 0 \;\; 4.0\times10^{-5} \;\; 8.0\times10^{-5}]$

for the velocity

$\dot{x} \Rightarrow [-5.0\times10^{-2} \;\; 0 \;\; 5.0\times10^{-2}]$

Outputs
for all of the regulating magnetomotive forces
$u_j \Rightarrow [-700.0 \;\; 0 \;\; 700.0]$, j=1,2,3,4.

The characteristic surface of the fuzzy controller is thus shown in Fig. 5.

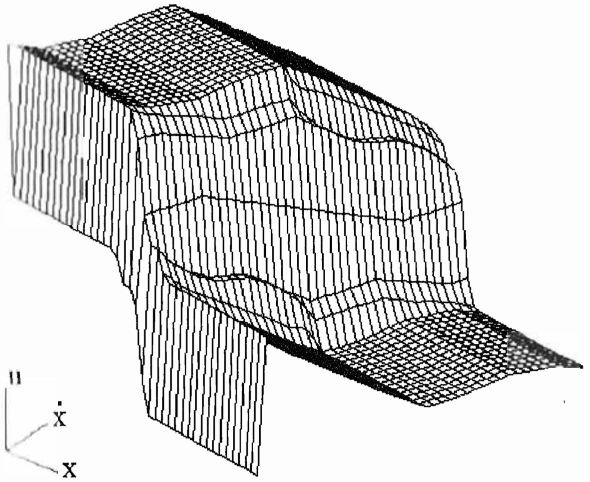


Fig. 5 Characteristic surface of the fuzzy controller.

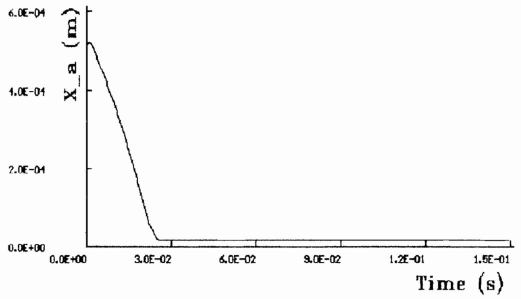
The building-up process of the rotor shaft was simulated; there the initial conditions of the magnetic bearing a (see Fig. 2) are given as follows:

initial displacement $x_a = 5.0 \times 10^{-4}$ $[m]$

initial velocity $\dot{x}_a = 5.0 \times 10^{-2} \left[\frac{m}{s}\right]$

constant disturbing force at the x axis
$F_{dis} = 2000$ N

The simulating results for a rotational speed range from 0 to 100000 rpm are shown in figures 6-8.



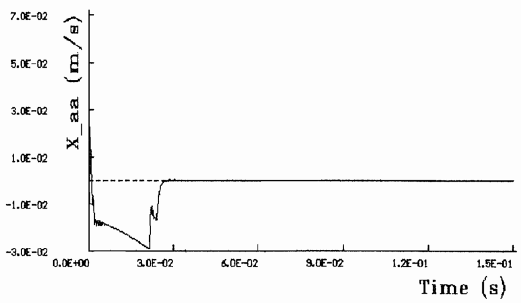
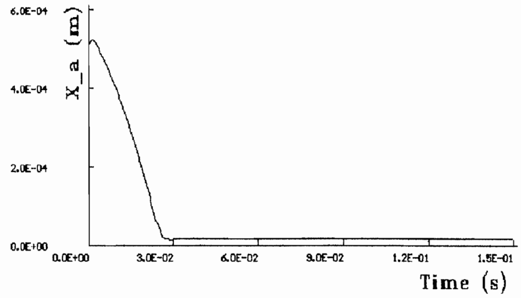


Fig. 6 Magnetic bearing a: rotational speed = 0 rpm.



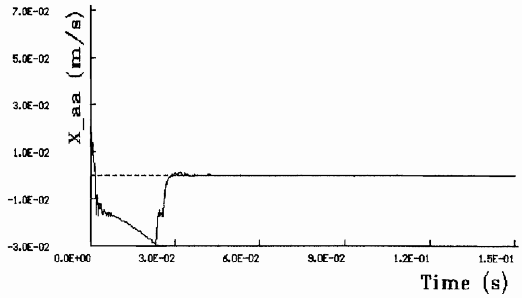
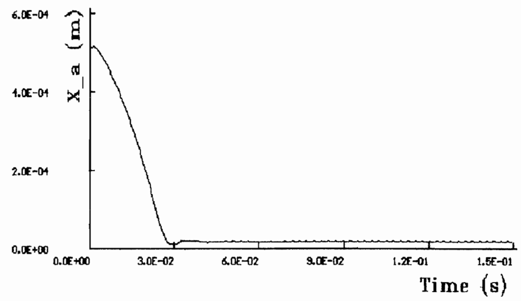


Fig. 7 Magnetic bearing a: rotational speed = 50000 rpm.



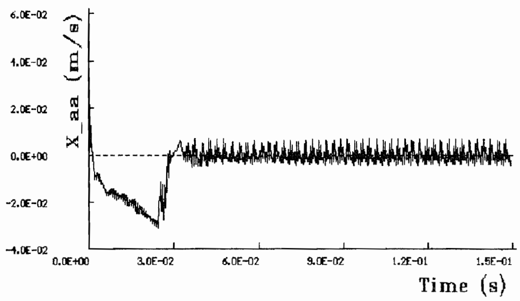


Fig. 8 Magnetic bearing a: rotational speed = 100000 rpm.

CONCLUSIONS

Fuzzy logic offers the possibility to develop a nonlinear fuzzy controller for the magnetic bearing system without premagnetization. By adjusting the rule base and the distribution of the membership functions, the controller performance can be improved. It follows from the discretizing vectors and the characteristic surface given above that the fuzzy controller for magnetic bearings without premagnetization works in a way of a saturation function. The displacement and the velocity of certain values are assumed as a limit set, where an especially small value is taken for the displacement. If the limit is exceeded, the output of the controller is then saturated. Only within the limit area the controller works fine and continuously.

The fuzzy controller is expressed by a general system of fuzzy equations, the so-called boundary conditions can therefore be satisfied. Owing to the simplicity, the t-norm $\mathbf{t}_G$ = min is chosen as the conjunction operator.

According to the statement of Omron Corporation [5] in 1991 about the digital fuzzy processor FP-3000,which has an external clock speed of 24 MHz, the operating time amounts to 650 μs, where 20 rules with 5 variables and 2 outputs are included and the center-of-gravity method is utilized for defuzzification. With a quick signal processor, such as the TMS 320C40 (external clock speed : 50 MHz), and the defuzzification by the fuzzy-mean method it is possible to apply the decentral fuzzy controller discussed above to magnetic bearing systems practically. It is pointed out that the fuzzy-mean method is much simpler than the center-of-gravity method. Therefore, by using the fuzzy-mean method much computing time can be saved.

The coupling between the subsystems is treated as disturbing signals. The robustness of the controller depends not only on the coupling strength but also on the controller construction. The simulation has shown that the decentral fuzzy controller for magnetic bearing systems without premagnetization remains stable in a wide range of rotational speed (0-100000 rpm), where nearly the same building-up time is achieved for different rotational speeds.

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