DISTURBANCE OBSERVER BASED CONTROLLER FOR FLEXIBLE ROTOR SUPPORTED BY MAGNETIC BEARINGS

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ABSTRACT

A simple bending vibration control technique is introduced which is based on a disturbance observer. The bending force of shaft is estimated which is fedback to each magnet. Combining the local PD feedback to this bending force feedback, the damping of the bending vibration is improved. This technique requires only the collocated sensor and is applied to three mass rotor which is supported by two radial magnetic bearings. The system is simulated on a computer and the controller parameters are designed; so that, the resonant vibration is well reduced. The proposed system indicates good combination of high stiffness and good damping for the flexible rotor compared to the traditional PD controller. A simple experimental setup is made to confirm the validity of the proposed system.

INTRODUCTION

Magnetic bearing has a noncontact supporting capability which is adequate for a highspeed rotor [1]. Sometimes the system becomes unstable when the rotating speed increases to the resonant frequency of the bending mode. Several control techniques have been proposed and tested; for example, modal control, H^{∞} control and sliding mode control. However, the controller becomes complicated [2],[3],[4].

This paper proposes a simple bending vibration control technique based on a disturbance observer. In the previous work, the same observer has been applied to the rigid rotor [5]. The disturbing force to the rigid rotor was canceled out. The rotor was supported rigidly and the system became robust to the parameter changes. In this paper, this observer based controller is applied to increase the damping to the bending vibration of the three mass flexible rotor. This technique is originally developed and reported by Yuki, et. al. for controlling the elastic manipulator [6]. The similar method is applicable to the flexible rotor which is supported by two radial magnetic bearings. If the bending force of shaft is fedback to each magnet, the bending vibration of the rotor can be controlled. This bending force is easily estimated by a simple observer. Combining the local PD feedback and the bending force feedback, the damping of the bending vibration is improved.

The proposed observer based controller is analytically designed on a simplified rotor dynamics. Actually two bearing controllers interact adversely each other. Also the observer has its dynamics, hence the system is simulated on a computer and the controller parameters are designed; so that, the resonant vibration is well reduced with the bending force feedback. The proposed system indicates good combination of stiffness and damping for the bending mode compared to the traditional PD controller. A simple experimental setup is made to confirm the validity of the proposed system.

FLEXIBLE ROTOR AND OBSERVER BASED CONTROLLER

In this paper, three mass rotor is treated which is supported by two magnetic bearings. The system is shown schematically in Fig. 1. For simplicity, rotor is assumed to be symmetric to the midspan mass m_c. Active vibration control scheme is introduced.

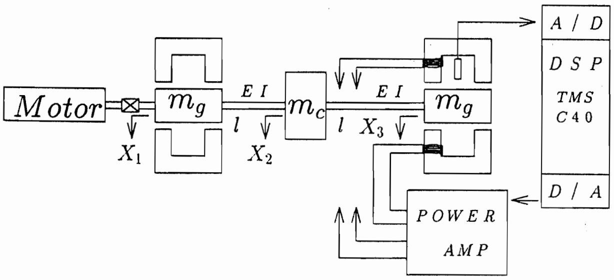

FIGURE 1: Flexible rotor and control system

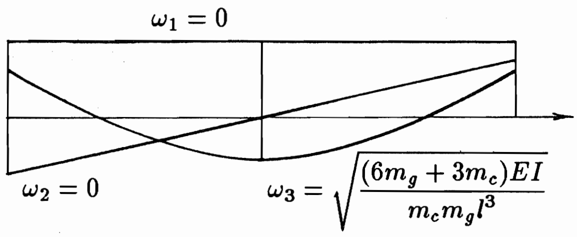

FIGURE 2: Vibrating modes

Modeling of the flexible rotor

Equation of vertical motion of the shaft in matrix form is written by

$$\begin{bmatrix} m_g & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_g \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} \frac{3EI}{2l^3} & -\frac{3EI}{l^3} & \frac{3EI}{2l^3} \\ -\frac{3EI}{l^3} & \frac{6EI}{l^3} & -\frac{3EI}{l^3} \\ \frac{3EI}{2l^3} & -\frac{3EI}{l^3} & \frac{3EI}{2l^3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \quad (1)$$

where

m_g	:	mass of bearing disk [0.39 kg]
m_c	:	mass of center disk [1.35 kg]
l	:	length between disks [0.2 m]
EI	:	stiffness of shaft [41.21 Nm^2]
x_1, x_2, x_3	:	displacement of each disks [m]
f_1, f_2, f_3	:	applied force to each disks [N]

If the stiffness of the shaft is high enough not to cause any bending vibration, the controller design is easy. Derivative gain of the PD controller improves the system damping well. For the flexible rotor, however, we need to analyze Eqn. (1) to solve the resonant frequencies. Then we have $\omega_1 = \omega_2 = 0$ and $\omega_3 = \sqrt{(6m_g + 3m_c)EI/m_c m_g l^3}$. The corresponding modes are shown in Fig. 2. The third mode is difficult to be controlled.

Bending force estimator

For simplicity, the rotor is assumed to be slender and each end of bearing controller can be designed individually. Suppose that the right end is simply supported and the left bearing controller can be designed individually. Then we have the following equations:

$$m_g \ddot{x}_1 = f_1 - f_k \quad (2)$$
$$m_c \ddot{x}_2 = 2f_k \quad (3)$$

where f_k is the bending force of the shaft and is given by

$$f_k = \frac{3EI}{2l^3}(x_1 - 2x_2) = k(x_1 - 2x_2) \quad (4)$$

Notice that the external force is ignored and only f_k is acting on the bearing disk. The collocated gap sensor detects the displacement x_1. From this displacement and the actuator force, the bending force can be estimated. The actuator force is given by

$$f_1 = K_f i + k_m x \quad (5)$$

where K_f is the force factor [N/A] and k_m is the negative stiffness of magnetic bearing [N/m]. The mass m_g and the force factor K_f are considered to be the addition of nominal values and their variations.

$$m_g = m_{gn} + \Delta m_g \quad (6)$$
$$K_f = K_{fn} + \Delta K_f \quad (7)$$

Then Eqn.(2) can be expanded as

$$m_{gn}\ddot{x} = K_{fn} i - \{f_k + \Delta m_g \ddot{x} - \Delta K_f i - k_m x\} = K_{fn} i - f_{kt} \quad (8)$$

where f_{kt} is the total bending force

$$f_{kt} = f_k + \Delta m_g \ddot{x} - \Delta K_f i - k_m x \quad (9)$$

This bending force can be estimated using the reduced order estimator [5],[6].

RESONANCE RATIO CONTROL

The bending force feedback can control the resonance ratio of the elastic shaft [6]. Combining the bending force feedback with local PD feedback, the bending vibration is well controlled. This method only requires the collocated sensor.

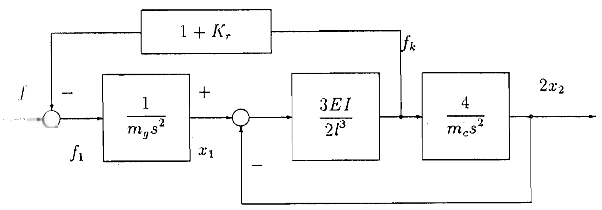

FIGURE 3: Block diagram of bending force feedback

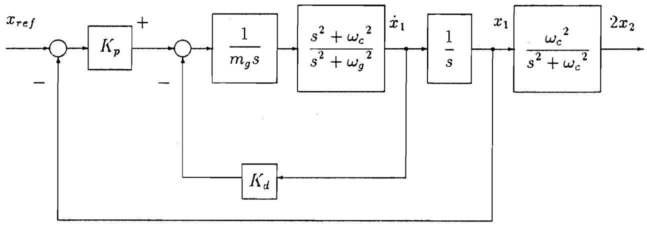

FIGURE 4: Block diagram of local PD feedback

Bending force feedback

Let us consider that the bending force f_k is estimated and fedback to the actuator force f_1:

$$f_1 = -K_r f_k \tag{10}$$

Then we have the following equations:

$$m_g \ddot{x}_1 = -(K_r+1) f_k \tag{11}$$

$$\frac{m_c}{4}(2\ddot{x}_2) = f_k \tag{12}$$

$$f_k = \frac{3EI}{2l^3}(x_1 - 2x_2) \tag{13}$$

The block diagram of this system is shown in Fig. 3. The transfer function from the virtual force f to the displacements x_1 and x_2 is defined as:

$$\frac{x_1}{f} = \frac{m_c s^2 + 4k}{s^2(m_g m_c s^2 + 4m_g k + m_c k(1+K_r))} \tag{14}$$

$$\frac{2x_2}{x_1} = \frac{4k}{m_c s^2 + 4k} \tag{15}$$

Define the resonant frequencies ω_c and ω_g as

$$\omega_c = \sqrt{\frac{4k}{m_c}} \tag{16}$$

$$\omega_g = \sqrt{\frac{4m_g k + m_c k(1+K_r)}{m_c m_g}} = \alpha\omega_c \tag{17}$$

where α is the resonance ratio and is given by

$$\alpha = \sqrt{1 + \frac{m_c(1+K_r)}{4m_g}} \tag{18}$$

Notice that ω_c is determined by the physical parameter of the shaft, while ω_g can be controlled by the bending force feedback K_r. By using the resonant frequency of Eqns. (16) and (17), Eqns. (14) and (15) are modified:

$$\frac{x_1}{f} = \frac{s^2+\omega_c^2}{m_g s^2(s^2+\omega_g^2)} \tag{19}$$

$$\frac{2x_2}{x_1} = \frac{\omega_c^2}{s^2+\omega_c^2} \tag{20}$$

Local PD feedback

For the bearing controller, local PD feedback and the bending force feedback is considered.

$$f_1 = -K_p x_1 - K_d \dot{x}_1 - K_r f_k \tag{21}$$

The system block diagram is indicated in Fig. 4. Then the transfer functions are given by

$$\frac{x_1}{x_{ref}} = \frac{K_p(s^2+\omega_c^2)}{\Delta} \tag{22}$$

$$\frac{2x_2}{x_{ref}} = \frac{K_p \omega_c^2}{\Delta} \tag{23}$$

where Δ is the characteristic equation of the system. We can select the poles by determining the resonant frequencies ω_1 and ω_2 and the damping ratios ζ_1 and ζ_2 as

$$\begin{aligned}\Delta &= m_g s^4 + K_d s^3 + (m_g \omega_g^2 + K_p)s^2 \\ &+ K_d \omega_c^2 s + K_p \omega_c^2 \\ &= m_g(s^2 + 2\zeta_1\omega_1 s + \omega_1^2) \\ &\times (s^2 + 2\zeta_2\omega_2 s + \omega_2^2)\end{aligned} \tag{24}$$

Then we have the following relations:

$$K_d = 2m_g(\zeta_1\omega_1 + \zeta_1\omega_2) \tag{25}$$

$$K_p + m_g\omega_g^2 = m_g(\omega_1^2 + \omega_2^2 + 4\zeta_1\zeta_2\omega_1\omega_2) \tag{26}$$

$$K_d\omega_c^2 = 2m_g(\zeta_1\omega_1\omega_2^2 + \zeta_2\omega_2\omega_1^2) \tag{27}$$

$$K_p\omega_c^2 = m_g\omega_1^2\omega_2^2 \tag{28}$$

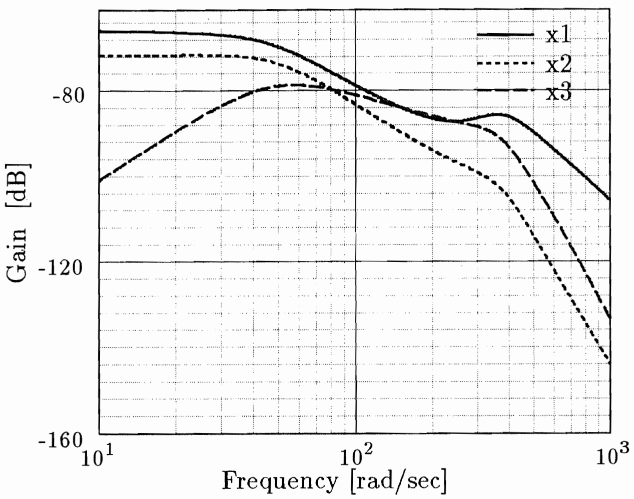

FIGURE 5: Simulated frequency response

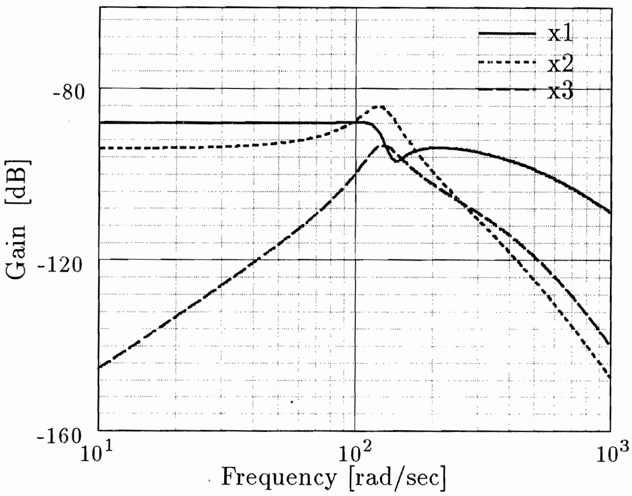

FIGURE 6: Simulated frequency response

Notice that ω_2 is not independent and determined by ω_1. Hence the design equations of the controller are given as the followings:

$$\omega_2 = \frac{-\zeta_2({\omega_1}^2 - {\omega_c}^2)}{2\zeta_1\omega_1} + \frac{\sqrt{{\zeta_2}^2({\omega_1}^2 - {\omega_c}^2)^2 + 4{\zeta_1}^2{\omega_1}^2{\omega_c}^2}}{2\zeta_1\omega_1} \quad (29)$$

$$K_d = 2m_g(\zeta_1\omega_1 + \zeta_2\omega_2) \quad (30)$$

$$K_p = m_g{\omega_1}^2{\omega_2}^2/{\omega_c}^2 \quad (31)$$

$$\omega_g = \sqrt{-\frac{{\omega_1}^2{\omega_2}^2}{{\omega_c}^2} + {\omega_1}^2 + {\omega_2}^2 + 4\zeta_1\zeta_2\omega_1\omega_2} \quad (32)$$

Determination of quadruple poles

One of the best gain selection is that the closed loop has quadruple poles so that the damping is good and the bandwidth is wide. Insert $\zeta_1 = \zeta_2 = 1.0$ and $\omega_1 = \omega_2$ into Eqn.(24), we have

$$0 = (\omega_1\omega_2 - {\omega_c}^2)(\omega_1 + \omega_2) \quad (33)$$

Then, we get the following resonance ratio and the feedback gains

$$\alpha = \sqrt{5} = 2.236 \quad (34)$$

$$K_r = \frac{16m_g - m_c}{m_c} \quad (35)$$

$$K_p = m_g{\omega_c}^2 \quad (36)$$

$$K_d = 4m_g\omega_c \quad (37)$$

SIMULATION

The previous solution is the simplified one where each bearing controller can be designed individually. Actually, two magnetic bearings interact adversely. Also the bending force should be estimated with the 2nd order observer. Hence the system is simulated on a computer to evaluate the proposed control method and to determine the practical feedback gains.

Equation (1) is expanded to the state equation

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{m}^{-1}\mathbf{k} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{v} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{m}^{-1} \end{bmatrix} \mathbf{f} \quad (38)$$

The following reduced-order observer is designed to each control freedom of magnetic bearings

$$\begin{bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{bmatrix} = \begin{bmatrix} -l_1 & -\frac{1}{m_n} \\ -l_2 & 0 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} + \begin{bmatrix} \frac{K_{fn}}{m_n} \\ 0 \end{bmatrix} f + \begin{bmatrix} -l_1^2 - \frac{l_2}{m_n} \\ -l_1 l_2 \end{bmatrix} x \quad (39)$$

$$\begin{bmatrix} \hat{x} \\ \hat{v} \\ \hat{f}_{dt} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} + \begin{bmatrix} 1 \\ l_1 \\ l_2 \end{bmatrix} x \quad (40)$$

where η_1 and η_2 are the internal variables, and l_1 and l_2 are the design parameters. l_1 and l_2 are determined by selecting the observer poles at $-1,000$. The magnetic bearings are controlled by the collocated feedback

$$f = -K_p\hat{x} - K_d\hat{v} - K_r\hat{f}_{kt} \quad (41)$$

These equations are simulated on Macintosh-Matlab to evaluate the proposed method. Inserting

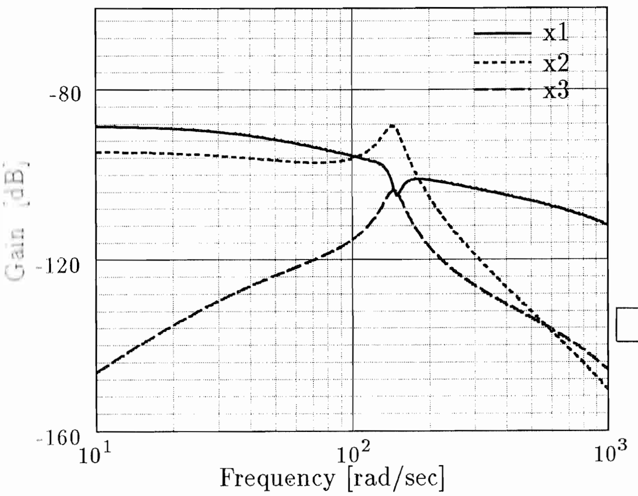

FIGURE 7: Simulated frequency response

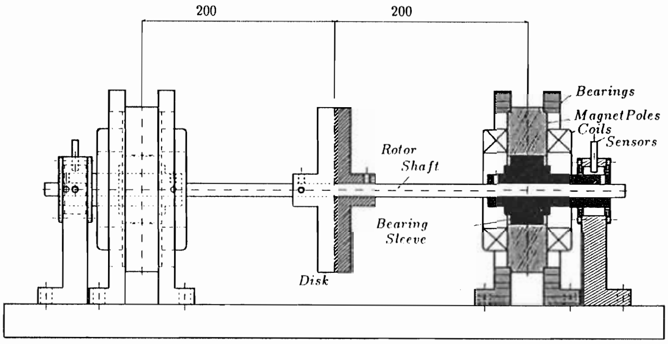

FIGURE 8: The experimental setup

the physical parameters into Eqns. (35), (36) and (37), we have the control gains $K_p = 8892$, $K_d = 236$ and $K_r = 3.62$. The simulated frequency responses are shown in Fig. 5. The responses are calculated by enforcing the left-end bearing disk and evaluating the three displacements x_1, x_2 and x_3. In the previous simple analysis, the feedback gains are determined to have the quadruple poles. The simulated results, however, have the real poles of -40.4, -564 and the complex conjugate poles of $\omega_1 = 49.9$, $\zeta_1 = 0.6$ and $\omega_2 = 380$, $\zeta_2 = 0.3$. This is considered because that two magnetic bearings are affect adversely. The static response is -66 dB (the equivalent stiffness is 1,995 N/m) which is not enough to support the rotor weight.

The bending force feedback affects adversely the bearing stiffness. Hence the feedback gains are determined through simulation such that the PD gains K_p, K_d are relatively large and the bending force feedback K_r is small. One example of $K_p = 100{,}000$ $K_d = 600$ and $K_r = 3$ gives us relatively good responses which are shown in Fig. 6. The worst resonant pole is $\omega_1 = 126$, $\zeta_1 = 0.2$, which means relatively good damping. The static response is -88 dB (the equivalent stiffness is 25,118 N/m), which can support the rotor weight. Figure 7 indicates the response of $K_p = 26{,}000$, $K_d = 600$ and $K_r = 0$. This is the PD controller with the same static response of the previous one. The midspan disk (x_2) indicates the resonant peak of $\omega = 143$ and $\zeta = 0.1$. This means that only PD feedback cannot improve both of stiffness and damping of the bending vibration.

EXPERIMENTAL RESULTS AND CONSIDERATIONS

To confirm the proposed method, a test apparatus is made, as shown schematically in Fig. 8. The observer based controller of Eqns. (39), (40) and (41) is designed to each control freedom of magnetic bearings x_1, y_1, x_3 and y_3. They are converted to the digital controller by the bilinear transformation with the sampling interval of $\tau = 10\mu$ s and installed in a DSP (TMS320C40). The unbalance responses are shown in Figs. 9 and 10. The controller gains are adjusted experimentally. The experimental bending force controller has the gains about $K_p = 50{,}000$, $K_d = 200$ and $K_r = 0.5$, while the PD controller means that the bending force feedback is changed to $K_r = 0$ without changing the PD values K_p and K_d. In the both figures, the peaks of rigid modes are not recognized. However, the bending mode has the peak at the rotating speed near 1,500 rpm. This bending mode peak is reduced half by adding the bending force feedback. However, the effect is limited to this level, because of our poor experimental setup. The dynamic response will be improved by improving the experimental setup.

CONCLUSIONS

A simple observer based controller is introduced to improve the dynamic property of the flexible rotor supported by magnetic bearings which is applied to the 3 mass rotor. The system only requires the collocated sensor. The controller parameters are designed on a computer simulation. The responses indicate a good combination of bearing stiffness and bending mode damping. Experimental results also clarify the validity of the proposed method, but the effect is limited because of our poor experimental setup. Further

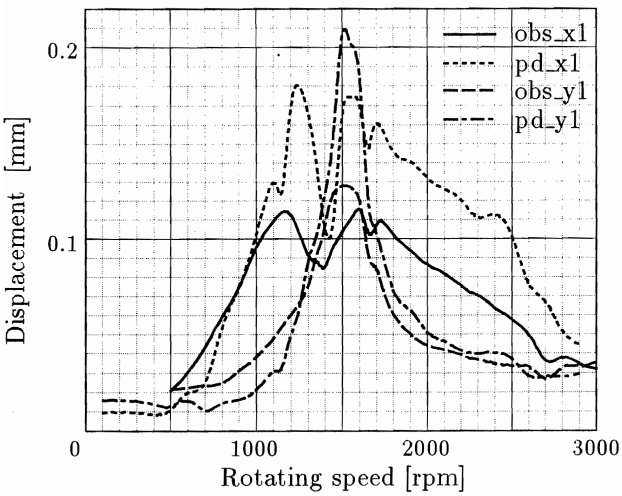

FIGURE 9: The unbalance response of outboard disk

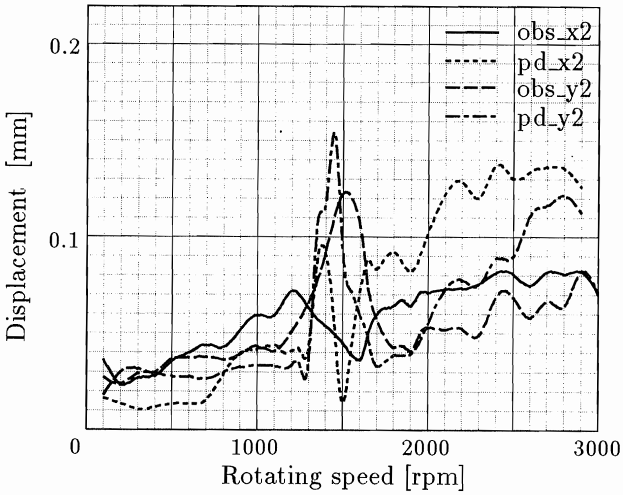

FIGURE 10: The unbalance response of inboard disk

work is continuing to clarify the effects and the limitation of the proposed method.

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