Optimum Design of Decentralized Magnetic Bearings for Rotor Systems

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ABSTRACT

Magnetic bearings are currently being explored as viable alternatives to existing hydrodynamic or rolling element support devices for high speed rotating machinery in various applications. Incorporation of active magnetic bearings into rotor systems can lead to enhanced stability characteristics by proper choice of the feedback control law. Traditionally low–order PD type controllers or high–order LQR/LQG based controllers have been used for such bearings. In this paper, a method of designing low–order decentralized magnetic bearing controllers for high–order plants is proposed. The controller is represented by a minimum set of design parameters (numerator and denominator coefficients of the transfer functions of the controller), and the optimum controller parameters are obtained by means of a numerical search in the parameter space. The various design specifications for such a design could be insurance of stability, boundedness of design parameters within upper and lower limits, placement of closed–loop eigenvalues within an acceptable region in the complex plane, and avoidance of closed–loop eigenfrequencies from an envelope around the rotor operating speed. Satisfaction of the multiple specifications is attempted by solving the problem as a sequence of constrained minimization problems, with more and more constraints being introduced in each subsequent stage. The design requirements specified in terms of the closed–loop characteristics of the system are achieved through a step by step process. The algorithm utilizes the method of feasible directions to solve the nonlinear constrained minimization problem at each stage. This methodology emphasizes the designer's interaction with the algorithm to generate acceptable controller designs by changing various specifications and altering the initial guesses interactively. A graphical interface has been developed to facilitate design interaction by the user.

INTRODUCTION

Magnetic bearings are being used in high speed rotor systems as an attractive and viable alternative to currently existing hydrodynamic or rolling element support devices in various applications. From merely the control standpoint, the ability to alter the magnetic bearing transfer function can lead to enhanced closed–loop stability characteristics and achievement of various closed–loop design criteria. Active magnetic bearing systems offer the designer enormous flexibility to choose the nature of the feedback control law and the parameters that govern the law. However, this flexibility cannot be put to effective use due to the limited availability of practical and implementable feedback design methods to exploit the range of available bearing parameters for rotor systems.

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Research in control system design and optimization applied to rotor systems has followed a number of different paths depending on the methodology and the final objective. Eigenvalue placement and eigenstructure assignment has been used as one control strategy. The problems associated with such a method are i) choice of a desirable eigenstructure is not obvious, ii) inability to achieve such a structure by output feedback, and iii) lack of a control magnitude penalty.

Controller design based on state space methods minimizing a quadratic cost function to obtain an optimal control law is another alternative approach [1,2]. Such designs, classified under the title of Linear Quadratic Regulator/Gaussian (LQR/LQG) problem, remedy some of the problems faced by eigenstructure assignment. Moreover, the guarantee of a closed–loop unique and stabilizing solution of this problem via the solution of the Ricatti equation is an attractive proposition. However, the main disadvantages of this method are i) the resulting controller requires access to all the rotor states, or to a state–observer, ii) the controller order is the same as the rotor model, iii) actual design specifications must be translated into a choice of weighting matrices, iv) LQR based controller generally exhibits poor robustness, and v) the scalar quadratic cost function may be inadequate to represent certain design objectives.

One approach to overcome some of these problems has been the development of decentralized fixed–order (typically low–order) controllers by prespecifying the feedback controller structure and minimizing a scalar quadratic cost function of the weighted output states and the weighted control inputs. The design of such controllers, often called output feedback controllers, involves a numerical parameter optimization of the controller structure, since a closed–loop solution to the problem is not available [3–6]. The main problems associated with such a method include i) requirement of an initially feasible starting guess, ii) inadequacy of the scaler quadratic cost function to represent different design objectives, and iii) handling of multiple local optima encountered during optimization.

This paper extends the concept of decentralized low–order controllers for the nominal higher order model of the rotor, with design objectives to be minimized that reflect practical needs of the rotor system designer. Numerical optimization methods have been used to determine the controller parameters, starting from an initial guess. Emphasis has been put on issues involving bad initial guesses resulting in unfeasible/unacceptable local maxima. A design example has also been included to demonstrate the methodology.

DESIGN VIA PARAMETER OPTIMIZATION

The basic objective of the methodology is to develop a design procedure for decentralized low–order controller for rotor systems to achieve certain specifications regarding the stability and performance of the closed–loop system. To do so, we need to briefly describe the concept of decentralized control. Decentralized, or local control is defined as a control mechanism based only on local state information, and can be regarded as a particular form of output feedback where certain elements of the feedback matrix are constrained to be zero [7–11]. A low–order decentralized controller for rotor system would consist of a bearing with simple dynamics, where the force at the bearing is dependent only on the measurement at the sensor location, and the relationship between the bearing force and the local measurement is a low–order transfer function. One of the simplest examples of such a scheme is selecting the "stiffness" and "damping" coefficients of a Proportional Derivative (PD) controller in order to control rotor response and stability.

The rotor system or the plant is represented by a second–order matrix differential equation, which is a fair approximation of the continuous system for modeling purposes provided the number of degrees of freedom chosen is large enough.

This can be readily converted to a first–order state–space form

$$\dot{x} = Ax + Bu \qquad (1)$$
$$y = Cx$$

where x = states of the rotor
u = input force vector for the rotor
and y = output vector for the rotor

For plants with high system order, a model reduction may be deemed necessary to improve the computational efficiency in the subsequent numerical optimization, and also to reflect the facts that the finite–element model is unable to correctly represent the high frequency dynamic behavior of the rotor, and that the bandwidth of the controller is limited by practical considerations. A reduced order model can be constructed by modal truncation method, dynamic condensation, or internal balancing.

The magnetic bearings are represented as low–order dynamic systems, with state–space description

$$\dot{x}_c = A_c x_c + B_c y \qquad (2)$$
$$u = C_c x_c + D_c y$$

where x_c = internal states of the controller
y = input vector for the controller
and u = output vector for the controller

In general, the order of the controller is chosen *a priori* to the design process. The state–space description of the closed–loop system is given as

$$\begin{bmatrix} \dot{x} \\ \dot{x}_c \end{bmatrix} = \begin{bmatrix} A + BD_cC & BD_c \\ B_cD & A_c \end{bmatrix} \begin{bmatrix} x \\ x_c \end{bmatrix} \qquad (3)$$

$$= \left\{ \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} B & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} D_c & C_c \\ B_c & A_c \end{bmatrix} \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix} \right\} \begin{bmatrix} x \\ x_c \end{bmatrix}$$

or, $\dot{\tilde{x}} = [\tilde{A} + \tilde{B}\ K\ \tilde{C}]\ \tilde{x} = A_{CL}\ \tilde{x}$

The problem has been converted to a static output feedback form, and the objective is to find the controller matrix K to satisfy the specified design requirements. To minimize the number of free parameters, it is necessary to convert $\{A_c, B_c, C_c, D_c\}$ to some canonical form. For this analysis, we have chosen the controller canonical form, where the controller matrices are represented as

$$A_c = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\beta_0 & -\beta_1 & -\beta_2 & \cdots & -\beta_{n-1} \end{bmatrix} \qquad B_c = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \qquad (4)$$

$$C_c = [\, \alpha_0 \;\; \alpha_1 \;\; \alpha_2 \cdots \;\; \alpha_{n-1} \,] \qquad\qquad D_c = \delta_0$$

and the corresponding transfer function is

$$G(s) = C_c(sI - A_c)^{-1} B_c + D_c \tag{5}$$

$$= \frac{\alpha_{n-1}\, s^{n-1} + \alpha_{n-2}s^{n-2} + \cdots\cdots + \alpha_1 s + \alpha_0}{s^{n-1} + \beta_{n-1}s^{n-1} + \beta_{n-2}s^{n-2} + \cdots\cdots + \beta_1\, s + \beta_0} + \delta_0$$

The problem of finding the controller matrix K is now translated into finding the vector of design parameters

$$z = [\alpha_{n-1}\; \alpha_{n-2} \cdots \alpha_1\; \alpha_0\; \beta_{n-1}\; \beta_{n-2} \cdots \beta_1\; \beta_0]$$

for each controller, which satisfies the design criteria. The controller canonical form may not be best suited for representing the transfer function since it may lead to numerical ill conditioning on the augmented system matrix A_{CL}. However, it has been chosen for the simplicity of representation. Any other canonical form could be used alternatively.

DESIGN SPECIFICATIONS

Specification of the objective function for minimization is one of the main issues of controller design for rotor systems. Selection of an appropriate objective function based on the closed–loop eigenvalues is a primary aspect of this paper. The general trend for control system designs based on LQR/LQG has been the minimization of the quadratic performance measure based on weighted state and control cost. However, such an objective function does not allow a direct specification based on the closed loop eigenvalues.

To overcome this limitation, we present performance measures and constraints defined in terms of the eigensolution of the closed–loop system. The performance index should ideally measure a sum of the stability margins of the individual eigenvalues, among other things. This leads to the formulation of a nonlinear constrained optimization problem where the feedback parameters are obtained by a vector search over the parameter space.

For rotor systems, the stability margin is often measured in terms of the logarithmic decrement of the damped eigenvalues defined as $\delta_i = -2\pi p_i/\omega_i$ where p_i, ω_i are the real and imaginary parts of the i^{th} damped mode. For most designs of rotor systems, it may suffice to have a minimum log decrement δ_L for modes below a certain lower cut–off frequency ω_L, and another value of log decrement δ_U for modes above another upper cut–off frequency ω_U, and a minimum log decrement $\delta_B(\omega_i)$ for modes in between. Also, it may be prescribed that no damped eigenvalue can lie within a specified envelope around the operating speed of the rotor (typically, 10% above and below the operating speed). These requirements translate into moving the closed–loop eigenvalues into an acceptable region of the complex plane. Attention is focused upon those eigenvalues that are outside the acceptable region and control effort is designed trying to bring them into this region [12]. From an optimization point of view, the objective can be formulated as the minimization of an acceptability function A, which is only required to be continuous and differentiable (almost everywhere) and have a value zero in the acceptable region and positive everywhere else.

$$\min A(z) \tag{6}$$

subject to the constraints

$$g_j(z) \leq 0$$

where z = vector of design parameters
and $g_j(z)$ = jth constraint

The acceptability function is *not* a performance index. It merely indicates if a solution is acceptable or not, and can be chosen at will by the designer to facilitate the optimization. Controller design will proceed based on a numerical search in the design parameter space, and a reduction in the value of *A* will occur at every iteration until a local minimum of A is reached. Thus, unless this occurs first, A will be eventually reduced to zero, yielding an acceptable solution to the problem. Convergence to a local non–zero minimum of A would call for restarting the search from a new initial guess, relaxing the constraints, or even a change cf the acceptability function to get out of the local minimum. Repeated failure to reduce A to zero will indicate the absence of an acceptable solution for the designer specified values.

The design problem requires the satisfaction of a set of specifications. Often, finding an acceptable solution considering all the specifications simultaneously as objective or constraints may become too costly from a computational point of view. This led to the idea of solving the design problem as a sequence of constrained minimization phases [13]. The order in which these phases occur in the sequence depend on the designer, though the 'harder' or more important constraints are put in as the initial phases. In our case, the optimization proceeds in four phases, with each phase consisting of a constrained (or unconstrained) minimization problem, and the objective function is converted into a constraint as it moves into the subsequent phase.

Phase I – Satisfaction of Stability Requirements

The optimization problem is

$$\min \left\{ \sum_{i=1}^{m} \max \left[0, p_i(z)\right] \right\} \tag{7}$$

subject to no constraints
where $p_i(z) = Re\, \lambda_i(z)$ and $\lambda_i(z)$ = i^{th} closed–loop eigenvalue

Phase II – Satisfaction of Lower and Upper Bounds on the Design Parameters

The optimization problem is

$$\min \left\{ \sum_{j=1}^{\ell} \max \left[0, (z_{jlow} - z_j), (z_j - z_{jup})\right] \right\} \tag{8}$$

subject to $p_i(z) \leq 0$ $i=1,\ldots,m$ Stability constraints
where z_j = jth element of the design vector z
and z_{jlow}, z_{jup} = lower and upper bounds on z_j

Phase III – Satisfaction of Logarithmic Decrement Requirements

The optimization problem is

$$\min \left\{ \sum_{i=1}^{m} \max \left[0, \left(\delta_{ispec} - \delta_i\right)\right] \right\} \tag{9}$$

subject to $p_i(z) \leq 0$ $i=1,\ldots,m$ Stability constraints

$z_{jlow} \leq z_j \leq z_{jup}$ $j=1,\ldots,\ell$ Design parameter constraints

where $\delta_i(z)$ = logarithmic decrement for the i^{th} eigenvalue

and δ_{ispec} = specified logarithmic decrement value for the i^{th} eigenvalue

Phase IV – Satisfaction of Operating Speed Requirements

The optimization problem is

$$\min \left\{ \sum_{i=1}^{m} \min \left[\left(\alpha_u \Omega - \omega_i\right), \left(\omega_i - \alpha_L \Omega\right)\right] \right\} \tag{10}$$

subject to $p_i(z) \leq 0$ $i=1,\ldots,m$ Stability constraints

$z_{jlow} \leq z_j \leq z_{up}$ $j=1,\ldots,\ell$ Design parameter constraints

$\delta_i(z) \geq \delta_{ispec}$ $i=1,\ldots,m$ Logarithmic decrement constraints

Constrained numerical search schemes can be used to minimize the acceptability function within each stage. Sequential unconstrained minimization techniques (SUMT) using penalty functions, the method of feasible directions, or the generalized reduced gradient method may be used [14, 15]. The method of feasible directions has been adopted here as the numerical search strategy, which starts from an initial guess and proceeds by iteratively searching along the feasible directions. If no constraints are violated, unconstrained methods like conjugate gradient (Fletcher–Reeves), variable metric (Davidson–Fletcher–Powell, Broyden–Fletcher–Goldfarb–Shanno), or nongradient (Powell) may be employed within the feasible directions method. The objective and the constraint functions are evaluated at each iteration and within the unidimensional line search for finding new estimates of the controller parameters, while the gradient information for the objective function and the active constraints is calculated at the end of each iteration.

APPLICATION TO ROTOR SYSTEMS

The rotor system chosen to illustrate the design methodology is a uniform symmetric beam 50 inches in length, and 4 inches in diameter. The rotor has been modeled by 11 lumped mass stations, and the order of the system is 44 (each mass station or node is associated with four degrees of freedom, two translational and two rotational). Assuming negligible rotational inertia and gyroscopic effects of the shaft, the rotational degrees of freedom can be condensed out by Guyan reduction, and the system order can be reduced by four to 11 by virtue of the symmetry in the horizontal and vertical coordinate directions. The rotor is supported at the two ends by magnetic bearings represented as two low–order decentralized controllers (Fig. 1). For this example case, the controllers are implemented as fourth–order strictly proper transfer functions, with the velocity and displacement at the two ends as the outputs and the control forces as the inputs. The coefficients of the transfer function for the initial guess are chosen randomly and the following example represents one successful case from these initial random guesses. The initial controller transfer function is

$$G_1^0(s) = \frac{7.208\text{x}10^{12}\ s^3 + 2.6618\text{x}10^{17}\ s^2 + 2.738\text{x}10^{21}\ s + 8.607 \text{ x } 10^{24}}{s^4 + 1.236\text{x}10^6\ s^3 + 3.930\text{x}10^{11}\ s^2 + 6.672\text{x}10^{15}\ s + 2.937\text{x}10^{19}} \quad (11)$$

$$= \frac{7.208\text{x}10^{12}\ (s + 2.229\text{x}10^4)(s + 7319 + i\,7.47)(s + 7319 - i7.47)}{(s + 6.104\text{x}10^5)(s + 6.083\text{x}10^5)(s + 8854 + i840)(s + 8854 - i840)}$$

$$G_2^0(s) = G_1^0(s)$$

The initial design parameter vector consists of the numerator and denominator coefficients of the transfer functions $G_1^0(s)$ and $G_2^0(s)$

z^0 = [7.208x10^12^ ...]

The pole–zero locations and the bode plot of the transfer function for this initial guess of the design parameters are shown in Fig. 2. The corresponding closed–loop system eigenvalues and logarithmic decrements are listed in Table 1.

The design specifications are laid down as follows:

I. Stability of the system must be insured. This implies

$$Re\ \lambda_i(z) \leq 0 \quad \text{or,} \quad p_i(z) \leq 0$$

II. Upper and lower bounds on the coefficients of the transfer function have been fixed. To limit the bandwidth of the system, the requirements are chosen as

$$1 \leq z_j \leq 1 \text{ x } 10^{30}$$

III. The logarithmic decrement δ requirements have been established

A minimum log dec $\delta_L = 2.0$ for modes below a lower cut–off frequency $\omega_L = 800$ rad/sec.

A minimum log dec $\delta_U = 0.01$ for modes above an upper cut–off frequency $\omega_U = 15{,}000$ rad/sec.

A minimum log dec $\delta_B(\omega_i)$ given by a straight line interpolation between δ_L and δ_U for modes with frequency $\omega_L \leq \omega_i \leq \omega_U$.

IV. No operating speed envelope requirements have been requested.

The design parameter vector is subjected to the constrained optimization procedure as described in the previous section. Within the feasible region, a conjugate–gradient (Fletcher–Reeves) scheme is adopted, and the optimization is terminated after 7 iterations, yielding the final design vector

$$z^* = [7.083\text{x}10^{12}\ 7.083\text{x}10^{12}\ 3.122\text{x}10^{17}\ 3.122\text{x}10^{17}\ 5.349\text{x}10^{21}\ 5.349\text{x}10^{21}\ 2.082\text{x}10^{23}\ 2.082\text{x}10^{23}\ 1.232\text{x}10^6\ 1.232\text{x}10^6\ 3.703\text{x}10^{11}\ 3.703\text{x}10^{11}\ 3.755\text{x}10^{15}\ 3.755\text{x}10^{15}\ 4.204\text{x}10^{19}\ 4.204\text{x}10^{19}]$$

Translated into the transfer function form, the resultant controllers are

$$G_1^* = \frac{7.083\text{x}10^{12}\, s^3 + 3.122\text{x}10^{17}\, s^2 + 5.349\text{x}10^{21}\, s + 2.082\text{x}10^{23}}{s^4 + 1.232\text{x}10^6\, s^3 + 3.703\text{x}10^{11}\, s^2 + 3.755\text{x}10^{15}\, s + 4.204\text{x}10^{19}} \tag{12}$$

$$= \frac{7.083\text{x}10^{12}(s+39)(s+2.202\text{x}10^4 + i1.639\text{x}10^4)(s+2.202\text{x}10^4 - i1.639\text{x}10^4)}{(s+7.356\text{x}10^5)(s+4.864\text{x}10^5)\ (s+5046+i9593)(s+5046-i9593)}$$

$$G_2^*(s) = G_1^*(s)$$

It is to be noted that symmetry is retained though it was not imposed explicitly during the optimization. The pole–zero locations and the bode plot of the transfer function for the resultant decentralized controller are shown in Fig. 3. Even though the structure of the bode plots remain similar over the dynamic range of the system, some loop shaping has occurred in the optimized controller leading to an improved design. The static gain shows a decrease, while there is an introduction of additional dynamics due to the complex controller poles at 9593 rad/sec. This would have the effect of having a frequency dependent damping coefficient on the plant. A simple PD type controller would not be able to provide such variable damping within the bandwidth of the system. The corresponding closed–loop system eigenvalues are listed in Table 2. The logarithmic decrement for the lower modes have been increased considerably to meet the requirements, and some of the modes have been damped out in the process. The results indicate that all the specifications have been met fully and the resultant design is an appreciable improvement over the initial guess.

A design oriented user interface is extremely important for engineering design optimizations such as this, to provide visual information to the user. Ideally, the information at the end of each iteration process should be available graphically to the user, and the ability to transfer control to the user to enable him/her to change various program variables must be provided. Fig. 4 shows the graphical display at the beginning of the design process, for the initial guess of the design vector. The bottom half of the screen shows the acceptability regions for the closed–loop eigenvalues in terms of the logarithmic decrements and the real and imaginary parts of the eigenvalues. The top half displays the upper and lower bounds on the design parameters and their values at the initial guess and at the end of each iteration. The corresponding display at the end of the optimization process is shown in Fig. 5, clearly displaying the results of the particular optimization we have discussed.

CONCLUSIONS

A method has been presented for the design of low–order decentralized controllers for rotor systems by constrained parameter optimization. The controller has been represented in terms of a control canonical form, to reduce the number of free parameters or design variables. Instead of minimizing a performance index, the method emphasizes satisfying a set of specifications laid down by the designer represented by acceptability functions through a sequence of constrained minimization problems. The proposed methodology has been illustrated by means of an example, and the accompanying graphical user interface has been demonstrated. Although the method shares the problems of other parameter optimization techniques such as providing a good initial guess and not guaranteeing a solution if one exists, the reduced complexity and flexibility of the controller structure and the ability to handle different practical design constraints directly make it a very viable alternative to other design methods.

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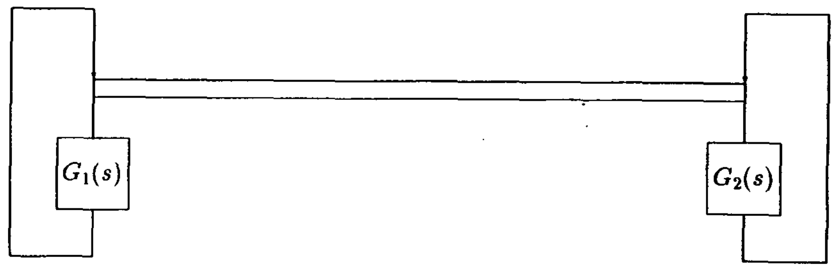

Fig. 1. Rotor supported on two magnetic bearings (low–order decentralized controllers)

Table 1. Initial guess design

PHASE 3 MINIMIZATION OF ACCEPTABILITY REGION VIOLATION WITH STABILITY AND BOX CONSTRAINTS

EIGENVALUE NO.	REAL PART (1/SEC)	IMAG PART (RAD/S)	COMPLEX MODULUS	LOG DECREMENT	ACCEPTABILITY REQUIREMENT	DIFFERENCE
1 (	-6.4030,	668.4637)	668.4944	.0602	2.0000	1.9398
2 (	-107.9883,	1747.2556)	1750.5895	.3883	1.8673	1.4789
3 (	-237.2705,	2908.6404)	2918.3019	.5125	1.7045	1.1919
4 (	-234.7262,	5262.9322)	5268.1639	.2802	1.3746	1.0943
5 (	-8736.2564,	941.1672)	8786.8067	58.3228	1.9802	.0000
6 (	-8736.4289,	941.2107)	8786.9829	58.3213	1.9802	.0000
7 (	-167.2305,	9374.5829)	9376.0744	.1121	.7984	.6863
8 (	-114.6667,	14951.9903)	14952.4299	.0482	.0167	.0000
9 (	-75.7163,	21826.0283)	21826.1596	.0218	.0100	.0000
10 (	-46.0429,	29824.0017)	29824.0372	.0097	.0100	.0003

ACCEPTABILITY MINIMIZATION FN = 6.39158111

Table 2. Final optimized design

PHASE 3 MINIMIZATION OF ACCEPTABILITY REGION VIOLATION WITH STABILITY AND BOX CONSTRAINTS

EIGENVALUE NO.	REAL PART (1/SEC)	IMAG PART (RAD/S)	COMPLEX MODULUS	LOG DECREMENT	ACCEPTABILITY REQUIREMENT	DIFFERENCE
1 (	-39.8749,	.0000)	39.8749			
2 (	-42.1978,	.0000)	42.1978			
3 (	-752.2274,	826.8541)	1117.8255	5.7161	1.9962	.0000
4 (	-1394.5733,	.0000)	1394.5733			
5 (	-1949.0420,	.0000)	1949.0420			
6 (	-1069.5539,	4295.7279)	4426.8752	1.5644	1.5101	.0000
7 (	-2583.7191,	8541.7922)	8924.0023	1.9005	.9151	.0000
8 (	-3204.2184,	9590.2315)	10111.3578	2.0993	.7681	.0000
9 (	-1315.2465,	10377.6490)	10460.6631	.7963	.6578	.0000
10 (	-122.3856,	15339.8155)	15340.3037	.0501	.0100	.0000

ACCEPTABILITY MINIMIZATION FN = .00000000

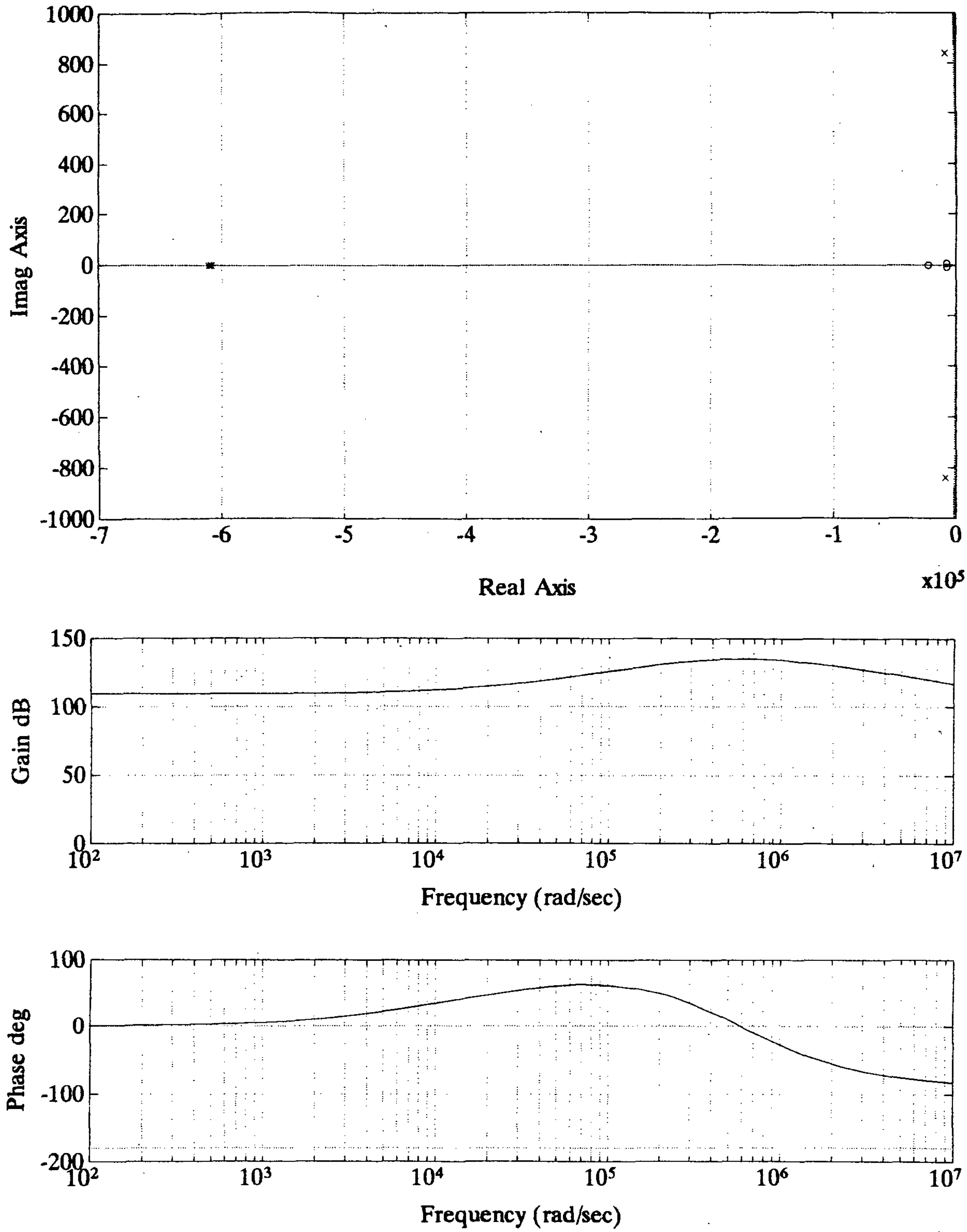

Fig. 2. Pole–zero locations and bode plot of the controller transfer function for initial guess design.

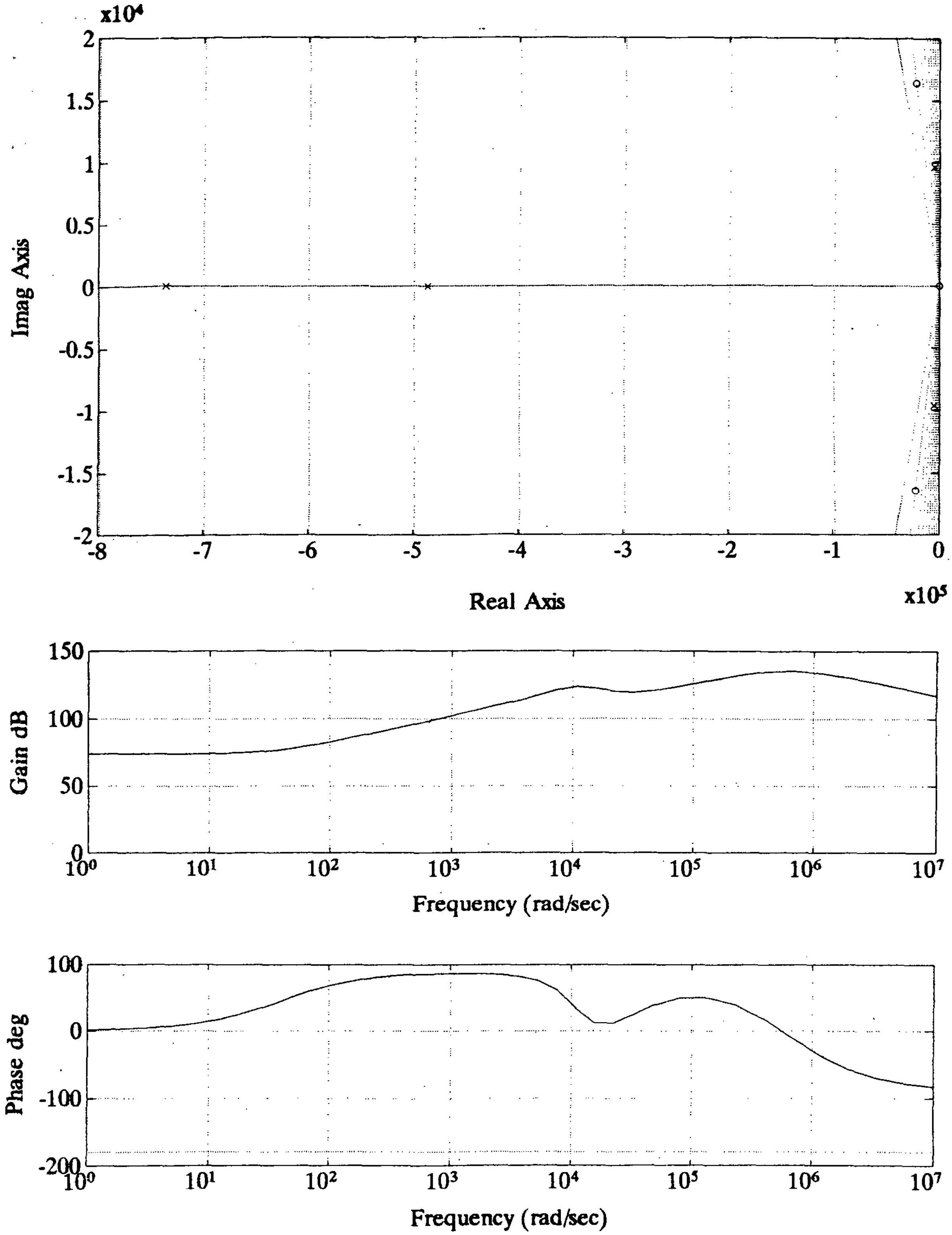

Fig. 3. Pole–zero locations and bode plot of the controller transfer function for initial guess design.

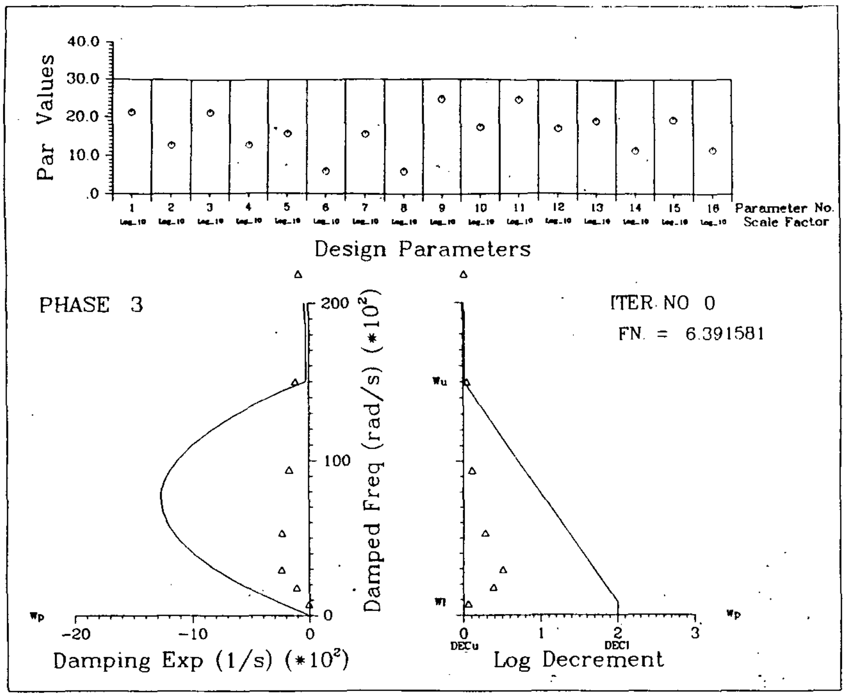

Fig. 4. Graphical display of design specifications, closed–loop eigenvalues and design parameters for initial guess design.

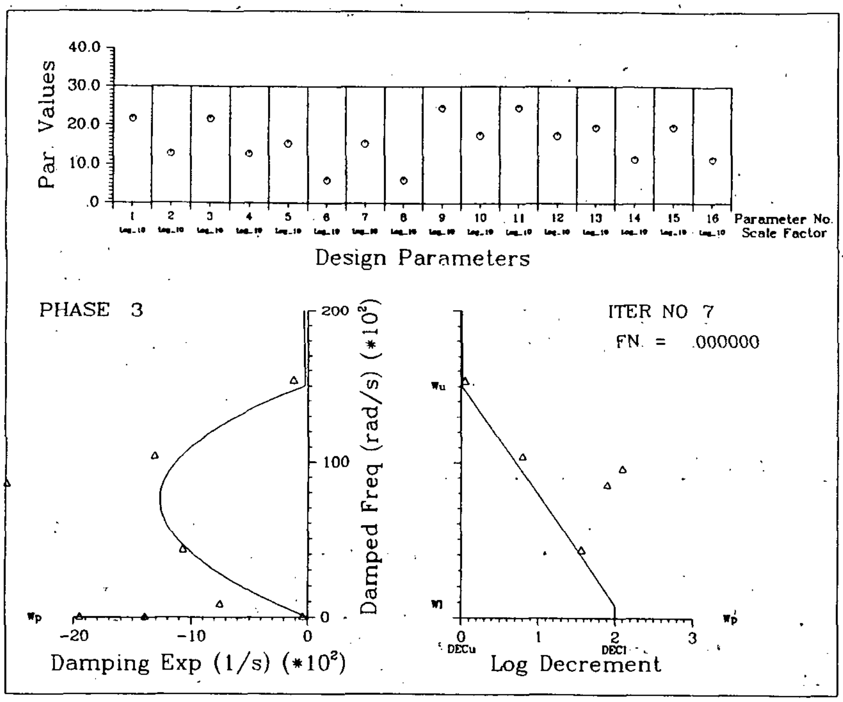

Fig. 5. Graphical display of design specifications, closed–loop eigenvalues and design parameters for optimized design.