Identification of Rotor Unbalance and Reduction of Housing Vibration by Periodic Learning Control in Magnetic Bearings

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ABSTRACT

Rotating machinery has two major vibration problems caused by unbalance of a rigid rotor --- whirling motion of the rotor and vibration of the housing. In order to solve these vibration problems in magnetic bearings, the authors have proposed applications of periodic learning control (PLC) with inverse transfer function compensation (ITFC). This paper describes a new technique of applying the PLC to reduce the housing vibration. In both the former and the new methods for the reduction of housing vibration, the compensatory input calculated according to the PLC is applied to eliminate the fluctuations of coil-currents synchronous to the rotation. One of the factors that determine the properties of the PLC is where within the magnetic bearing system the compensatory input is applied. While the input was added to the current control signal in the former method, the input is added to the signal of rotor displacement in the new method. The new method has some advantages: (1) The input to eliminate the fluctuations of coil-currents converges to the amount of unbalance as the rotational speed becomes large. (2) The unbalance can be approximately identified when the PLC with ITFC is carried out at a higher rotational speed. (3) When the identified value of the unbalance is used as the compensatory input,
the housing vibration is reduced at any higher rotational speeds. Several experiments have been carried out to demonstrate the advantages of the new method.

1. Introduction

Rotating machinery has two major vibration problems caused by unbalance of a rigid rotor. One is whirling motion of the rotor. The other is vibration of the housing. Magnetic bearings can solve these problems by controlling the forces of electromagnets. The authors have proposed applications of periodic learning control (PLC) with inverse transfer function compensation (ITFC) to magnetic bearings in order to solve such vibration problems [1]. The rotor is assumed to be rigid in this paper.

The unbalance of the rigid rotor, i.e., difference between geometrical axis and principal axis of inertia acts on magnetic bearings as disturbance synchronous to rotation.

It causes synchronous fluctuations of the forces of electromagnets. They result in the housing vibration.

The housing vibration does not occur if the synchronous fluctuations of the forces of electromagnets are eliminated. Under such condition, the axis of rotation coincides with the principal axis of inertia [2]. The authors have proposed control schemes to eliminate the synchronous fluctuations of the forces [3]. However, it is generally difficult to measure the forces of electromagnets. On the other hand, it is easy to measure coil-currents of electromagnets. If the coil-currents do not fluctuate, the forces of electromagnets fluctuate very little as long as the unbalance is relatively small. The authors have shown by experiments that the PLC with ITFC can eliminate the synchronous fluctuations of coil-currents, and that this elimination reduces the housing vibration [1]. This paper describes another technique of applying the PLC to eliminate the fluctuations of coil-currents, and the advantages of the new method.

2. Periodic Learning Control (PLC) with Inverse Transfer Function Compensation (ITFC)

Rotating machinery is subject to periodic disturbances. Periodic learning control (PLC) can make periodic output error converge to zero [1].

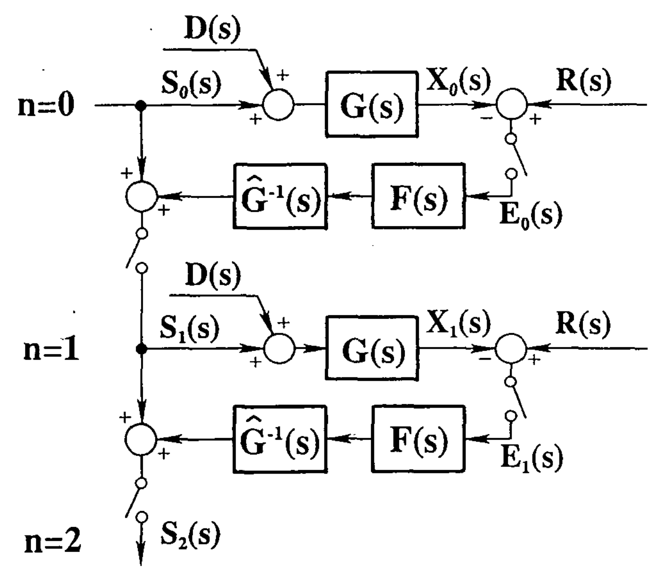


Fig.1 Block Diagram of Periodic Learning Control with Inverse Transfer Function Compensation

G(s): transfer function matrix of periodic controlled system;

In this paper, the transfer function matrix of the closed loop which consists of magnetic bearings and PID(PD)-compensator is G(s).

F(s): imaginary filter which extracts components fundamental in the controlled system

$\hat{G}^{-1}(s)$: inverse of G(s); ^ denotes that the identification of it has some error.

A computer functions as the element F(s) by means of Fourier series expansion, and also as $\hat{G}^{-1}(s)$.

X(s): output of the controlled system

E(s): output error; = R(s)-X(s)

S(s): periodic compensatory input ($S_0(s)=0$)

R(s): periodic reference input

D(s): periodic disturbance

n: trial number of periodic learning control

(All subscripts denote the trial number.)

Fig.1 shows a block diagram of PLC with ITFC. Let each variable in the diagram have N-DOF. N=4 in the magnetic bearing system, because 4-DOF of the rigid rotor are affected by the unbalance. Symbol ^ added to $G^{-1}(s)$ denotes that identification of $G^{-1}(s)$ may have some errors.

Provided that R(s)=0, the output error of the n^{th} trial is given by

$$E_n(s) = -G(s)\,\{I_n - \hat{G}^{-1}(s)F(s)G(s)\}^n\,W(s) \quad . \tag{1}$$

By means of Fourier series expansion, F(s) extracts specific components at 0, ω_f, $2\omega_f,\ldots,K\omega_f$, where ω_f is the fundamental angular frequency in the periodic system. In magnetic bearings, ω_f is rotational angular velocity. From eq.(1), the output error at $k\omega_f$ converges to zero if the identified $\hat{G}^{-1}(jk\omega_f)$ satisfies the following equation:

$$\| 1 - \hat{G}^{-1}(jk\omega_f)\,G(jk\omega_f) \|_2 < 1 \quad , \quad (k=0,1,2,\ldots,K) \quad , \tag{2}$$

where $\|\ldots\|_2$ denotes L2-norm. Therefore, even if the parameters of the controlled system are not accurately identified, the PLC with ITFC can make the output error converge to zero. When the aim of the PLC is the compensation of unbalance, k=1.

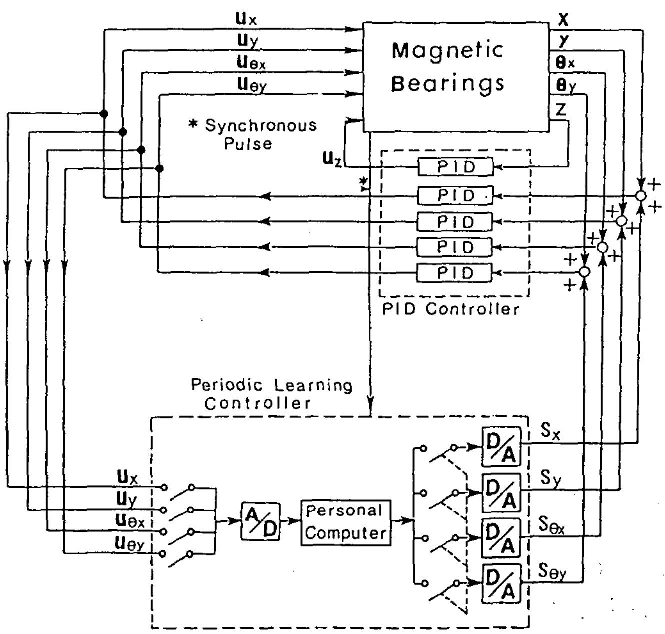


Fig.2 Structure of Control System

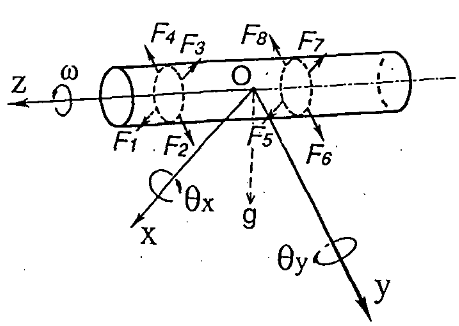


Fig.3 Definition of Coordinate System and Direction of Attractive Forces of Electromagnets

3. Application of PLC with ITFC in Magnetic Bearings

Fig.2 shows a structure of the control system. Fig.3 shows the definition of the coordinate system. The PID-controller stabilizes translational motion (x,y,z) and tilting motion (θ_x,θ_y) of the rotor, respectively; Such a control scheme is called centralized control. A symbol u_k in Fig.2 represents a current input which controls the electromagnetic force in the *k* direction (k=x,y,z,θ_x,θ_y). Coil-current in each electromagnet is determined from these current input through geometrical conversion.

As mentioned in chapter 2, the PLC with ITFC makes the periodic output error of the controlled object converge to zero. The closed loop composed of magnetic bearings and the PID-controller is treated as the controlled object (Fig.2).

There are several factors that determine the properties of applications of the PLC. One of them is the choice of the output variable. When the rotor displacement is treated as the output, the PLC suppresses the whirling motion. When the current input is treated as the output, the PLC reduces the fluctuations of coil-currents [1]. Another factor is where in the controlled object the compensatory input is applied. The authors have reported about the PLC with ITFC in which the compensatory input is added to the current input [1]. In this paper, the compensatory input is added to the signal of rotor displacement. The properties of this form of PLC will be mentioned in the following chapters.

In order to carry out the PLC with ITFC, a periodic learning (PL) controller is added to the controlled object (Fig.2). It works independently of the PID-controller. Consequently the PLC does not affect the stability of the magnetic bearings at all. The PL-controller samples the current input through an A/D-converter, carries out Fourier series expansion and multiplication of $\hat{G}^{-1}(s)$ (Fig.1), and applies the compensatory inputs through four D/A-converters which are added to the displacement signals. D/A-conversion is carried out 256 times a rotation with phase locked loop (PLL) in use. And so the PL-controller can generate synchronous signals even when the rotational speed is changing.

4. Identification of Unbalance

Fig.4 shows a block diagram of the subsystem (θ_x,θ_y) [4]. The subsystem (x,y) has the same structure if c=0 and K_c=0. The output of the controlled object is the current input U(s). The compensatory input S(s) is added to the rotor displacement Θ(s). Unbalance forces are given by

$$W(s) = (1-c)\tau\omega_f^2\ \mathcal{L}[\cos(\omega_f t+\beta)+j\cdot\sin(\omega_f t+\beta)] \quad , \qquad (3)$$

where τ is the amount of dynamic unbalance and ω_f is rotational angular velocity. Because the objective of the compensation by the PLC is to eliminate the synchronous fluctuations of U(s), the analysis only at the frequency $s=j\omega_f$ is required. If the fluctuation of $U(j\omega_f)$ is eliminated, the relationship between the input and the unbalance force is given by

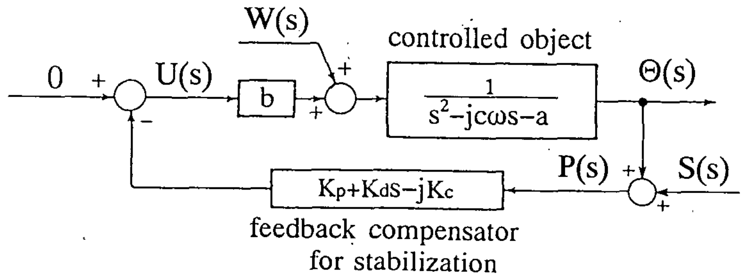


Fig.4 Block Diagram of Subsystem (θ_x,θ_y) in Complex Variable Representation
$\theta=\theta_x+j\theta_y$: angular rotor displacement , $u=u_{\theta x}+ju_{\theta y}$: current input
$w=w_{\theta x}+jw_{\theta y}$: unbalance force , $s=s_{\theta x}+js_{\theta y}$: compensatory input
a,b: coefficients of electromagnets , c : gyro-factor

$$S(j\omega_f) = \frac{1}{(1-c)\omega_f^2+a} W(j\omega_f) \quad . \tag{4}$$

Substitution of eq.(3) into eq.(4) yields

$$\begin{aligned} S(j\omega_f) &= \tau \frac{(1-c)\omega_f^2}{(1-c)\omega_f^2+a} \mathcal{L}[\cos(\omega_f t+\beta)+j\cdot\sin(\omega_f t+\beta)] \\ &\to \tau\, \mathcal{L}[\cos(\omega_f t+\beta)+j\cdot\sin(\omega_f t+\beta)] \quad (\omega_f \to \infty) \quad . \end{aligned} \tag{5}$$

Therefore, the amplitude and phase of $S(j\omega_f)$ to eliminate the fluctuation of $U(j\omega_f)$ vary little even if ω_f varies. And the execution of the PLC is approximately equivalent to the identification of the unbalance if ω_f is large. Moreover, it is expected that the input $S(j\omega_f)$ obtained at a certain higher rotational speed ω_f can reduce the housing vibration at other higher rotational speeds [2].

5. Experimental Result

The authors carried out an experiment to reduce the fluctuation of current input at the rotor speed of 950[rps] (51,000[rpm]). A section view of the magnetic bearing spindle used in the experiment is shown in Fig.5.

The procedure of the PLC with ITFC at the trial number 'n' is
[n.1] Compensation with s_n begins, (s_0=0),
[n.2] $\mathbf{u}_n$ is sampled by the A/D converter every 10[μs] throughout one rotation,
[n.3] $\mathbf{u}_n^*$ is calculated by means of Fourier series expansion,
[n.4] The next input s_{n+1} is calculated by the following equations:

$$\begin{aligned} \Delta s_n &= T^{-1}\cdot(-\mathbf{u}) \quad , \\ s_{n+1} &= s_n + \Delta s_n \quad , \end{aligned} \tag{6}$$

where, $\mathbf{u}^*(t)=[u_x^*,u_y^*,u_{\theta x}^*,u_{\theta y}^*]^T$: component of current input synchronous to rotation,
$s(t) =[s_x,s_y,s_{\theta x},s_{\theta y}]^T$: compensatory input,

$T_f^{-1}=(c_{PQ})$: 4×4 complex inverse transfer function matrix at f[rps],

$c_{PQ} = a_{PQ}\cdot\exp(j\cdot p_{PQ})$, (P,Q=1,..,4).

The input s_n is applied continuously during [n.2],..,[n.4].

"n=0→1" in Fig.6 (a) indicates the time when the compensation with s_1 began [1.1]. The figure shows that the residual synchronous fluctuations of the current inputs are not negligible. This estimation implies that the T_f^{-1} has some errors. "n=1→2" in Fig.6 (b) indicates the time when the input changed from s_1 to s_2 [2.1]. The figure shows that the fluctuations of the current inputs synchronous to the rotation became very small.

Fig.7 shows that the housing vibration is reduced when the synchronous fluctuations of the current inputs are reduced.

Similar experiments were carried out successfully at several rotational speeds. The compensatory inputs to eliminate the synchronous fluctuations of current inputs at each rotational speed is shown in fig.8. It shows that the input $S(j\omega_f)$ varies very little even if ω_f varies.

Table 1 shows the compensatory input s_2 by which the synchronous fluctuations of current inputs was eliminated at 950[rps] (Fig.6). Fig.9 shows the reduction of housing vibrations compensated with this input s_2 at various rotational speeds. The result shows that s_2 is equal approximately to the unbalance, and that it can reduce the housing vibration at other higher rotational speeds. Owing to the installation of phase locked loop (PLL) in the PL-controller, the housing vibration was reduced by s_2 even when the rotational speed is changing.

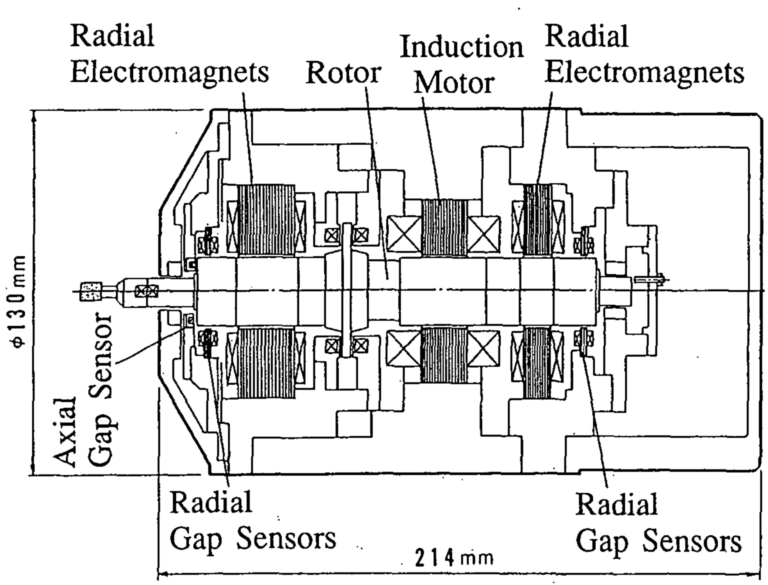
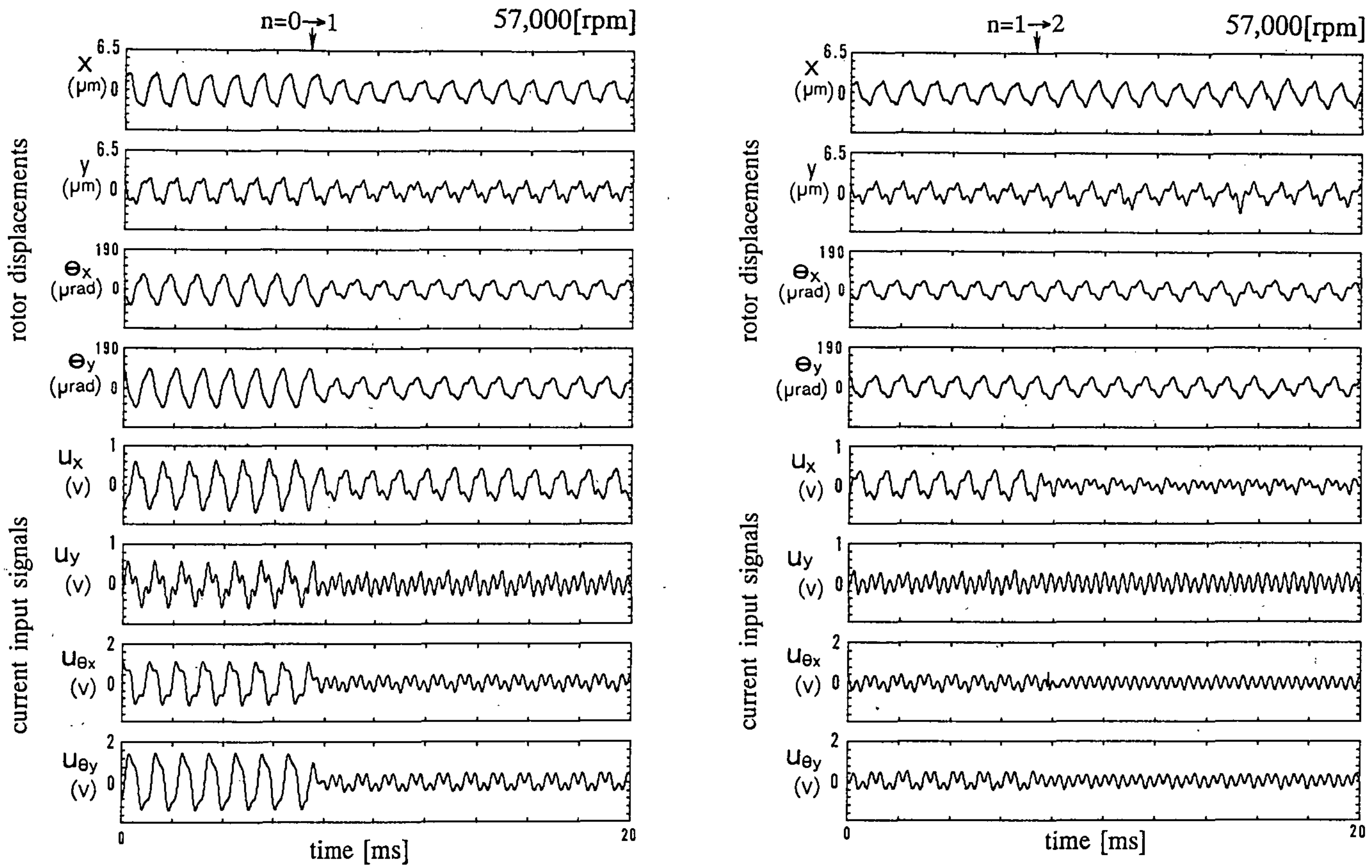


Fig.5 Cross Section of Magnetic Bearing Spindle
(Manufactured by Seiko Seiki Co., Ltd. , Japan)

Table 1 Compensatory Input at 950[rps] (57,000[rpm])
(Static and Dynamic Unbalance Identified)

$s_x + j\cdot s_y = (1.5[\mu m])\cdot\exp\{j(\omega_f t+135°)\}$
$s_{\theta x}+j\cdot s_{\theta y} = (40[\mu rad])\cdot\exp\{j(\omega_f t+115°)\}$



(a) n=0→1

(b) n=1→2

Fig.6 Reduction of Fluctuations of Current Inputs (950[rps]).

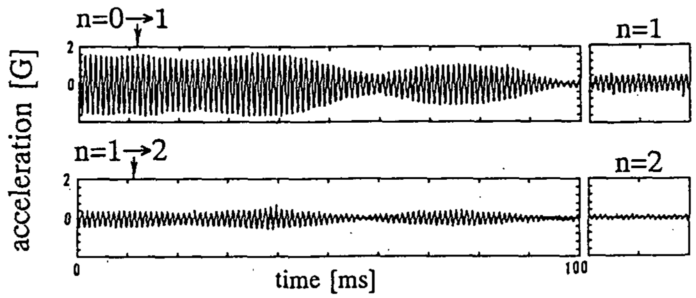


Fig.7 Housing vibration (950[rps]).
The accelerometer was attached to the metal band which fixes the spindle housing to the base.

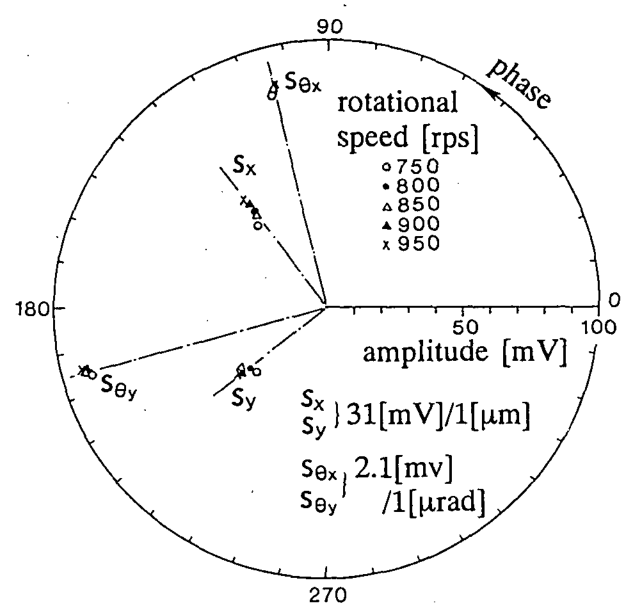


Fig.8 Compensatory Inputs to Eliminate the Fluctuations of Current Input at Various Rotational Speeds

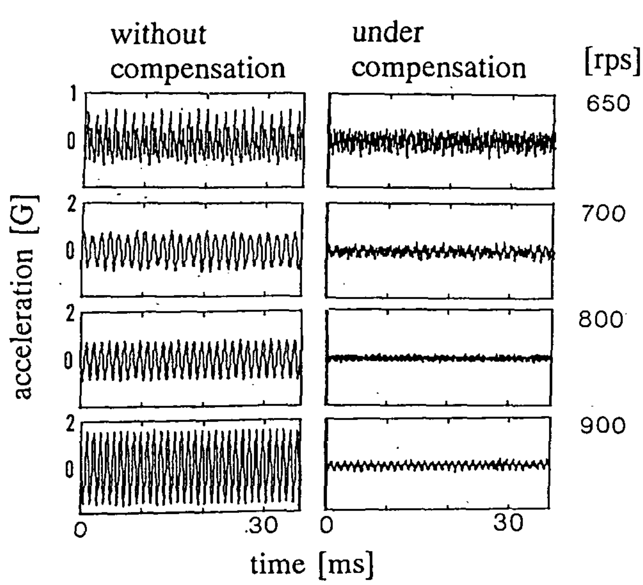


Fig.9 Housing vibrations with compensatory input shown in Table 1.

6. Conclusion

The authors proposed to apply the PLC with ITFC to magnetic bearings for the reduction of housing vibration. The PLC can carry out accurate compensation even if parameters of the magnetic bearings are not accurately identified. And the PLC does not affect the stability of the magnetic bearings at all. The compensatory input to reduce the housing vibration at a higher rotational speed is approximately equal to the unbalance of the rigid rotor. Experimental results assure the effectiveness of the PLC with ITFC:
(1) The PLC with ITFC can reduce the housing vibration by eliminating the fluctuations of coil-currents. (2) The compensatory input for the elimination at a higher rotational speed can reduce the housing vibrations at other higher rotational speeds. (3) The input for the elimination at a higher rotational speed is approximately the unbalance itself.

The implementation of the PLC with ITFC is easily accomplished by adding a digital periodic learning (PL-)controller to a conventional PID controller. Owing to the installation of phase locked loop (PLL) in the PL-controller, the housing vibration can be reduced even when the rotational speed is changing.

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