Maximum Stability Area Optimal Control for Active Magnetic Bearings

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ABSTRACT

Industrial use of magnetic bearings and suspensions leads to new requirements for actuators, sensor systems and especially for control systems. This paper presents synthesis methods for magnetic bearings control algorithms according to the criterion of maximum stability area in space or phase coordinates, taking into account the limitations for values of control voltages on windings of electromagnets. The influence of eddy currents and resilient properties of construction on magnetic bearings dynamic characteristics is examined.

INTRODUCTION

Modern level of development of engineering puts forward more and more contradictory requirements for bearings of rotor machines which in a number of cases are practically unrealizable if we use existing rolling-contact bearings and plain bearings. Magnetic bearings with external control system are regarded as an alternative to existing rolling-contact bearings and plain bearings. Owing to absence of mechanical contact there is no necessity for lubrication, the magnetic bearings can operate in vacuum and corrosive mediums as well as in wide range of temperatures, there are no problems of friction, wear and noise; these bearings allow for developing maximum rotational speeds. From recent publications [1,2,3,4] it is clear that magnetic bearings and suspensions are beginning to be used in significant numbers of rotating machinery and other applications. Industrial use of magnetic bearings and suspensions leads to new requirements for actuators, sensor systems and especially for control systems.

Peculiarity of regulation processes in electromagnetic suspension systems consists of self-unstability of the suspended body. Area of stability of a body, suspended in

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conrolled electromagnetic field is always limited because of voltage limitations on electromagnets windings. Area of stability in state variables space appears to be an important characteristic of stabilization quality and it defines the optimum criterion for control algorithms synthesis. Magnetic bearings pertain to the systems with small gaps between electromagnets and the suspended body where limitations of control voltages are not so considerable because admissible rotor displacements are small. However in the present paper the optimum criterion is described which can be used in small gaps systems because algorithms optimum as regards to dimensions of stability area allow one to achieve maximum permissible phase coordinates deviations for given limitations of control voltage. Also they allow for the estimations of maximum permissible phase coordinates deviations and the estimations of maximum permissible external disturbances. For instance, estimation of permissible force impulse acting on rotor can be obtained on the base of permissible initial velocity of rotor displacement.

THEORETICAL BACKGROUND

Synthesis methods consist in the following. Considered are dynamics equations of EMS system which are linearized.

$$\frac{dx}{dt} = A\cdot x + B\cdot U + F \quad , \tag{1}$$

where: x-vector of the state of order n; U-vector of the control voltage; $|U|\leq U_o$; F-vector of the disturbing influence;

A,B - matrixes of order (n×n),(n×r) accordingly.

Equations (1) using linear coordinates transformation

$$x = L\cdot z \tag{2}$$

can be brought to a canonical form

$$\frac{dz}{dt} = \Lambda\cdot z + B'\cdot U + F' \quad , \tag{3}$$

where: $\Lambda = L^{-1}\cdot A\cdot L$ - canonical form of matrix A.

$$A\cdot L = L\cdot\Lambda \; ; \; z = L^{-1}\cdot x \; ; \; B' = L^{-1}\cdot B \; ; \; F' = L^{-1}\cdot F \; . \tag{4}$$

Canonical form of state equations contains two undependent subsystems

$$\frac{dz^-}{dt} = \Lambda^-\cdot z^- + B'^-\cdot U + F'^- \; ; \tag{5}$$

$$\frac{dz^+}{dt} = \Lambda^+\cdot z^+ + B'^+\cdot U + F'^+ \; ; \tag{6}$$

where stable subsystem (5) conforms to matrix A eigenvalues with negative real parts. Unstable subsystem (6) conforms to matrix A eigenvalues with positive real parts. Area of system controllability (number of initial states, from which the system can be brought to desirable state) consists of a number, limited for canonical variables, which corresponds to positive eigenvalues [5]. System stability area (number of points in state space, from which the object of control comes to the beginning of coordinates within a limited period of time when control is organized as a function of state coordinates) is a part of controllability area or coincide with it. According to [6] to satisfy the criterion of the maximum stability area it is enough to control in an optimum way the unstable subsystem (6) of system (3), and we may consider the speed-optimum partly-constant relay control algorithm as an optimum according to the dimensions of stability area algorithm. The latter exists always if there is even one control, which brings a phase point from initial position to final.

OPTIMIZATION OF CONTROL FOR ONE-DEGREE-OF-FREEDOM SYSTEM

Let's consider the one-degree-of-freedom system of electromagnetic suspension. This system conforms to thrust active magnetic bearing (AMB) and radial AMB when reciprocal influence of vertical and horizontal coordinates is absent (Fig.1).

The equations describing the EMS system without controller are like these:

$$\begin{cases} m\cdot\dfrac{d^2x}{dt^2} = F - F_m;\ F_m = F_m(i,x) \cong F_m(i_0,x_0) + ai + bx; \\ U = R\cdot i + L\cdot\dfrac{di}{dt} + i\cdot\dfrac{\partial L}{\partial x}\cdot\dfrac{dx}{dt}, \end{cases} \tag{7}$$

where: m-rotor mass; x - regulating coordinate; F - external disturbing force; F_m - magnetic force;

$a = \dfrac{\partial F_m}{\partial i}$ -current stiffness; $b = -\dfrac{\partial F_m}{\partial x}$ -position stiffness;

R,L-resistance and inductance of winding; i-current;
U - control voltage ($|U| \le U_0$).

Let's think that $C = i_0\dfrac{\partial L}{\partial x} = 0$.

linearized system of equations (7) is like this:

$$\begin{cases} \dot{x}_1 = x_2 = \dot{x};\ \dot{x}_2 = x_3; \\ \dot{x}_3 = \dfrac{R\cdot b}{L_0\cdot m}\cdot x_1 + \dfrac{b}{m}\cdot x_2 - \dfrac{R}{L_0}\cdot x_3 - \dfrac{a}{L_0\cdot m}\cdot U + \dfrac{R}{L_0\cdot m}\cdot F \end{cases} \tag{8}$$

System (8) has eigenvalues: $\lambda_1=\sqrt{\frac{b}{m}}$; $\lambda_2=-\sqrt{\frac{b}{m}}$; $\lambda_3=-\frac{R}{L_o}$, where $\lambda_1>0$ – is determined by the own unstability of the rotor according to the x coordinate. According to the methods of previous part we get the speed-optimum algorithm for unstable z_1 coordinate as an optimum according to the dimensions of stability area algorithm:

$$U=-U_o\cdot \text{sign}\ z_1. \tag{9}$$

For real coordinates:

$$U=-U_o\cdot \text{sign}\ \frac{1}{2\cdot\sqrt{\frac{b}{m}}\cdot\left(\frac{R}{L_o}+\sqrt{\frac{b}{m}}\right)}\cdot\left[\frac{R}{L_o}\cdot\sqrt{\frac{b}{m}}\cdot x+\left(\frac{R}{L_o}+\sqrt{\frac{b}{m}}\right)\cdot\dot{x}+\ddot{x}\right] \tag{10}$$

Stability area of rotor in space or phase coordinates $x, \dot{x}, \ddot{x}$ is the number of points between two infinite surfaces (Fig.2) which is determined as follows:

$$\left|\frac{R}{L_o}\cdot\sqrt{\frac{b}{m}}\cdot x+\left(\frac{R}{L_o}+\sqrt{\frac{b}{m}}\right)\cdot\dot{x}+\ddot{x}\right|\le\frac{a\cdot U_o}{L_o\cdot m\cdot\sqrt{\frac{b}{m}}} \tag{11}$$

This equation allows oneto have the estimations of maximum permissible phase coordinates deviations.

Static load capacity of bearing is like this:

$$F_{max}=\frac{a\cdot U_o}{R} \tag{12}$$

The Figure 3 presents a structural scheme of the AMB control system.

INFLUENCE OF EDDY CURRENTS ON ROTOR DYNAMIC CHARACTERISTICS

Eddy currents occur in electro-conductive ferromagnetic parts of AMB magnetic system if there exists variable magnetic flux. Secondary field, induced by eddy currents, acts opposite the initial exciting field. Eddy current value depends on electric conductivity and magnetic permeability of material and also on magnetic system geometrical parameters and speed of magnetic flux variation. Dynamic characteristics of magnetic cores are defined mainly by eddy currents.

Real eddy current can be presented as a current in equivalent secondary winding on magnetic core surface. Here the investigation of AMB dynamics transforms to investigation of dynamic characteristics of two-windings transformer with variable air gap to which equation of motion of mobile mass is added.

Relatively to magnetic circuit of AMB the eddy current is a source of magnetomotive force and it can be presented by equivalent current of the same value in one-turn secondary

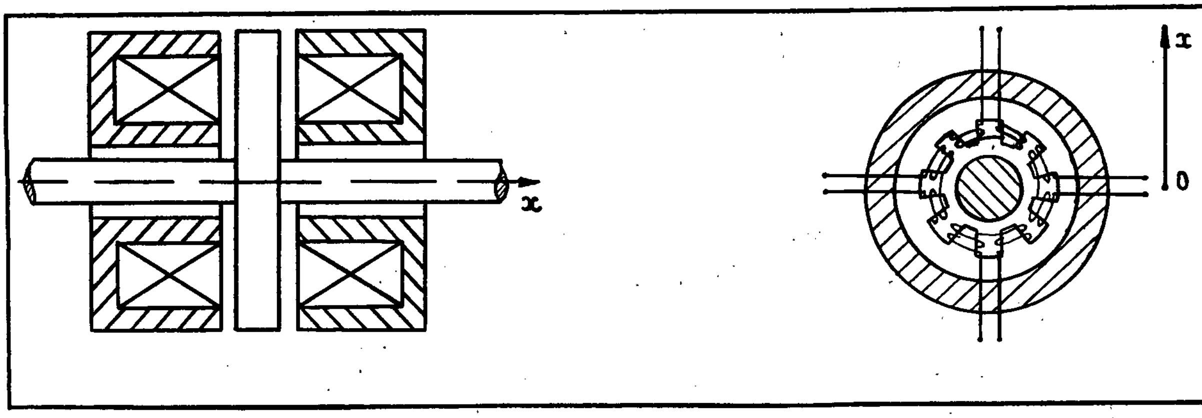

Figure 1.

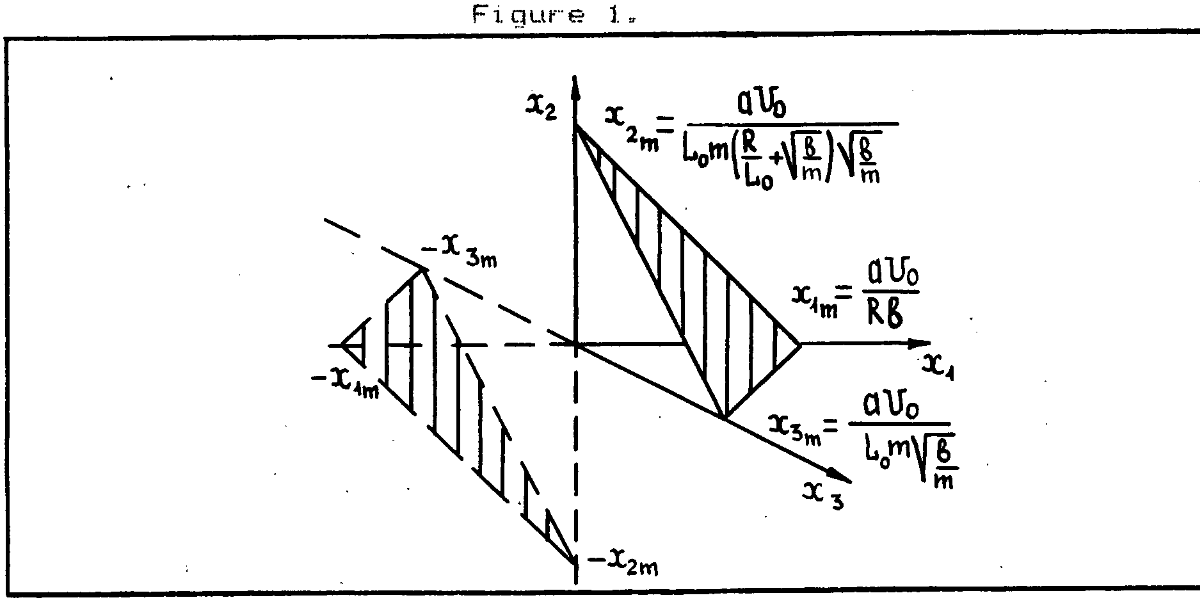

Figure 2.

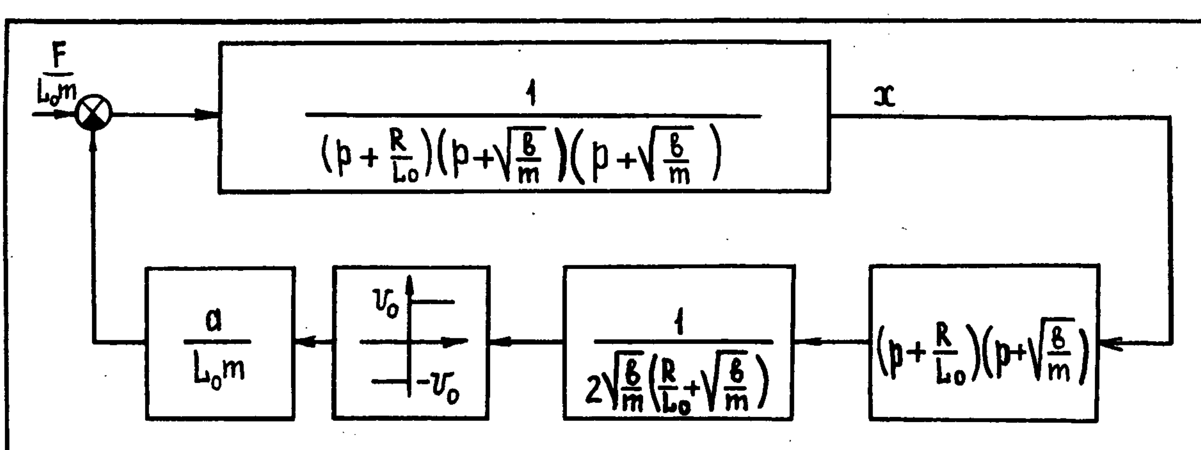

Figure 3.

winding. Fig.4 presents equivalent electric schemes of AMB. The parameters of equivalent scheme (Fig.4.b) are like these [7]:

$$L_1 - K\cdot M = W_1^2\cdot G - \frac{W_1 (W_1 W_2 G)}{W_2} = 0;\quad L_2 - \frac{M}{K} = W_2^2\cdot G - \frac{W_1 W_2 G\cdot W_2}{W_1} = 0\,, \quad (13)$$

where L_1, L_2, M – inductances of windings W_1, W_2 and their reciprocal inductance accordingly; G – total conductivity of magnetic circuit.

Taking into account (13) the equivalent scheme of AMB winding can be presented as in Fig.5. Electromechanical processes dynamics can be described by the following equations system:

$$\begin{cases} U = R_1 i_1 + L\cdot\dfrac{di_m}{dt} + i_m\dfrac{\partial L}{\partial i_m}\cdot\dfrac{di_m}{dt} + i_m\dfrac{\partial L}{\partial x}\cdot\dfrac{dx}{dt} \quad ;\; U = R_1 i_1 + R_2' i_2' \\ i_m = i_1 - i_2' \;;\; m\cdot\dfrac{d^2x}{dt^2} = F - F_m \;;\; F_m = F_m(i_m, x) \;;\; L = L(i_m, x) \end{cases} \quad (14)$$

where i_1–current of electromagnet winding; i_2'–current of secondary winding W_2, which is equivalent to eddy current; i_m – magnetization current.

This case differs from previous case because the time constant T of AMB winding includes time constant T_2 which characterizes the transient process in secondary winding W_2.

$$T = \frac{L_e}{R_e} = \frac{R_1 + R_2'}{R_1\cdot R_2'}\cdot\left[L_o + i_{mo}\cdot\frac{\partial L}{\partial i_m}\right] = \frac{1}{R_1}\cdot\left[L_o + i_{mo}\cdot\frac{\partial L}{\partial i_m}\right] + \frac{1}{R_2'}\cdot\left[L_o + i_{mo}\cdot\frac{\partial L}{\partial i_m}\right] = T_1 + T_2. \quad (15)$$

Let's think as in previous case that C=0. Following the methods of optimization we get optimum control algorithm like (9), where

$$z_1 = \left[x + \frac{1}{\sqrt{\frac{b}{m}}}\cdot\dot{x} - \frac{a/m}{\sqrt{\frac{b}{m}}\cdot\left(\sqrt{\frac{b}{m}} + \frac{R_e}{L_e}\right)}\cdot i_m \right] \quad (16)$$

Taking into account connection between magnetization current i_m and real electromagnet current i_1, we get a structural scheme of the AMB optimum control system which is presented in Fig.6.

Peculiarity of this case consists of sluggishness of current i_1 feedback, because ordinary current feedback does not influence the time constant T_2 which is conditioned by eddy currents influence.

INFLUENCE OF CONSTRUCTION RESILIENT PROPERTIES ON MAGNETIC BEARING DYNAMIC CHARACTERISTICS

Increase of mass and dimensions of structural elements of AMB call for necessity to take account of resilient properties of construction when control systems of AMB are designed.

Taking into account the resilient properties of AMB construction the equations describing the AMB system (Fig.7) without controller look like these:

$$\begin{cases} m_1 \ddot{x}_{m1} + k_1 \dot{x}_{m1} + c_1 x_{m1} = a \cdot i - b \cdot (x_{m2} - x_{m1}); \\ m_2 \ddot{x}_{m2} = -a \cdot i + b \cdot (x_{m2} - x_{m1}); \quad U = R \cdot i + L \cdot \dfrac{di}{dt} + C \cdot (\dot{x}_{m2} - \dot{x}_{m1}), \end{cases} \tag{17}$$

where m_1-mass of construction; m_2-rotor mass; k_1, c_1- elastic and dissipative coefficients; x_{m1} - mass m_1 coordinate; x_{m2}- rotor position; x_3 - gap between rotor and electromagnets.

As in previous cases:

$$z_1 = \sum_{i=1}^{5} l'_{1i} \cdot x_i \quad , \tag{18}$$

where l'_{1i}- components of L^{-1} matrix which are determined from (4).

Taking into account that we can measure real phase coordinates x_3 (gap) and x_5 (current) the equation for optimum control algorithm look like this:

$$U = -U_0 \cdot \text{sign} \left\{ \left[l'_{13} \left(1 + \frac{l'_{14}}{l'_{13}} \cdot p \right) - l'_{11} \cdot \frac{1 + \left(l'_{12} / l'_{11} \right) \cdot p}{p^2 + c_1 p + k_1} \right] \cdot x_3 - T \cdot \left[1 - \frac{l'_{11}}{T} \cdot \frac{+\left(l'_{12} / l'_{11} \right) \cdot p}{p^2 + c_1 p + k_1} \right] \cdot x_5 \right\} \quad , \tag{19}$$

where p - differentiation statement, $T = \dfrac{L_0}{R}$

Characteristic feature of the controller providing maximum area of stability in the discussed case when only gap and current are measured consists by necessity for the introduction of additional correction of signals of their sensors which is

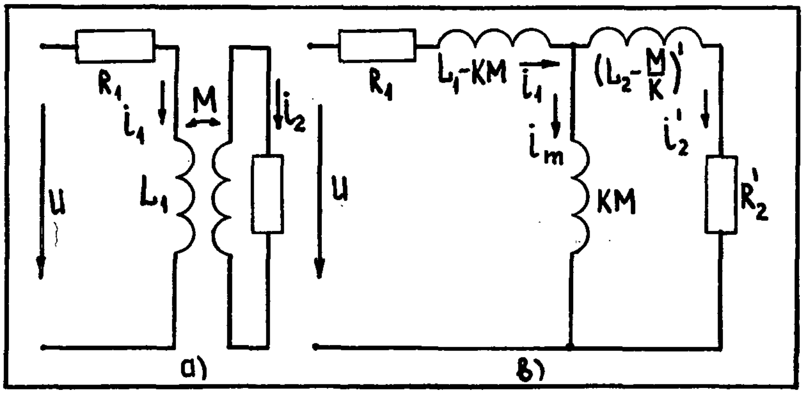

Figure 4.

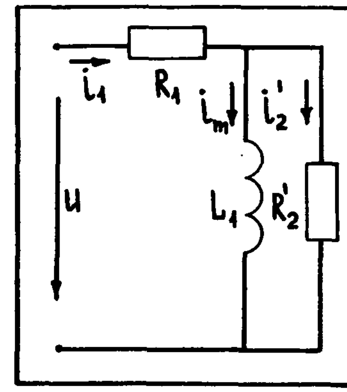

Figure 5.

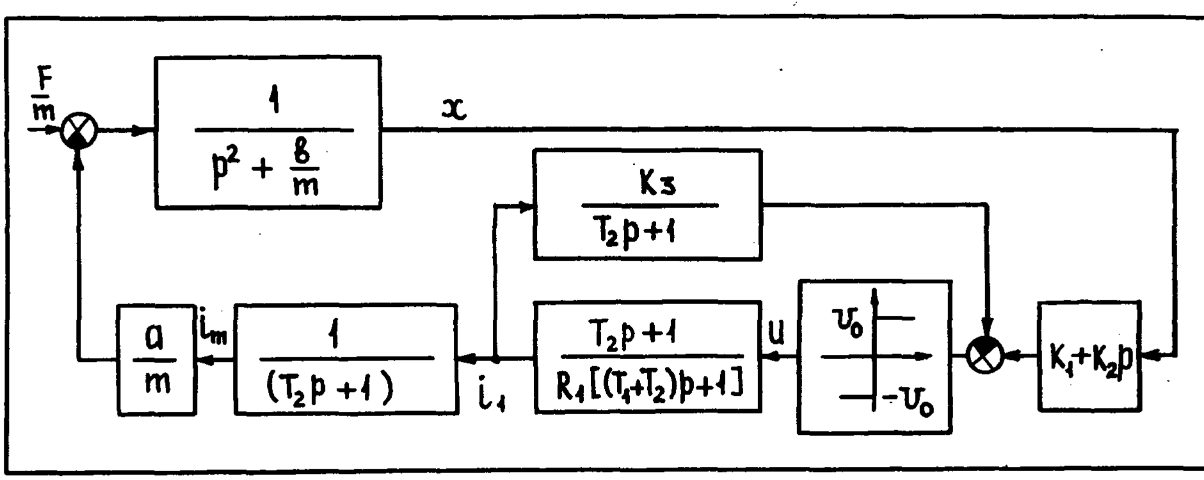

Figure 6.

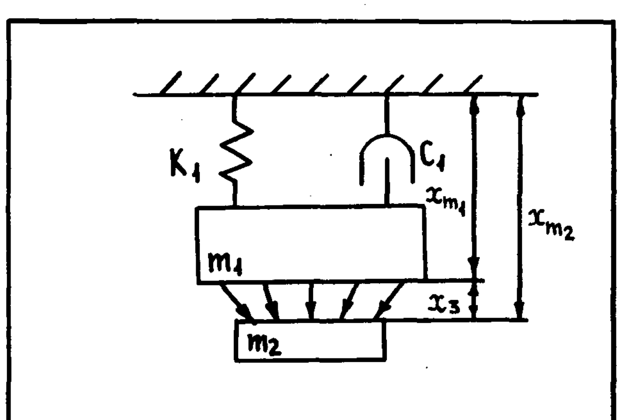

Figure 7.

realized when second order active filters tuned to own frequency of elastic vibration of the bearing are used.

CONCLUSIONS

The analysis of the results, that were given, allows one to draw the conclusion that this method of synthesis of the control algorithms for magnetic bearings permits one to use effectively the available resource of control and maximise the stability area of the rotor in space or the phase coordinates for given limitations to the value of the control votages. Also it allows one to have the estimations of maximum permissible phase coordinates deviations and the estimations of maximum permissible external disturbances. According to this technique the control system of the magnetic suspension for a wind tunnel was realized [8] and the control system for a pump with magnetic bearings is being realized now. This method which was used for one-degree-of-freedom system synthesis, taking into account the influence of eddy currents and resilient properties of construction, can be employed for multiple-degree-of-freedom system [9,10,11].

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