Robustness of Unbalance Response Controllers

C. R. KNOSPE

ABSTRACT

This paper examines two methods for the reduction of synchronous response, feedback control and open loop control, with an emphasis on stability robustness. Modern control theoretic measures of robustness are introduced and are used to analyze two ad hoc filter feedback methods. The stability robustness of open loop controllers is then examined and experimental results demonstrating the efficacy of open loop control are presented.

INTRODUCTION

One of the principle advantages of active magnetic bearings is the high degree of vibration control obtainable. This paper examines the use of active magnetic bearings for the reduction of unbalance response. Two methods of attenuating machine unbalance response have been advocated: feedback and open loop control. Feedback control of unbalance response relies upon the tailoring of the magnetic bearing's impedance to rotor motion at the operating speed. The feedback controller, in addition to reducing unbalance response, must robustly stabilize the rotor in suspension and have adequate performance with respect to excitations other than unbalance. Investigators have examined two kinds of feedback controllers for reducing unbalance response: ad hoc filters and synthesis-based controllers. Ad hoc methods introduce a filter in series with a stabilizing feedback controller (typically P.I.D.). The filter is designed to act over a narrow range of frequencies around the operating speed, either increasing (bandpass) or decreasing (bandstop) the effective stiffness to unbalance response. Bandpass filters are used to suppress the rotor's unbalance response while bandstop filters are used to attenuate the casing's unbalance response. An important issue with the ad hoc filter methods is the effect of the filter's introduction on system stability and robustness.

The ad hoc filter in greatest use in industry today is the notch filter [1, 2] . The notch filter technique is often referred to as 'automatic balancing'. This name is misleading since the reduced bearing stiffness and damping at the operating speed are not analogous to conventional rotor balancing. Since notch filters achieve synchronous vibration reduction in the casing through reducing the magnetic bearing's stiffness and

[1]Carl R. Knospe, Center for Magnetic Bearings, Department of Mechanical and Aerospace Engineering, University of Virginia, Charlottesville, VA, 22901

damping at the system's operating speed, the stability and the robustness of this approach becomes an important issue. It has been shown that the introduction of a notch filter may result in poor robustness or nominal instability [2, 3, 4].

Synthesis-based feedback methods use a controller derived with H^2 or H^∞ controller synthesis tools and employ either disturbance accommodation or frequency weighting. These feedback controllers have been investigated by a number of researchers including [5, 6, 7]. Stability robustness as with any feedback method is an important consideration with these controllers. While some robustness is guaranteed with a few of these methods (notably 60° multivariable in-the-loop phase margin with LQR and frequency weighted states) this will be lost with the introduction of the required observer.

The open loop control method adds a synchronous force on top of a stabilizing feedback control to cancel the unbalance response. This is dynamically equivalent to mechanical balancing in conventional machinery. Open loop control can be implemented adaptively to accommodate changes in the unbalance,rotor dynamics, or bearing characteristics. This method can cancel synchronous (and asynchronous periodic) vibrations over the entire operating speed range. It should be noted that open loop control can neither stabilize nor destabilize magnetic suspension - it does not affect system stability. Thus, a feedback controller can be designed so as to yield maximum stability (with unbalance response performance ignored) and an open loop controller can be added to achieve unbalance attenuation. Many researchers have investigated this method [8, 9, 10, 11, 12].

In this paper, these methods of attenuating unbalance response are examined in detail with an emphasis on stability robustness. First, useful measures of stability robustness are discussed and previous research on ad hoc feedback methods is summarized. Next, the stability of open loop methods is briefly discussed. Finally, experimental results are presented demonstrating the efficacy of open loop control.

ROBUSTNESS

In order to examine the stability of various methods of attenuating unbalance response, measures of robustness are introduced here. The property of a controller to retain closed loop system stability in spite of variations in the plant is known as *stability robustness.* The importance of the robustness of a magnetically suspended rotor lies in the accuracy of the mathematical description of the rotor, magnetic actuators, sensors, and power amplifiers used in controller design. The robustness of a controller is measured by the 'size' of the smallest variation in the plant which compromises system stability.

For a rotor in magnetic bearings there are two types of variations to be considered: in-the-loop and cross-coupled. In-the-loop variations are due to differences in the sensors, actuators, and electronics. Examples include inaccuracy in actuator gain and phase due to magnetic circuit modelling, inaccuracy in bearing clearances, and thermal changes in magnetic properties. The robustness of a magnetic bearing system to in-the-loop variations may be easily tested via multiplicative structured singular value analysis [13, 4]. Cross-coupled variations change the way the plant model couples channels of input and output and represent mainly inaccuracies in the rotor system model. While the rotor itself may be modelled highly accurately, some phenomena (e.g., the stiffness and damping of seals, the impedance of the process fluid in pumps, aerodynamic cross coupling in compressors) may not be accurately reflected in the model. In this paper

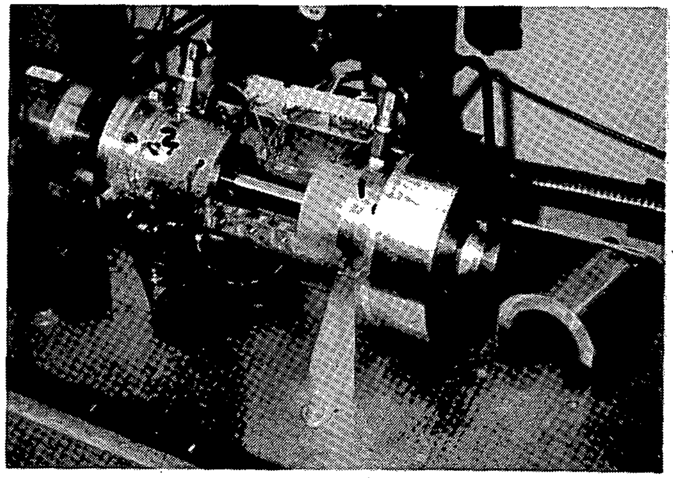

Figure 1: High speed test rig at the University of Virginia

only robustness with respect to in-the-loop variations is examined.

Due to space limitations, the various measures of stability robustness will not be reviewed in this paper. Interested readers are urged to consult [4] for a full explanation. The robustness is measured at each frequency by the smallest maximum singular value $\mu(j\omega)$ achievable over all complex matrices of a predefined structure which destabilizes the system. This is the *structured singular value*, a natural generalization for multiple-input-multiple-output (MIMO) problem of the SISO gain and phase margins. Large μ indicates poor robustness since μ is the reciprocal of the 'size' of the smallest destabilizing variation. Here, the structured singular value μ is employed in a multiplicative in-the-loop fashion [4] where the assigned structure of the complex matrix is diagonal. In this case, each channel of feedback receives an independent gain/phase change. This allows the testing of the magnetic bearing system's relative stability with respect to simultaneous and independent gain/phase variations in each of the feedback loops. This form of variation very accurately tests for uncertainty in the nominal models of the actuators, sensors, and electronics.

The rotor model chosen for analysis is a 22 mass station model of a high speed test rig supported with two radial magnetic bearings and one axial magnetic bearing. This rig, shown in Figure 1, was constructed at the University of Virginia for an industrial sponsor to simulate a small high speed compressor [14]. The designed operating speed range for this machine is 30,000 to 70,000 rpm. In the model employed for this analysis several simplifying assumptions are employed. Modal damping of 0.2% is added to the rotor model (see [4] for a detailed discussion). The model has collocated sensors and actuators and local phase lead control. For this analysis, it will be assumed that the negative stiffness of the open loop bearings can be precisely cancelled with a minor loop so as to achieve an actuator which is neutrally stable. While this is clearly not possible, this assumption simplifies the following analysis and no generality is lost. The magnetic bearings and sensors are assumed to have infinite bandwidth. Gyroscopic terms are not included in the model; therefore, vibration in only a single plane is examined. These assumptions simplify the analysis and allow the attribution of any stability robustness problems to the ad hoc filter controller itself and not to other destabilizing effects (e.g. non-collocation or controller roll-off). The first five free-free natural frequencies of the

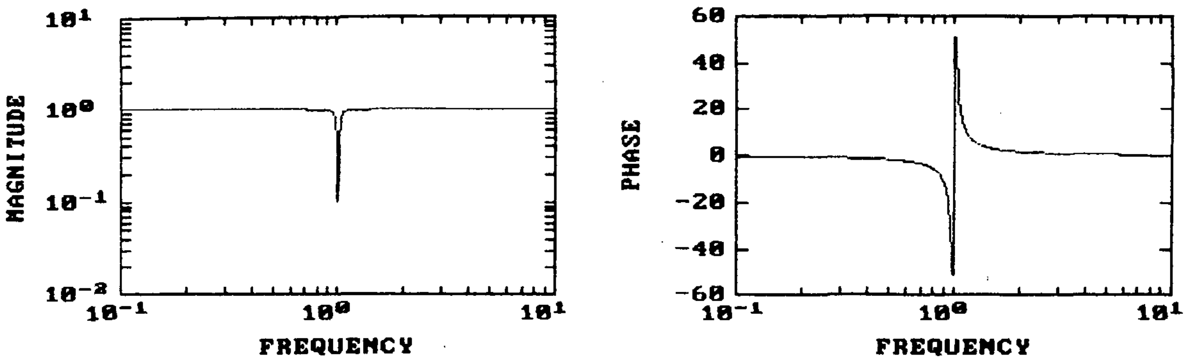

Figure 2: Magnitude and phase of notch filter

rotor are listed in Table 1. With phase lead feedback (no ad hoc filter) the first five natural frequencies of the closed loop system are listed in Table 2. The lead-lag controller employed for this analysis has a static stiffness of 1000 lbf/in, a zero at 180 rad/s, and a pole at 2000 rad/s.

AD HOC FEEDBACK METHODS

Two ad hoc filter methods are examined for robustness in this section: a conventional notch filter and an alternative filter proposed by the author [4]. The notch filter method for attenuating transmitted synchronous vibration has been employed for many years [1]. The transfer function for a notch filter centered at operating speed ω_o is

$$N(s) = \frac{s^2 + g\,w\,\omega_o\,s + \omega_o^2}{s^2 + w\,\omega_o\,s + \omega_o^2}$$

where g is the notch filter's gain at the center frequency and w is the width of the notch (approximately the width of the frequency band where the attenuation is greater than -3 dB. The gain and phase of this transfer function are shown in Figure 2. In theory, if the notch gain is zero, no synchronous response will be transmitted through the bearings (if the open loop bearing stiffness is zero). This transfer function may be realized with an analog circuit composed of a single operational amplifier and several resistors and capacitors [15] . It is important to point out that the transfer function of this circuit is highly sensitive to operational amplifier roll-off. This notch filter is phase lead above

Table 1: Free-Free Natural Frequencies

Mode	Natural Frequency (rpm)
1	0
2	0
3	21545
4	53944
5	115250

Table 2: Closed Loop Natural Frequencies

Mode	Damped Natural Frequency (rpm)	Log Decrement
1	7140	6.316
2	11156	4.760
3	22463	0.245
4	55320	0.064
5	115685	0.016

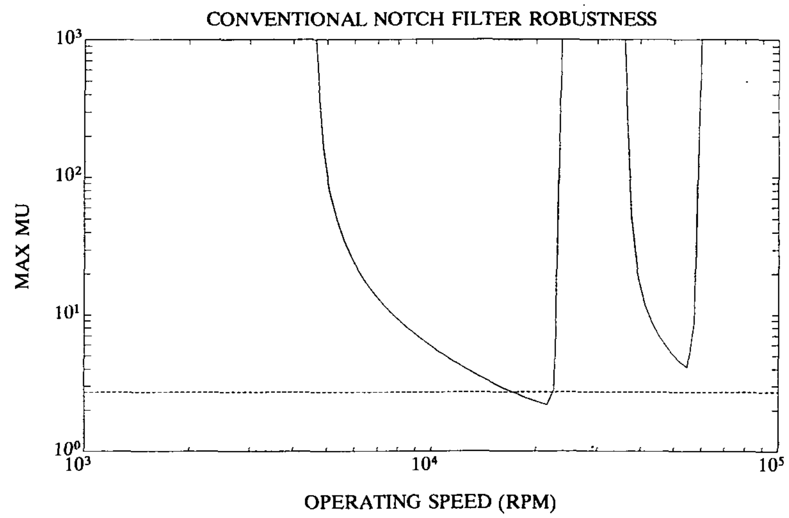

Figure 3: Maximum structured singular value vs. operating speed, with and without notch filter

the center frequency and phase lag below. The maximum phase lag of the filter is

$$\phi_{\text{max lag}} = \arctan(\sqrt{g}) - \arctan(\frac{1}{\sqrt{g}})$$

The maximum phase lag is a function only of the attenuation at the center frequency. For a notch filter with 20 dB attenuation, the phase lag is 51°. From a control theory viewpoint, the introduction of notch filters into the feedback loops seems unwise, since the collocated Bode plot of the open loop system contains many regions where the loop gain is greater than one and the phase is −180°. Thus, phase lag introduced by the notch filter at these frequencies may result in instability. However, the notch filter also reduces loop gain in precisely the same frequency range where it introduces phase lag. If the loop gain is lowered sufficiently, the phase lag becomes unimportant.

To test the effect of the insertion of notch filters on in-the-loop robustness, the maximum structured singular value was computed as a function of notch center frequency. The method used was as follows: an operating speed was chosen and a notch filter model was constructed with center frequency at the operating speed; the notch filter was appended in series with the lead-lag controller to each of the rotor model's outputs; the output of the notch filter was connected to the bearing inputs and the closed loop system derived; the nominal system was tested for stability by an eigenvalue check; if the system was stable, the structured singular value was computed as a function of frequency using a minimization method [16] and the maximum singular value over frequency was determined; operating speed was incremented and the procedure used again. In the tests presented here, the notch attenuates at the center frequency by 20 dB ($g = 0.1$) and has a -3 dB width of approximately 5% of center frequency ($w = 0.05$).

Figure 3 shows the robustness measure μ_{max} plotted as a function of operating speed (notch center frequency). Also shown is a line indicating the robustness measure of the magnetic bearing system without notch filters. Note from Figure 3 that an unstable system will result if the notch filter is used at operating speeds between 1000

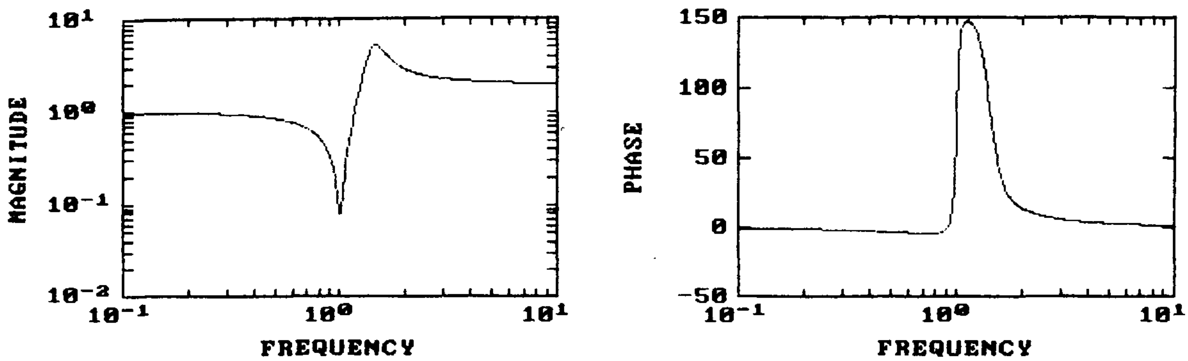

Figure 4: Magnitude and phase of alternative filter

and approximately 4500 rpm ($\mu_{max} = \infty$). The notch filter's phase lag below the center frequency results in the destabilization of the system. This instability is interesting since it results from the interaction of both notch filter controllers through the rotor and does not appear in analogous SISO problems. (The other instability mechanisms in this paper can all be explained through a SISO analogy.) The system will be stable but will have reduced in-the-loop robustness if a notch is placed between 4500 and 18000 rpm. For notch center frequencies of approximately 18000 to 22000 rpm, the notch filter will increase robustness. This is due to the phase lead above the notch center frequency adding damping to the third and fourth critical speeds. Above the third critical speed, the phase lag below the center frequency results in instability of the third critical speed for rotor systems with notches placed between 23000 and 34000 rpm. Above 34000 rpm, stability robustness is quickly recovered as the phase lag introduced by the notch at the third critical speed reduces with increasing center frequency. Notches placed for operating speeds between 60000 and 100000 rpm result in instability as the phase lag introduced below the center frequency destabilizes the fourth critical speed of the system.

An alternative ad hoc filter should work if it can greatly attenuate a narrow frequency band without resulting in excessive phase lag. If such a filter could be built, it would serve as a low cost, 'low tech' alternative to open loop control for some applications. In analog form, this circuit must be insensitive to variation in its components. A discrete time equivalent of this filter might also be useful for certain digital control applications. Such a filter was proposed by the author in [4] and re-examined here,

$$F(s) = \frac{2s^2 + 0.08\,\omega_o\,s + 2\,\omega_o^2}{s^2 + 0.28\,\omega_o\,s + 2\,\omega_o^2}$$

where ω_o is the bandstop frequency (operating speed). The magnitude and phase of this filter as a function of frequency are shown in Figure 4. Note that over 20 dB attenuation occurs at ω_o yet the maximum phase lag is only 4.4° which occurs at $0.76\omega_o$. While this filter may be built with resistors, capacitors, and four operational amplifiers, it may be highly sensitive to the gain-bandwidth product of the operational amplifier since the Q of the poles is 5 [15]. This transfer function could also be easily realized in a digital controller with a simple difference equation.

The effect on in-the-loop robustness of the insertion of this filter into each feedback

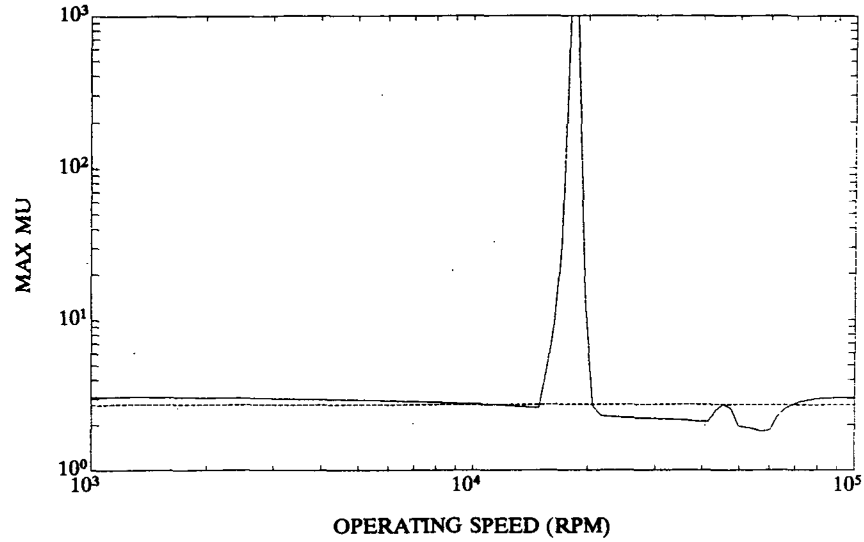

Figure 5: Maximum structured singular value vs. operating speed, with and without alternative filter

loop was examined in the same manner as in the previous section: μ_{max} was calculated for bandstop filters over a range of operating speeds. Figure 5 shows the robustness parameter μ_{max} as a function of operating speed. In comparison to Figure 3, it can be seen that the alternative filter is much more robust (smaller μ_{max}) than the conventional notch filter. For example, for operating speeds below 4500 rpm, the alternative filter has almost the same in-the-loop robustness as the rotor system with lead-lag feedback (which has no reduction in transmitted synchronous vibration) while a conventional notch filter results in instability. This is also true for operating speeds between 23000 and 34000 rpm. Indeed, the alternative filter yields in-the-loop robustness on par with lead-lag feedback (and better than the conventional notch filter) over the entire operating speed range. However, the alternative filter will result in poor robustness and even instability when used at operating speeds just below the third critical speed, between approximately 15000 and 20000 This seems surprising since the filter results in phase lead above the center frequency; one suspects that this should act to stabilize the third critical speed.

This phenomena can be understood from a simple Nyquist analysis of a SISO system. What is occurring is a rather unique circumstance: large phase lead followed by an increase in gain and then a sudden decrease in phase. This occurs, for example, with our system when a bandstop filter centered at 18000 rpm is used (an unstable system). A maximum phase lead in each loop greater than 180° is achieved below 21000 rpm. Above this frequency, the loop gain quickly rises as the phase decreases; then the zeros in the collocated transfer function associated with the first free-free bending mode results in a loss of gain. This yields encirclement of the critical point in the Nyquist diagram. Note that this type of instability does not occur at frequencies just below the other critical speeds. This is because at higher frequencies the lead-lag controller is not providing sufficient phase lead to permit encirclement. At lower frequencies the loop phase lead is insufficient to provide encirclement; the rotor's collocated transfer function at the inboard end is phase lag until a pair of imaginary zeros are encountered at approximately

1800 rad/s.

The in-the-loop robustness of the alternative feedback method provides a good example of the care that must be taken in interpreting the result of μ robustness tests. In the case just examined phase lead resulted in instability in a narrow operating speed range ($\approx$ 17000-18000 rpm). The poor robustness indicated ($\approx$ 15000-20000 rpm) was due to sensitivity to unmodeled phase lead in the plant. However, the actual plant is very unlikely to have unmodeled phase lead just below the third critical speed. In fact, the actual system is likely to have significantly less phase lead at these frequencies than the model due to actuator, sensor, and amplifier dynamics as well as control system roll-off. Therefore, the model may be considered to be robust with respect to the anticipated in-the-loop variations. This demonstrates the importance of a proper interpretation of the results of these robustness tests. It should be noted that any unmodelled phase lag in the actual plant would stabilize those cases where the alternative filter destabilized the nominal system.

OPEN LOOP CONTROL

An alternative to using a feedback method to reduce the transmitted synchronous response is to use open loop control. This method uses the addition of synchronous control currents to the feedback control currents. The magnitude and phase of the synchronous currents are adjusted so as to minimize the synchronous housing vibration. [1] The bearings exert no synchronous forces; the open loop control force cancels the feedback control and passive bearing (negative stiffness) forces. Thus, the transmitted synchronous vibration is reduced *without feedback*. The stability and the robustness of the rotor is unaffected by the introduction of open loop control. In theory, the open loop control method works in the same manner as conventional single speed N-plane balancing where the balancing planes are the N bearing locations. Thus, the unbalance response at any N locations on the shaft may be simultaneously reduced to zero.

Using the high speed model compressor, the efficacy of open loop control has been demonstrated in a variety of experiments [12]. The open loop control was implemented through a simple but effective setup. A key phasor signal was used to generate a harmonic signal synchronous with rotor speed for each bearing axis. The magnitude and phases of each harmonic signal could be adjusted manualy. These open loop control signals are added as a perturbation to the feedback control signals for each bearing axis.

In this paper, recent results demonstrating the reduction in transmitted synchronous force are presented. The open loop control currents were adjusted so as to minimize synchronous acceleration measured on the housing with the rotor operating at 42300 rpm. This operating speed was a natural frequency of the housing. The resulting reduction in vibration as measured by accelerometers mounted on the housing and on the foundation is shown in Figure 6. Better than 90 % reduction in synchronous vibration was obtained throughout the structure. The reduction in the transmitted synchronous vibration would have been greater except for the transmission of synchronous forces through the conventional bearings in the air turbine used to drive the magnetically suspended rotor.

[1]The open loop control currents may be adjusted to minimize the rotor's synchronous response. See [11, 12] for details.

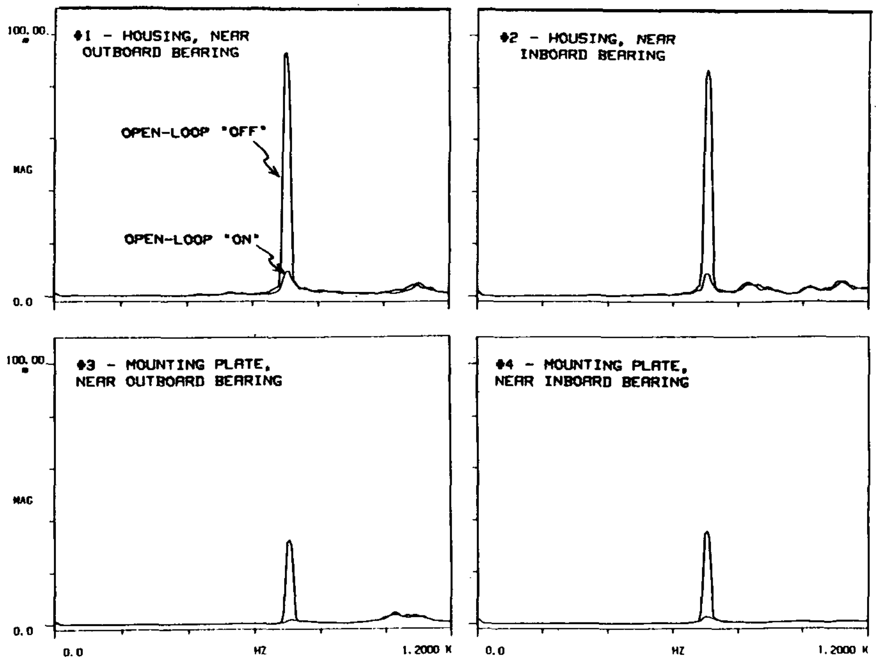

Figure 6: Frequency spectra of vibration amplitudes at several locations on the housing and foundation. With and without open loop control adjusted for minimum synchronous vibration on the housing near the outboard bearing. Rotor operating at 42300 rpm.

CONCLUSIONS

The robustness of several methods of attenuating the transmitted synchronous forces in magnetic bearing systems was examined. A 22 mass station model of a high speed model compressor was used for this investigation. The notch filter feedback approach was evaluated with respect to in-the-loop variations as a function of operating speed (notch center frequency) using structured singular value techniques. Poor robustness and, in some cases, even nominal instability resulted due to the excessive phase lag introduced below the notch center frequency. An alternative bandstop filter was then examined for in-the-loop robustness via the same structured singular value method. The same level of robustness as obtained with the nominal lead-lag controller was obtained with this feedback method of attenuation throughout the operating speed range.

Finally, open loop control experimental results from the high speed model compressor were presented. These demonstrated that the open loop method could produce over 20 dB attenuation in the synchronous vibration in the machine's housing and foundation. In the author's opinion, the open loop method is preferable to feedback methods for the reduction of transmitted synchronous response as it separates the rotor suspension and synchronous vibration control problems. As digital controllers for magnetic bearing systems become more prevalent, open loop control methods will move into a greater number of applications. Feedback controllers will then be designed solely to provide maximum stability. New tools for stability robustness, like the structured singular value test employed here, will prove useful in the design of these feedback control systems.

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