A Versatile Controller Layout CAD Tool for AMBs

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ABSTRACT

The most familiar control concept in todays AMB applications is to just emulate spring-damper characteristics by PD controllers. For high performance AMB systems more powerful controller layout tools are needed.
QDES, a new CAD tool for controller layout is applied to magnetic bearing systems. The advantages of this powerful tool are explained and discussed. QDES is based on the so-called convex optimization over the set of all stabilizing controller, allowing to consider for most control specifications and to find the 'best' solution of all stabilizing controllers.

1. INTRODUCTION:

One major advantage of AMBs is the ability to actively control the bearing force and bearing characteristics. AMBs are therefore excellently suitable for vibration control, e.g. vibration isolation, force-free rotation, and unbalance cancellation to cross critical speeds.
Our presentation deals with a new controller layout tool for active magnetic bearings. In todays industrial applications of AMBs mostly the characteristics of well-known passive control elements such as springs and dampers (PD) are emulated. But AMBs are active elements including a much wider potential using more sophisticated control concepts for the main controller and the actuator [Siegwart et al. 91] .
The *feedback control* feeds back the sensed values (e.g. displacement) to the AMB force according to an *on-line* control law. There is no a priori limitation on the control law structure (e.g. PD) for AMBs. In this paper, *QDES*, a CAD-tool for controller layout based on the convex optimization of the set of all stabilizing controller, is applied to AMBs. QDES allows to numerically find all stabilizing controllers meeting given specifications. This can lead to much higher performance of the closed loop system than e.g. 'simple' PD controllers. Moreover this method allows to find *the best solution to your optimization problem which is satisfying all proposed specifications.*
The controller layout with QDES is discussed on a two-mass oscillator suspended by AMBs (figure 3-1).

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2. CONTROL PROBLEM

AMB-systems can usually be modelled as linear systems, even if the AMB-actuator is not absolutely linear. We therefore assume that the plant P and controller C are linear and time-invariant (LTI).

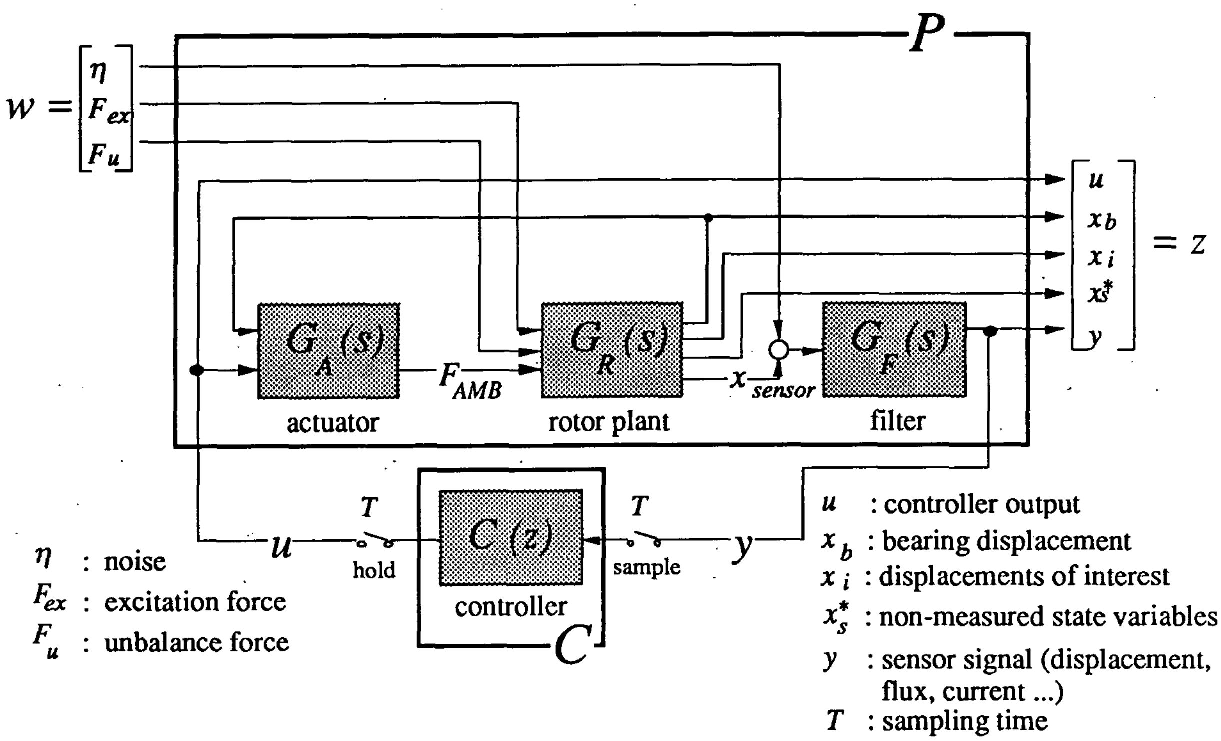

***Figure 2-1:** Block scheme of a typical AMB-system*

Figure 2-1 shows a general AMB system, where $G_A(s)$ is the transfer matrix of the AMB-actuator, $G_R(s)$ the transfer matrix of the rotor plant (suspended body) and $G_F(s)$ that of the sensor and signal filtering. The transfer matrices $G_A(s)$, $G_R(s)$ and $G_F(s)$ describe the AMB-plant P. $C(z)$ is the transfer matrix of the discrete controller C which has to be designed.

The inputs to the plant are divided into two vector signals:

- The *actuator input vector* $\boldsymbol{u}$, consisting of those inputs to the plant which can be manipulated by the controller.
- The *exogenous input vector* $\boldsymbol{w}$, consisting of all other input quantities such as noise, excitation forces, etc.

The output of the plant consists of two vector signals:

- The *measured output vector* $\boldsymbol{y}$, consisting of those measured signals which are accessible to the controller.
- The *regulated output vector* $\boldsymbol{z}$, consisting of all outputs of interest such as actuator input, rotor displacements, measured and non measured state variables.

This notation of the plant includes more details about the AMB system than it is common in classical control [Boyd et al. 90]. The exogenous inputs and regulated variables contain each signal subject to constraints or specification, whether it is measured or not.

Specifications for the Controller Layout

The controller layout of an AMB system is always restricted by different constraints and specifications. They have to be defined by the AMB-engineer and include e.g. physical aspects of the plant and specifications on the system performance. Common examples are *closed loop stability, maximum stiffness over a given frequency band, limitation of the amplifier's bandwith, noise rejection, noise filtering, force free rotation around the inertial axis, damping to cross critical speed, vibration rejection and robustness against to changes in the plant, modelling errors and nonlinearities.*
The constraints and specifications define upper and lower bounds for the input-output behavior of the AMB-system shown in figure 3-1.
Usually the different constraints and specifications are in opposition to each other and it is therefore not an easy task to find a solution for the control problem.

3. QDES: A VERSATILE CAD METHOD FOR AMB CONTROLLER DESIGN

QDES is a CAD computer code with an underlying controller design philosophy which has been elaborated by Stephen Boyd and his colleagues at Stanford University [Boyd et al. 88, 90]. QDES can deal with a great variety of control objectives and constraints like. frequency domain inequalities, time domain overshoot, settling time, rms disturbance response, LQG-type objective, H_∞-type objective and l_1-type objective. It offers numerical solutions to control problems where *no analytical* solution is known. The objectives and constraints are written in a control specification language (CSL), a language that comes natural to control engineers. QDES reads this CSL control problem specification and translates it into a standard convex optimization program which is then solved numerically by effective methods.

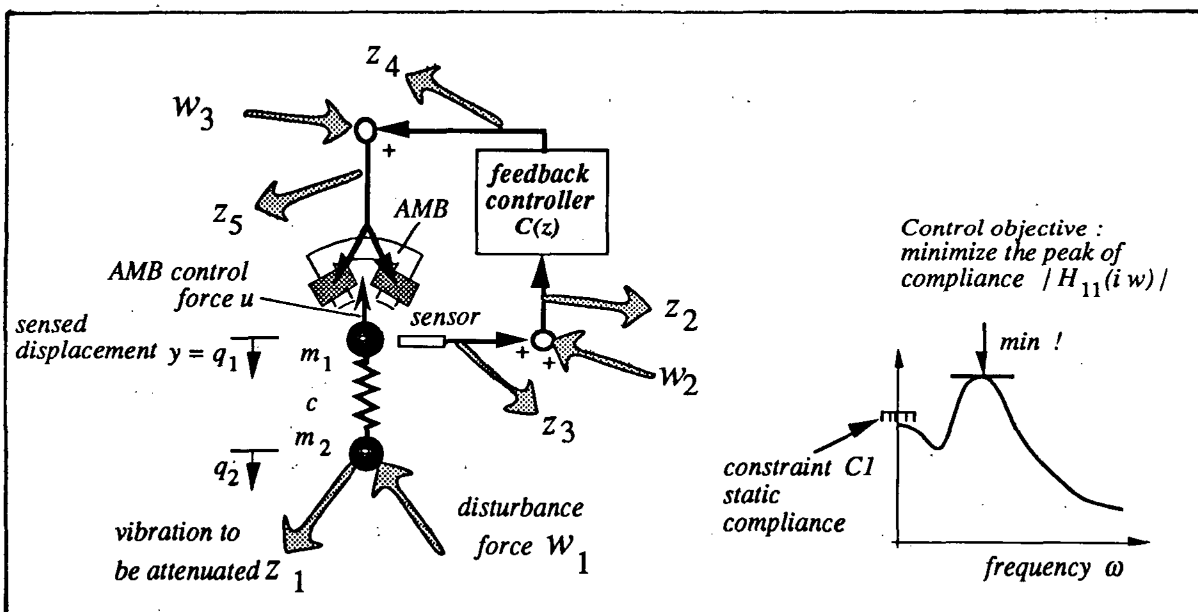

***Figure 3-1:** Two-mass oscillator with exogenous disturbances w_1, ..., w_3*
To verify new control concepts it is helpful to use a simple mechanical example e.g. a two mass oscillator. Two-mass oscillators (figure 3-1) have been used extensively for benchmark purposes in modern controller design. [Wie et al. 90,91] and can be regarded as very simple model of a flexible rotor.

The convex program solver first determines *whether or not* a controller exists which meets the given specifications. Therefore, *trade-offs* among different control specifications can be assessed.

In the next section we present an example of an AMB controller design obtained by QDES. The description of the QDES example below is limited to a superficial "user-point-of-view"; we intentionally hide the "inside-point-of-view". For an excellent description of the underlying controller design philosophy refer to [Boyd et al. 88, 90].

3.1 QDES Design Example for the Two-Mass Oscillator Benchmark Problem

Consider the controlled AMB system in figure 3-1. Both sensor and actuator are located at the top mass m_2. An unknown main disturbance force $z_1(t)$ acts on the bottom mass m_1. Let the objective of controller C be to minimize the frequency domain peak of vibration w_1 of the bottom mass m_1. Furthermore, let us impose a number of fairly realistic control constraints by considering additional disturbances w_2 and w_3, see figure 3-1. $H_{ij}(s)$ denotes the closed loop function from input w_j to output z_i. One constraint occuring frequently in practical AMB applications is static stiffness. Therefore, we have to impose an upper bound on $H_{11}(0)$. Furthermore, the Nyquist plot of the loop gain $PC(j\omega)$ should not pass the critical point $s=+1$ too closely. This is a very common way to incorporate some stability robustness in the controller design. The minimal distance of the loop gain to the critical point is precisely inverse to the peak of sensitivity function $H_{22}=H_{53}=1/(1-PC)$. Therefore, a simple way of considering "robustness" is to impose an upper bound on the peak of $|H_{22}(j\omega)|$. One more constraint that often makes sense in AMB control is to filter out some harmonic signal (e.g. unbalance force) with known frequency ω_0. If the corresponding amplitude and phase information is available, this can be achieved easily by "feedforward" techniques [Larsonneur et al. 92].

```
minimize  {
          max_Mag_H[1][1];                              /* specification of the design objective    */
          }

subject_to
          {                                             /* spec. of the design constraints          */
                                                        /* The 4 constr. are labeled C1,...,C4.     */
          Mag_H[1][1](0)      <=      1.5;              /* C1: static stiffness constraint          */
          max_Mag_H[2][2]     <=      10;               /* C2: loop margin constraint = 0.1         */
          for w=0.97*%pi/2 to 1.03*%pi/2 step freq_step do
               Mag_H[4][2](w) <=      0.01;             /* C3: -40 dB gain "around" omega_0         */
          for w=0.8*%pi to %pi step freq_step do
               Mag_H[4][2](w) <=      0.1+10*(%pi-w);   /* C4: high frequency gain constraint       */
          }
```

Table 3-1: CSL specification example for the two-mass oscillator.

Otherwise, an additional controller constraint has to be incorporated in the design, that is, $|H_{42}(j\omega)| = |H_{52}(j\omega)| = C/(1-PC)$ should be very small around ω_0. Finally, we consider a constraint regarding the controller gain at high frequencies. This is very important, since the modeling of a flexible rotor often has substantial uncertainties at high frequencies residual modes. At high frequencies, $C \approx C/(1-PC)$ since the plant is usually low-pass. Therefore, a constraint of the high frequency gain of controller C can be replaced by an upper bound on $|H_{42}(j\omega)| = |H_{52}(j\omega)|$ at high frequencies. In summary, the above control problem specifications consist in one objective term max $|H_{11}(j\omega)|$ and four constraints, that are, upper bounds (inequalities) regarding $H_{11}(0)$, max $|H_{22}(j\omega)|$, $|H_{42}(j\omega_0)|$ and $|H_{42}(j\omega : \omega >> 1)|$. This control problem specification is part of the input data for QDES. It can easily be written in the control specification language (CSL) (see table 3-1).

We fix the physical parameters to $m_1 = m_2 = c = 1$. The continuous time plant description has a double pole at $s = 0$ (rigid body mode) and "flexible motion" poles at $s = \pm i \sqrt{2}$. Furthermore, there are plant transmission zeros at $s = \pm$ i. Note that the CSL specifications refer to a discrete time description. We choose a sampling period of $T = 0.3$. The z-domain poles are all on the unit circle, namely a double pole at $\varphi = 0$ and a pair at $\varphi = \omega T = \pm 0.3 \sqrt{2} \approx 0.424$. It turns out, that there is also a complex-conjugate pair of z-domain plant transmission zeros on the unit circle, interlaced between the poles. A theoretical justification for this observation was given in [Herzog et al. 91]. Note that the exact location of the zero is *not* given by the "pole-transformation" formula $\varphi = \omega T$. However, in this case the location of the transmission zeros turns out to be almost at $\varphi \approx \omega T \approx 0.3$ since we have a quite fast sampling rate.

We now show the QDES result corresponding to the above plant data and the CSL specifications of table 3-1. Following the successful QDES translation phase, the convex program solver detected the *feasibility* of the above control constraints. Note that this is by *no* means a self-evident fact. If QDES would have detected the *unfeasibility* of the control problem, then the constraints should be relaxed or the control configuration should be changed. The convex program solver completed the constrained optimization task with a final performance value of about ≈ 2.4, i.e. the minimal peak of dynamic compliance $|H_{11}(j\omega)|$ is $\approx$ 2.4. There is *no* stabilizing *controller* enabling a better performance and meeting the above constraints. See figures 3-2, 3-3 and 3-4 for the optimization results and note, that all of our constraint specifications are met.

Although QDES is a very effective and versatile CAD tool for this kind of control problems, there remains an implementation problem since the order of a QDES controller is generally much *higher* than the order of the plant. That is why there remains a strong need to keep up with the latest developments in special purpose controller architectures and in modern "closed-loop" controller reduction schemes [Anderson & Liu 89], [Mustafa & Glover 91].

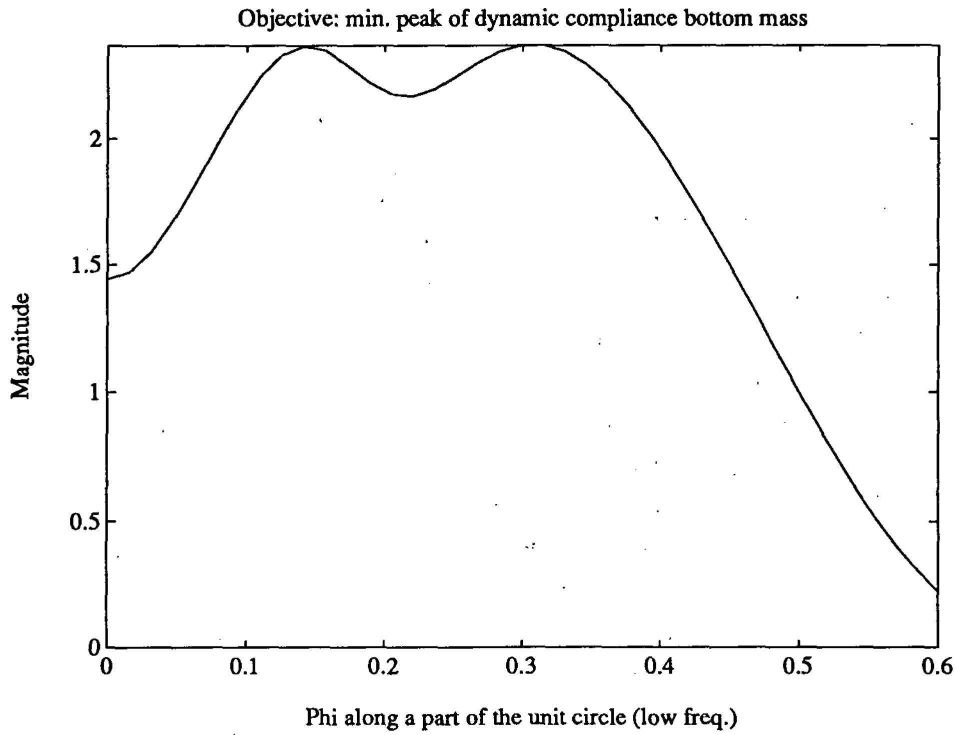

Figure 3-2: *Objective function H_{11}. Note the constraint C1 which imposes a static compliance below 1.5.*

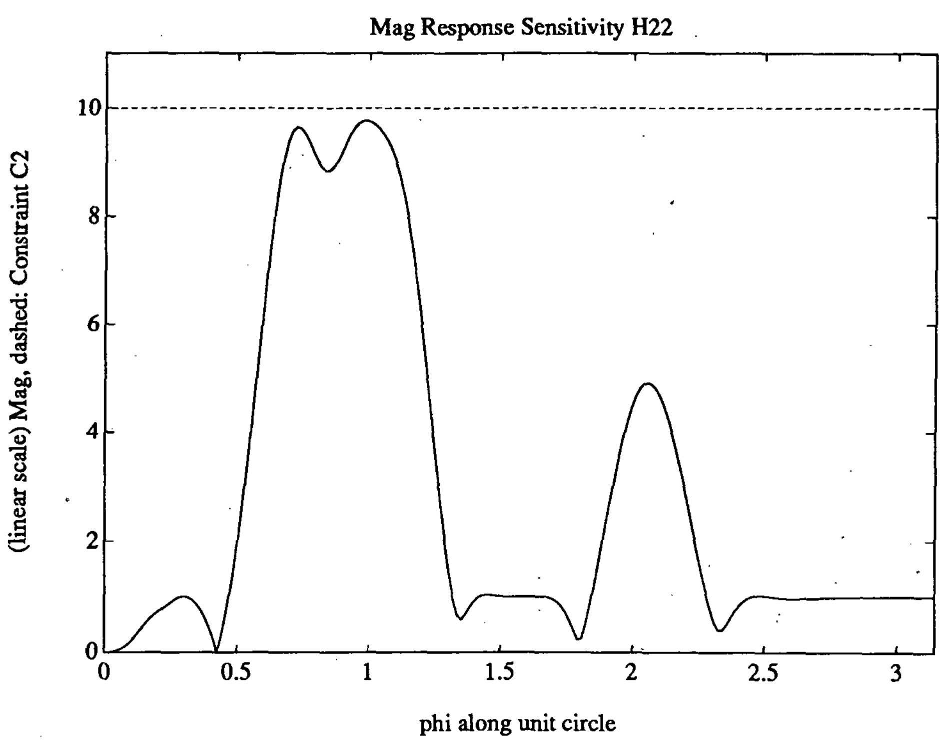

Figure 3-3: *Sensitivity function H_{22}. Note the constraint C2 : the peak of H_{22} is below 10. Therefore, the minimal distance of the loop gain PC to the critical point s=+1 is greater than 0.1.*

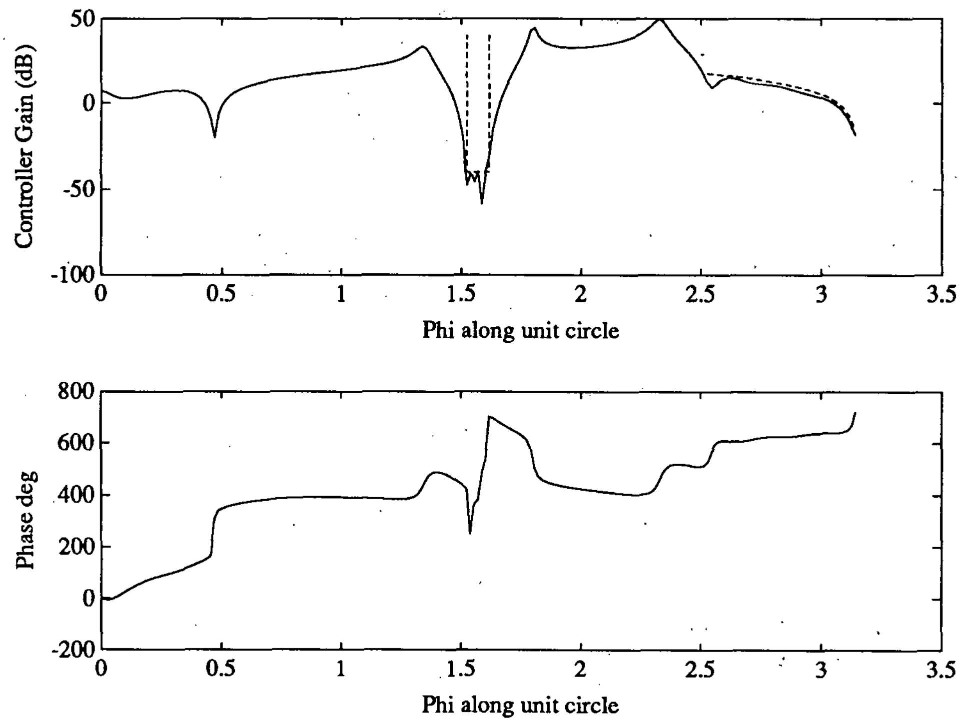

Figure 3-4: *Magnitude and phase response of the resulting QDES controller. Note the narrow-band constraint C3 and the high gain constraint C4. Although C3 and C4 are* closed-loop *shaping constraints, they also apply to the controller itself since $C \approx C/(1-PC)$ at high frequencies.*

5. SUMMARY AND OUTLOOK

A CAD-tool (QDES), based on convex optimization over the set of all stabilizing controller was applied to the two-mass model. The theoretical results show the efficiency of the presented controller layout tool and let us expect a big impact on future AMB applications. Presently, the effectiveness of the described approache is being experimentally verified. Test results are expected soon.

6. ACKNOWLEDGEMENTS

We would like to thank Professor Stephen Boyd, Stanford University, allowing us to use QDES.

7. REFERENCES

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