Adaptive Forced Balancing for Magnetic Bearing Systems

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ABSTRACT This paper presents an autobalancing algorithm with several unique features that differentiate it from traditional (notch filter) and model based autobalancing algorithms. The algorithm, called Adaptive Forced Balancing with Frequency Tracking (AFB/FT), has the following features: 1) there is no effect on control system bandwidth or stability, 2) a dynamic model of the magnetic bearing system is not needed, and 3) regulation of the entire shaft-actuator-control system in a minimum energy state is achieved. This algorithm includes a frequency tracking section which automatically estimates the shaft rotational rate. The combined algorithm simultaneously tracks the amplitude, phase and frequency of a synchronous disturbance in real-time. AFB/FT was implemented on a TI Digital Signal Processor (DSP) as part of a complete control system designed to support a single end suspension test rig. This paper presents the AFB/FT algorithm, describes the test rig and control system components and presents test data which demonstrates the effectiveness of the algorithm. Preliminary results show that AFB/FT provides at least 30 dB reduction in actuator control current at 3600 rpm.

INTRODUCTION

The application of magnetic bearings to rotating systems provides a means to actively control bearing stiffness and damping which, in turn, can modify the dynamic response of the system. The use of a magnetic bearing actuator also permits the compensation of rotor mass unbalance. This "compensation" can be applied in two distinct ways causing the shaft to either rotate around its center of mass or center of geometry, depending on the desired result. Shaft rotation around the center of mass termed *autobalancing* is the subject of this research.

A technique to accomplish autobalancing called Adaptive Forced Balancing (AFB) has been developed, implemented and tested using laboratory hardware. AFB implements an energy minimization algorithm to estimate the mass unbalance magnitude and phase. Unbalance amplitude and phase estimates are used to remove the synchronous component from the sensor output in defining the control system error. The energy of the synchronous component contained in the control system output is minimized, thereby allowing the shaft to rotate about its inertial axis. AFB also adapts to changes in the amplitude and/or phase of the unbalance, always driving the system to a minimum energy state in the process.

Adaptive Forced Balancing has several unique features which differentiate it from former autobalancing techniques. Previous autobalancing techniques include tracking notch filter designs [1], model based observer designs [2], and off-line least square approximation designs [3], just to name a few. AFB, the research topic of Beale [4], differs from notch filter designs in that it operates external to the stabilizing control loop, does not affect loop gain or phase, and can operate within the loop bandwidth. It differs from model based observer designs in that it requires no model and is therefore robust to modeling errors and time-varying plant parameters. The AFB algorithm is run in real-time, in parallel with the digital control laws, and requires no off-line computations.

Previous autobalancing research has always contained the implied need for external knowledge of the frequency of shaft rotation and hence the unbalance disturbance. This information may come from resolvers or tachometers or perhaps synchronous driven interrupts. The AFB algorithm is no exception. It requires precise shaft rotational rate information to provide adequate autobalancing. However, AFB also provides the means to adaptively estimate shaft rotation rate without any external knowledge. A Frequency Tracking algorithm developed to operate in synergy with AFB provides accurate shaft rate information. The combined algorithm, *Adaptive Forced Balancing with Frequency Tracking (AFB/FT) simultaneously tracks mass unbalance amplitude, phase and frequency in one integrated algorithm.*

This paper presents the AFB/FT algorithm and describes its operation. Its energy minimization properties are derived and discussed. The paper then describes the single end magnetic suspension test rig used to validate and verify AFB/FT operation. The test rig includes a DSP based control system which provides digital control to stabilize the rotating shaft. Hardware results are presented to show the effectiveness of the Adaptive Forced Balancing with Frequency Tracking algorithm.

PROBLEM DEFINITION AND ANALYSIS

A control loop for one axis of the magnetic bearing with unbalanced rotor, as shown in Figure 1, is represented by a single-input single-output (SISO), linear time-invariant (LTI) system. The plant P(s) represents electrical amplification, the linearized transfer characteristic of the magnetic bearing, and rotor dynamics. Plant input u(s) represents magnetic bearing actuator current command, and output y(s) represents the rotor cross sectional center of mass position relative to the radial axis of the magnetic bearing stator. The origin of a Y-Z coordinate axes, defined on the radial plane of the stator, is at the geometric center of the stator. When the geometric center of the rotor, $y_g(s)$, is not coincident with the rotor center of mass position y(s), $y_g(s)$ contains a synchronous measurement disturbance $d_\Omega(s)$ at frequency ω_x, the rotational frequency of the shaft. In the time domain $d_\Omega(s)$ has the form

$$d_\Omega(t) = \alpha_d \sin(\omega_x t) + \beta_d \cos(\omega_x t), \tag{1}$$

where α_d, β_d are the Fourier coefficients of the synchronous disturbance. Compensator C(s) stabilizes the system and, together with the plant, has infinite position error coefficient so that y(s) tracks step reference command r(s) with zero steady state error. An identical control loop operates on and stabilizes the Z axis of the system. Crosscoupling between the two control loops, which occurs through the rotor dynamics, is negligible since the rotor shaft is rigid and magnetic bearing actuation occurs at one end of the shaft only.

Compensation for rotor mass unbalance can be achieved by adding a second synchronous signal $r_\Omega(s)$ to the reference input. The system output is then decomposed into the sum of three terms:

$$y(s) = T(s)r(s) + T(s)(r_\Omega(s) - d_\Omega(s)), \tag{2}$$

where T(s) is the complementary sensitivity function given by

$$T(s) = P(s)C(s)/(1 + P(s)C(s)). \tag{3}$$

Typically $|T(j\omega)|$ is unity (0 dB) below the system bandwidth[1] ω_{BW}, and rolls-off at a minimum of –20 dB/decade above ω_{BW}.

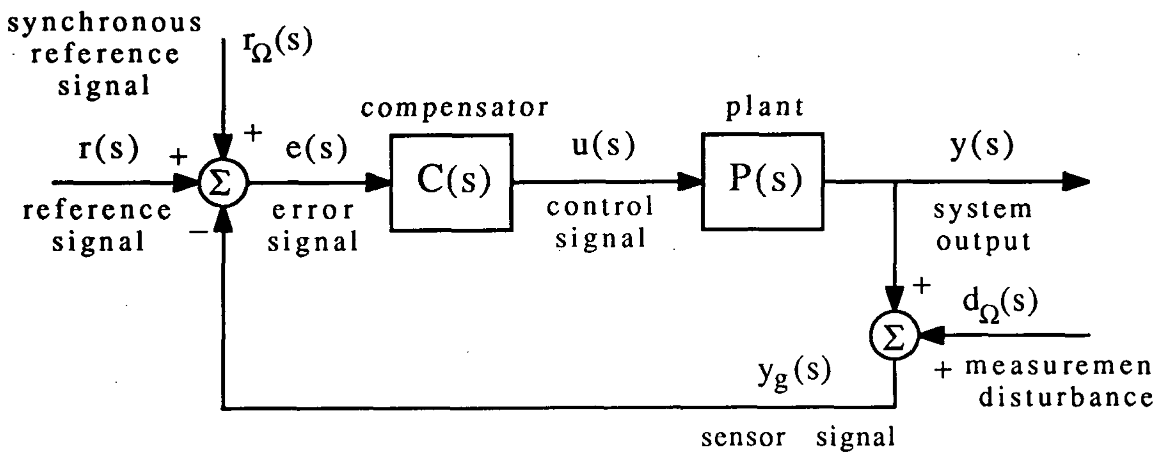

Figure 1. SISO, LTI Model of Magnetic Bearing Control Loop

In conventional mass unbalance compensation techniques ($r_\Omega(s)\equiv 0$), the system output response due to input $d_\Omega(s)$ is eliminated or "filtered out" by one of two methods: inserting a notch filter into the control loop or, in the case of model-based mass unbalance compensation, adding a second, observer-type compensator to the system. Both methods essentially alter the resulting system complementary sensitivity function so that either a "notch" in $|T(j\omega)|$ is present at the frequency of rotation ω_x or $|T(j\omega)|$ rolls-off before ω_x (i.e., so that the resulting system bandwidth is well below ω_x). Unfortunately, making a "notch" in $|T(j\omega)|$ within or near system bandwidth can eliminate the stability margin of the system because of the associated changes in phase; furthermore, sensitivity reduction properties are severely reduced at frequencies near ω_x. On the other hand, making $\omega_{BW} << \omega_x$ will severely degrade the desired transient response and plant disturbance rejection properties of the system.

AFB ANALYSIS AND DESIGN

To overcome the problems associated with the conventional methods a synchronous reference signal $r_\Omega(s)$ is computed and inserted

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[1]The system bandwidth in this case is defined as the frequency at which $|T(j\omega)|$ is –3 dB.

at the reference summing point as shown in Figure 1. The effect of $r_\Omega(s)$ is to cancel the component of the system output due to $d_\Omega(s)$ by making the factor $r_\Omega(s)-d_\Omega(s)=0$ in Equation (2) without changing the frequency response of T(s). This approach is desirable because the compensator C(s) can now be designed with the assumption that the rotor is perfectly balanced ($d_\Omega(s)\equiv 0$).

The reference signal has the form:

$$r_\Omega(t) = \alpha_r(t)\sin(\omega_x t) + \beta_r(t)\cos(\omega_x t), \tag{4}$$

where $\alpha_r(t)\equiv\alpha_r(kT_s)\equiv\alpha(k)$, $\beta_r(t)\equiv\beta_r(kT_s)\equiv\beta(k)$, k=0,1,2,... are discrete time-varying Fourier coefficients computed on-line and updated at a sampling period T_s. The adaptive laws governing $\alpha(k)$, $\beta(k)$ are given by

$$\alpha(k+1) = \alpha(k) + \nu q(k)p_\alpha(k)n(k), \quad k=0,1,2,... \tag{5a}$$

$$\beta(k+1) = \beta(k) + \nu(1-q(k))p_\beta(k)n(k), \quad k=0,1,2,... \tag{5b}$$

respectively, where $n(k)\equiv n_u(kT_s)$ is the norm or energy of the synchronous component of the control signal u(t) at sample time k. The norm function $n_u(t)$ is obtained by demodulation and filtering of u(t) followed by a complex magnitude calculation. The functional blocks of the AFB compensator are shown in Figure 2. The cut-off frequency of the demod filters depends upon approximate knowledge of the rotational frequency ω_x. The enable function $q(k)\in\{0,1\}$, and polarity functions $p_\alpha(k)$, $p_\beta(k)\in\{+1,-1\}$ are computed by an algorithm which seeks to minimize the norm or energy function $n_u(t)$. Finally, the constant gain ν is chosen *a priori* from given, approximate knowledge of the gain $1/|S(j\omega_x)C(j\omega_x)|$, where S(s) is the sensitivity function of the system.

Consequently, the entire class of AFB compensators can be characterized by the two scalar parameters ω_x and ν. This characterization facilitates quick design and flexible implementation. For example, AFB can be applied to different magnetic bearing applications or diverse synchronous disturbance rejection problems simply by modifying the demod filter bandwidth (a function of ω_x) and the gain ν. The gain ν depends upon of the nominal value of the gain $1/|S(j\omega_x)C(j\omega_x)|$ and is robust to time-varying plant parameters and modeling errors.

It can be shown [4] that the norm function n(k) is proportional to

$$[(\alpha(k) - \alpha_d)^2 + (\beta(k) - \beta_d)^2]^{1/2}. \tag{6}$$

In Equation (6) there are two time-varying parameters, $\alpha(k)$ and $\beta(k)$. If $\beta(k)$ is held constant, a change in $n(k)$ represents a change in the error $|\alpha(k) - \alpha_d|$. An enable function $q(k)$ is used to hold one Fourier coefficient (e.g. $\beta(k)$) constant while the other is adapting, and the polarity functions $p_\alpha(k)$, $p_\beta(k)$ are used to insert the correct sign of this error in the adaptive laws of Equation (5). AFB tracks the amplitude

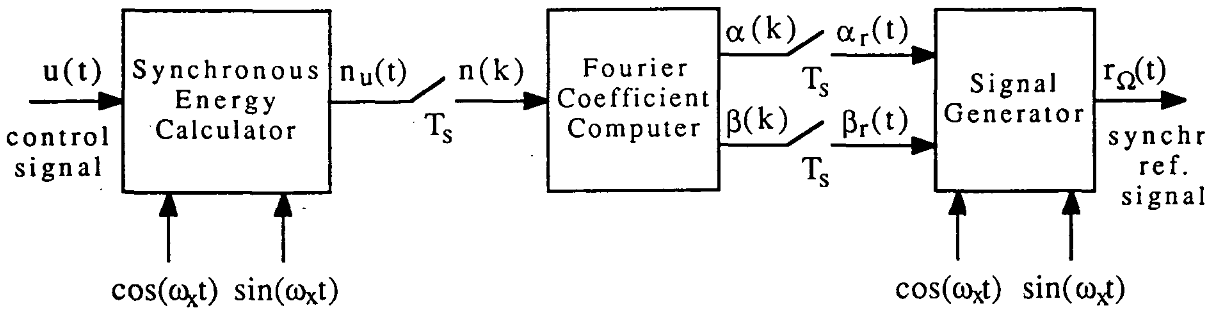

Figure 2. Adaptive Forced Balancing Compensation

and phase of the unbalance by driving $\alpha(k)\rightarrow\alpha_d$ and $\beta(k)\rightarrow\beta_d$ with the results that $r_\Omega(t)\rightarrow d_\Omega(t)$. Since $r_\Omega(t)=d_\Omega(t)$, the system output response due to the synchronous disturbance input is arbitrarily small in steady state. In practice the steady state disturbance rejection achieved depends only upon the level of random noise in the system and since the frequency response of T(s) remains unchanged, the AFB compensator successfully removes the synchronous component of the plant output without affecting the loop stability or the system bandwidth.

AFB WITH FREQUENCY TRACKING

In AFB the synchronous reference signal $r_\Omega(t)$ requires that a sine and cosine signal be generated at the frequency of shaft rotation ω_x (see Equation (4)). These sine and cosine waveforms can be generated from a tachometer measurement or other form of rotational sensor. If no hardware is available to provide ω_x, a frequency tracking (FT) algorithm must be included to provide estimates of ω_x. The combined AFB/FT algorithm, shown in Figure 3, simultaneously tracks the amplitude, phase and frequency of the mass unbalance in one unified algorithm.

Referring to Figure 3, the FT algorithm inputs are the Fourier coefficients $\alpha(k)$ and $\beta(k)$ and intermediate results from the synchronous energy calculation. FT produces one output, $\hat{\omega}_x$, which is used in the generation of the reference sine waves. The error $\hat{\omega}_x - \omega_x$ is driven to zero through the feedback paths which generate signals $r_\Omega(t)$ and $n_u(t)$.

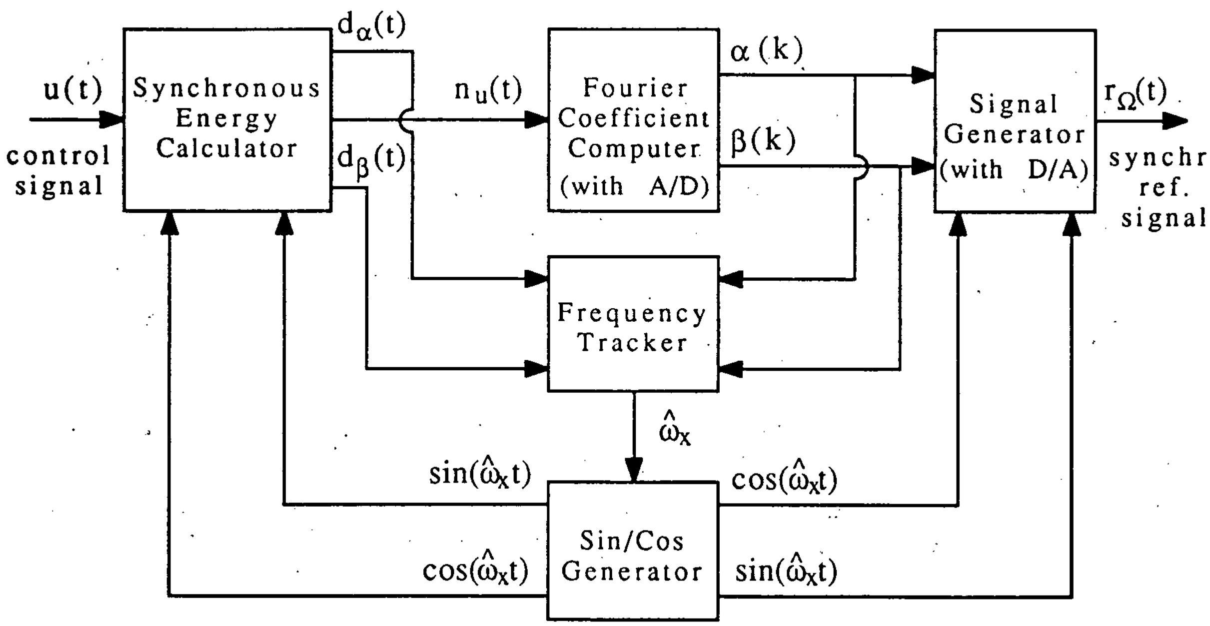

Figure 3. Adaptive Forced Balancing with Frequency Tracking

TEST RIG DESCRIPTION

The AFB/FT algorithm is combined with stabilizing digital control laws, implemented in a digital signal processor and tested on a single end, magnetic bearing suspension test rig. The test rig consists of a Draper designed, low power, magnetic bearing which supports a 150 lb shaft at one of its ends[2]. The other end is supported by rolling element spherical bearing. The shaft is driven by a motor capable of 8000 rpm. Commercial position sensors are used to provide position information to the stabilizing digital control laws. The position signals are digitized by an A/D converter, processed using a TI TMS320C30 DSP, and the resulting control current commands are sent to a pair of current controlled analog amplifiers via the D/A converters.

[2]The first critical of the shaft is approximately 270 Hertz.

The stabilizing control system consists of decoupled PID controllers operating at a 1 kHz sample rate. The bandwidth of the control system is selected to be 70 Hz. The 70 Hz bandwidth is selected intentionally to demonstrate that AFB/FT can operate within the system loop gain bandwidth without any adverse effects on system performance.

AFB/FT IMPLEMENTATION

The AFB/FT algorithm is coded in 'C' for real-time execution in tandem with the digital control laws. The Fourier coefficient computer section of the AFB/FT algorithm (see Figure 2) operates at subsample rates of the stabilizing control system. This allows $\alpha(k)$ and $\beta(k)$ to be computed as a background calculation to the real-time, stabilizing control system. The signal generator and synchronous energy calculator sections of the algorithm, which generate signals $r_\Omega(t)$ and $n_u(t)$*, respectively, are run at the same rate as the stabilizing control system.

Two AFB compensators are run independently; one for each axis or control loop. Operation of a single AFB compensator to provide two axis autobalancing has been simulated but not hardware tested. The frequency tracking algorithm is included with one of the AFB algorithms to provide rotational rate estimates for both axes.

EXPERIMENTAL RESULTS

With the rotor spinning at approximately 3720 rpm (62 Hz), AFB/FT compensation is enabled. The input reference command is set to zero mils ($r(t)\equiv 0$) so that the system error signal, e(t), represents the error in mils between the rotor position, y(t), and the center of geometry of the stator. The transient response of the Fourier coefficients ($\alpha_r(t)$, $\beta_r(t)$) and frequency ($\hat{\omega}_x$) parameters of the synchronous reference signal, $r_\Omega(t)$, along with the corresponding transient response of the system error signal are shown in Figure 4. The conversion factor to position in mils is given by .00030518 mils/quanta. At approximately 0.6 sec following application of AFB/FT, $\hat{\omega}_x$ converges to its steady state value. Following convergence of $\hat{\omega}_x$, the

* Since these signals are calculated in a digital computer $r_\Omega(t)$ and $n_u(t)$ are discrete signals $r_\Omega(k)$ and $n_u(k)$ respectively.

Fourier coefficients converge after about one second. In the ideal case the error converges to zero mils, which indicates that the shaft is rotating about the center of mass of the rotor. However, due to the random noise and higher harmonic disturbances (contributed by the fixed end of the shaft) present in the system, a random error exists in steady state as shown in Figure 4.

The steady state response of the system error signal is examined before and after AFB/FT is used. A snapshot of the time history of the error signal is shown in Figure 5(a), and the corresponding normalized fast Fourier transform (FFT) is shown in Figure 5(b). As can be seen elimination of the 62 Hz synchronous component from the error signal is in excess of 30 dB. Synchronous component rejection is limited by the fact that the intensity of random noise in the system is high relative to the degree of static mass unbalance of rotor.

CONCLUSION

An autobalancing technique was presented which simultaneously tracks rotor mass unbalance amplitude, phase and frequency. This algorithm, called Adaptive Forced Balancing with Frequency Tracking, does not alter system bandwidth nor destabilize the system when included in a stabilizing control loop. AFB/FT is also robust to modeling errors and time-varying plant parameters. The algorithm was tested on a single end suspension test rig operating near 60 Hz. AFB/FT successfully compensated mass unbalance at operating frequencies within the system bandwidth of 70 Hz with no adverse effects. Hardware results indicate that AFB/FT reduced the synchronous component of the system error signal and, hence, the magnetic bearing actuator current command by at least 30 dB.

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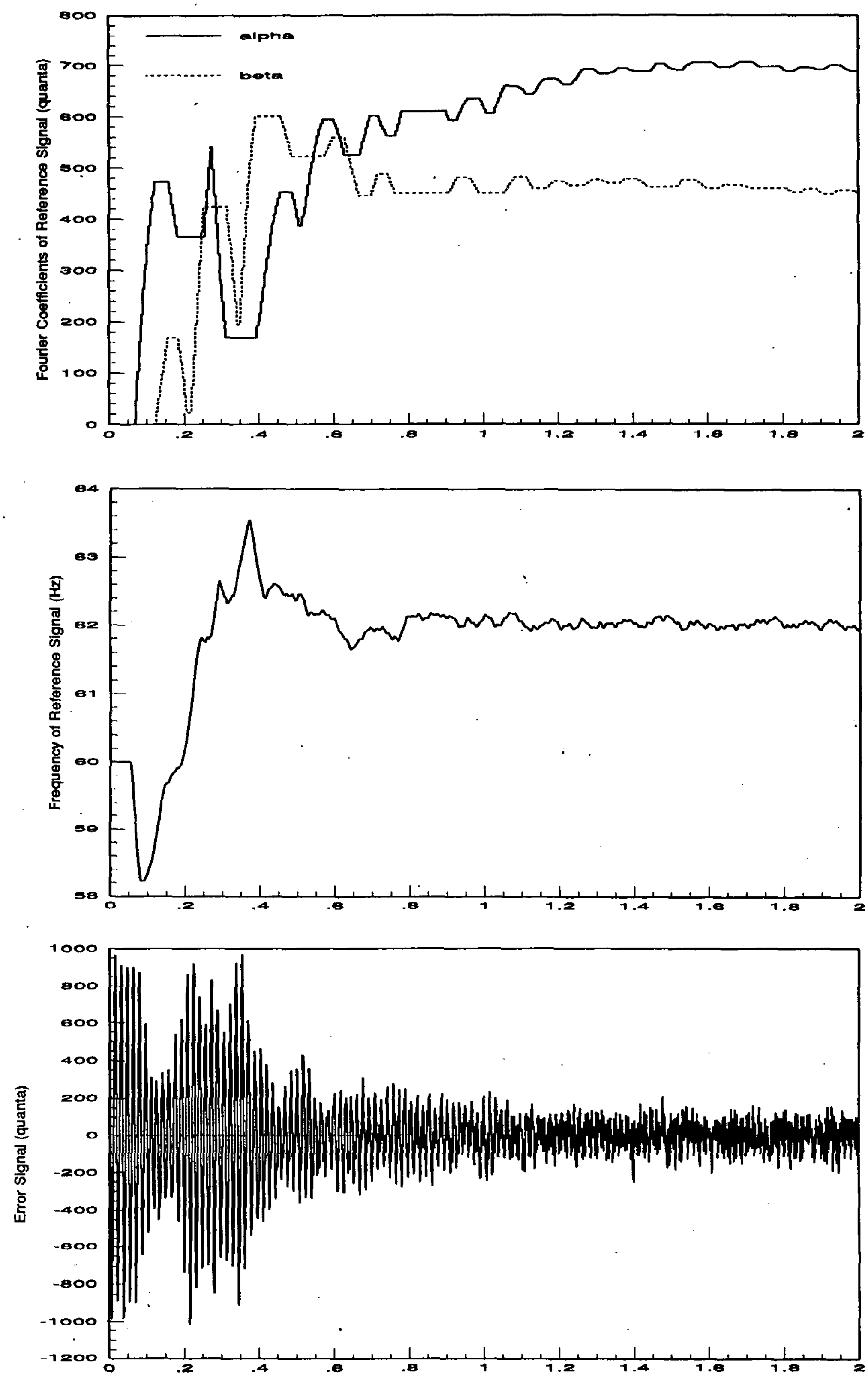

Figure 4. AFB/FT System Transient Response

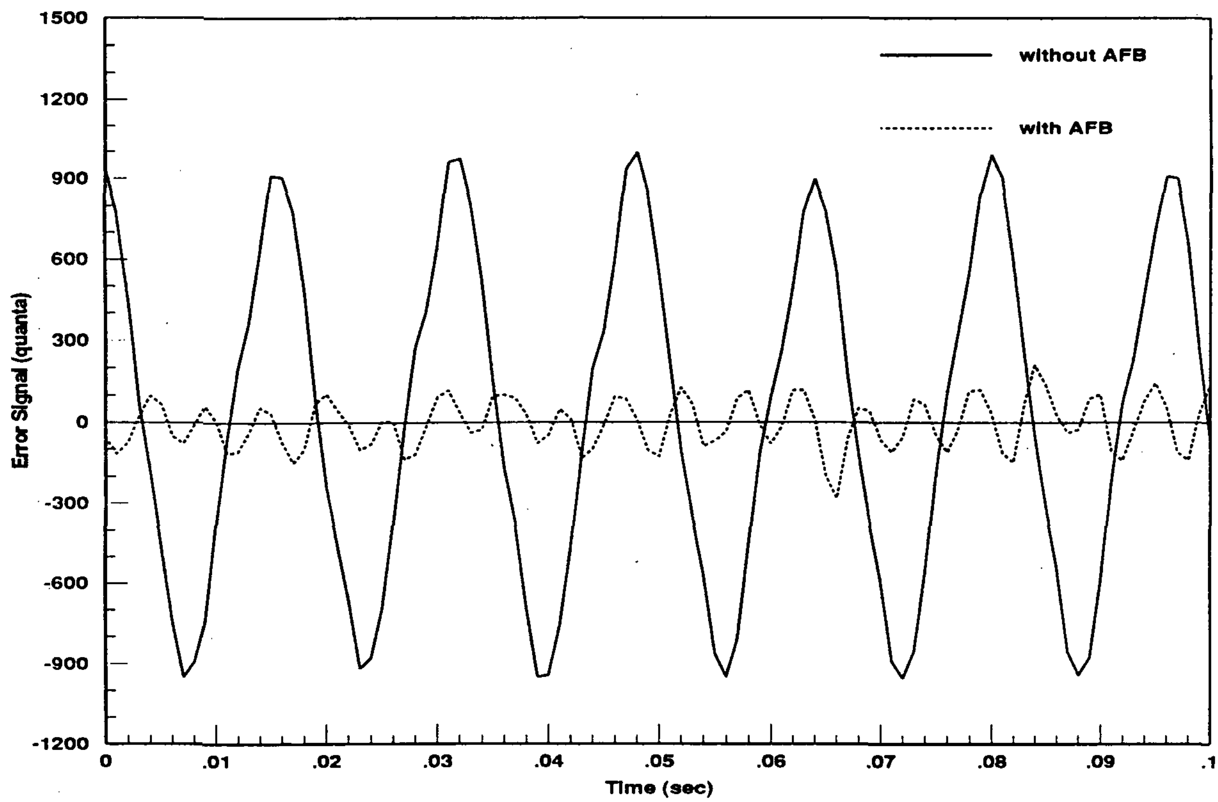

Figure 5a. System Error Signal in Steady State

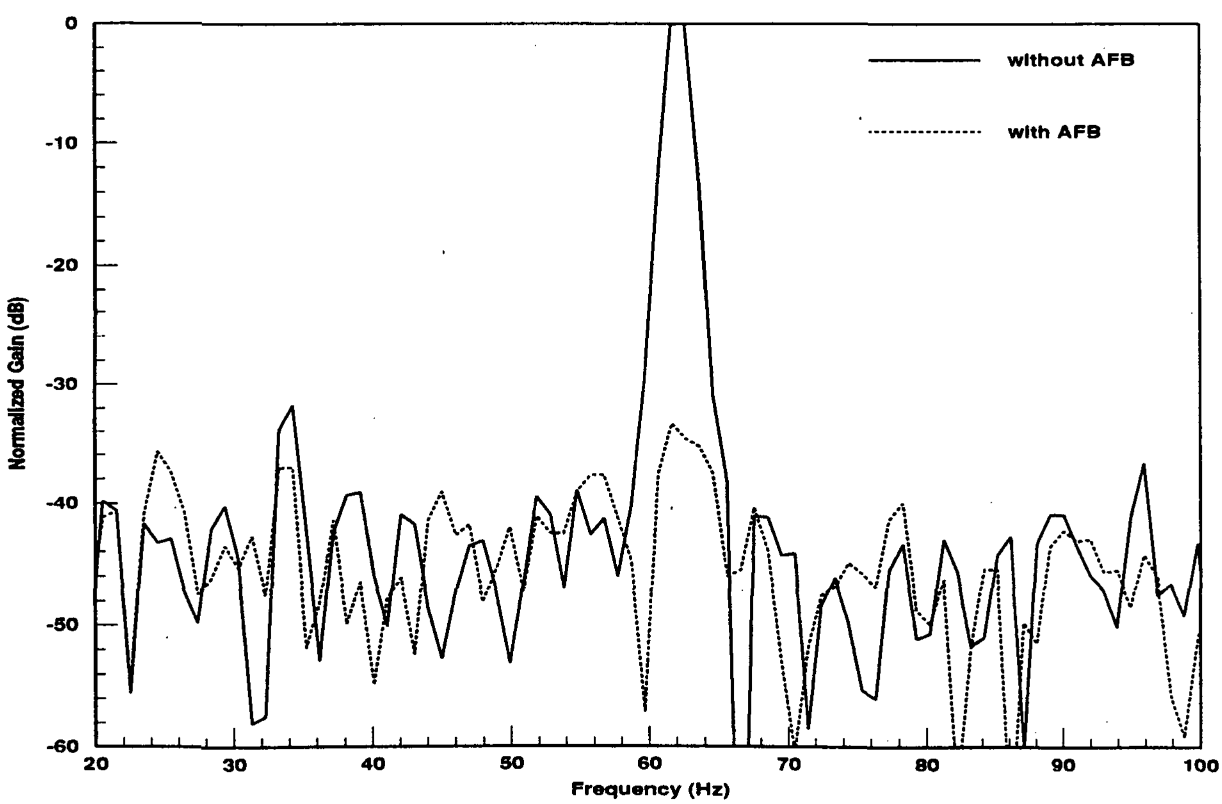

Figure 5b. FFT of the System Error Signal