Magnetic Bearing: Comparison between Linear and Nonlinear Functioning

A. CHARARA AND B. CARON

ABSTRACT

In this paper, we deal with the comparison between the linear functioning (with constant current in the coils of the actuators) and the nonlinear one of an Active Magnetic Bearing (AMB). We show that the nonlinear functioning is better in terms of energy savings. A nonlinear control is then applied on a real process. This method is based on differential geometry. The experimental results, in nonlinear functioning, are compared with those obtained with a numerical PID controller.

INTRODUCTION

The electromagnetic forces created by an AMB are proportional to the square of the current in the coils, and are inversely proportional to the square of the air-gap between the rotor and the stator [1]; in consequence, the model of an AMB is nonlinear. The figure 1 shows the static characteristic of the force as a function of the current. We can see that, when the equilibrium point is at the origin (where the current is equal to zero), the nonlinear model cannot be linearized. This is due to the fact that the tangent at this point is equal to zero, so the functioning is purely nonlinear. In order to obtain a linear model, a constant current I_0 is normally introduced in all the coils of the AMB. This current is generally taken equal to half of the maximum current I_{max}. When a change of the forces is desired, the currents $i = I_0 + \Delta i$ and $i = I_0 - \Delta i$ are sent to the two coils in opposite on the same axis. In that case, the consumption is always equal to I_{max} on the same axis. In the nonlinear case we only use one coil at a time on the same axis (the current in the other one is equal to zero) to control the bearing, so the worst consumption (I_{max}) occurs when the control is saturated. This phenomenon, added to those given in [2] guarantee that the energy savings of the nonlinear control are significant.

NONLINEAR MODEL OF THE PROCESS

The plant (Figure 2) is an inertial wheel suspended by a Passive Magnetic Bearing, which

A. CHARARA and B. CARON: LAMII, Université de Savoie, BP 806, 74016 Annecy cédex, France.

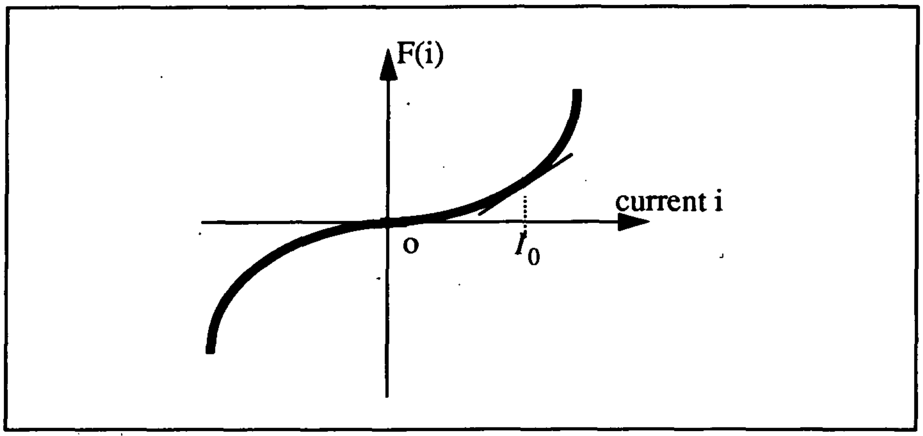

Figure 1: Electromagnetic force

controls three degrees of freedom: the displacement along axis z and the rotations around axes x and y. The centering of the rotor is assumed by two AMB which control the displacement along x and y. In nonlinear functioning, only one coil at a time is fed according to the direction of the desired force. A dynamic model taking into account the changeover of the current in the coils [2] has been elaborated ; this changeover appears with the sign of the current. This model is given by:

$$
\begin{aligned}
m\ddot{x}_g &= \frac{\mathrm{sign}(i_x) F_1 e_0^2 i_x^2}{(e_0 - \mathrm{sign}(i_x)\Delta e_x)^2} + F_{px} = F_x + F_{px} \\
m\ddot{y}_g &= \frac{\mathrm{sign}(i_y) F_1 e_0^2 i_y^2}{(e_0 - \mathrm{sign}(i_y)\Delta e_y)^2} + F_{py} = F_y + F_{py} \\
I\ddot{\alpha} + J\omega_0\dot{\beta} &= l_1 (F_y + F_{px}) \\
I\ddot{\beta} - J\omega_0\dot{\alpha} &= -l_1 (F_x + F_{py}) \\
\frac{di_x}{dt} &= -\frac{\mathrm{sign}(i_x)(\Delta\dot{e}_x) i_x}{(e_0 - \mathrm{sign}(i_x)\Delta e_x)} - \frac{R}{\lambda}(e_0 - \mathrm{sign}(i_x)\Delta e_x) i_x + \frac{1}{\lambda}(e_0 - \mathrm{sign}(i_x)\Delta e_x) E_x \\
\frac{di_y}{dt} &= -\frac{\mathrm{sign}(i_y)(\Delta\dot{e}_y) i_y}{(e_0 - \mathrm{sign}(i_y)\Delta e_y)} - \frac{R}{\lambda}(e_0 - \mathrm{sign}(i_y)\Delta e_y) i_y + \frac{1}{\lambda}(e_0 - \mathrm{sign}(i_y)\Delta e_y) E_y
\end{aligned} \qquad (1)
$$

With: $\mathrm{sign}(i) = \begin{bmatrix} 1 & \text{if} & (i>0) \\ -1 & \text{if} & (i<0) \end{bmatrix}$; $F_1 = (\lambda c)/(4e_0^2)$; $\lambda = \mu_0 n^2 s$

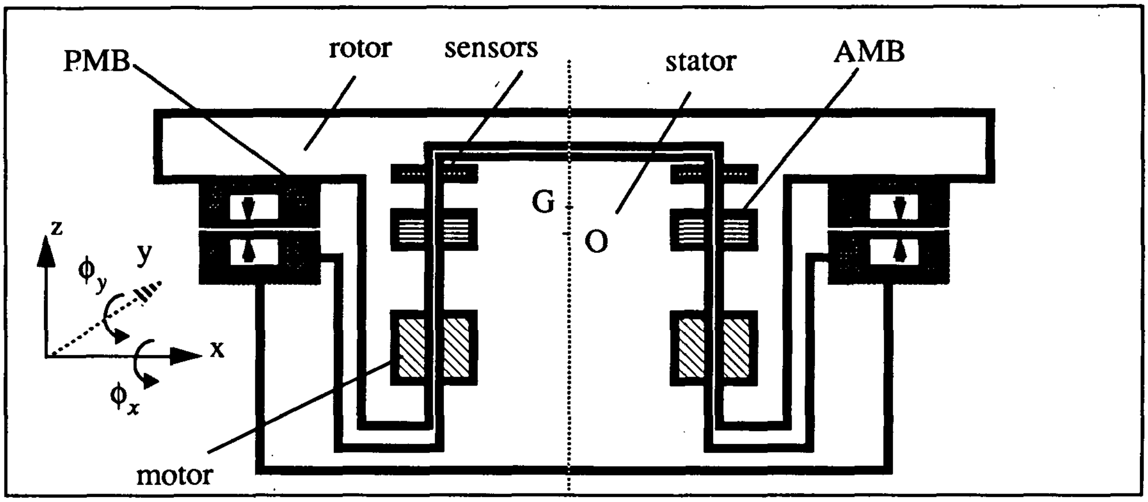

Figure 2: Cross section of the plant

$$F_{px} = ma_0\omega^2 \sin(\omega t) \; ; \; F_{py} = ma_0\omega_0^2 \cos(\omega t)$$

x and y are the two horizontal axes (see Figure 2.); x_g and y_g: the coordinates of G (centre of gravity of the rotor); μ_0: permeability of air; F_j: the electromagnetic force of the electromagnet j (the index j represents the x axis or the y axis); i_j: the current in the electromagnet j; ω: the speed of the rotation; R: resistor of the coil; E_j: applied voltage; I and J: radial and axial inertia momentum; **l**: the distance between G and the sensor; $l_1 = OG$; O: centre of the A.M.B. ; **m**: mass of the rotor; **s**: surface of a pole; **n**: number of turn; F_p: disturbance force (unbalance); a_0: the eccentricity (=0.01mm); e_0: the nominal air-gap(=0.5mm); Δe_j: the variation of the air-gap; $\Delta e_x = (x_g - \beta l)c$ and $\Delta e_y = (y_g - \alpha l)c$; α and β : angles of rotation around x and y axes; $c = \cos(\pi/8)$.

A lead compensator has been calculated using the Describing Function Approach (D.F.A.). This method allows to calculate a linear compensator taking into account static nonlinearities such as the force proportional to the square of the current (see Figure 1). The accuracy of the model has been tested by comparing the response of the model with the response of the real process in [2]. The trials show the soundness of the modelling.

NONLINEAR CONTROL

The D.F.A. only takes into account the parabolic characteristic of the force and neglects some other significant effects such as the coupling between each axis and the variation of the self - inductance. A nonlinear control method by immersion [3], [4] is then applied on the bearing in order to linearize the model. This method allows us to take into account all the nonlinearities of (1).

PRINCIPLE

Consider a nonlinear system described by equations of the form:

$$\begin{cases} \dot{\xi} = f(\xi) + \sum_{i=1}^{m} u_i(t) g_i[\xi(t)] \\ \Gamma_k(t) = h_k[\xi(t)] \end{cases} \quad (2)$$

where $\xi \in R^n$, $\Gamma \in R^m$. f and g are analytic vector fields, Γ is an analytic mapping. Note that this system has the same number of input and output components. We will compare the evolution of both (2) and the linear system (3) (where v are the new inputs):

$$\begin{cases} \dot{\eta}(t) = A\eta(t) + Bv(t) \\ \Gamma(t) = H(t) \end{cases} \quad (3)$$

This immersion is done by static feedback (4) and linear state space feedback (Figure 3):

$$u(t) = \Phi(\xi) + \Psi(\xi) v(t) \quad (4)$$

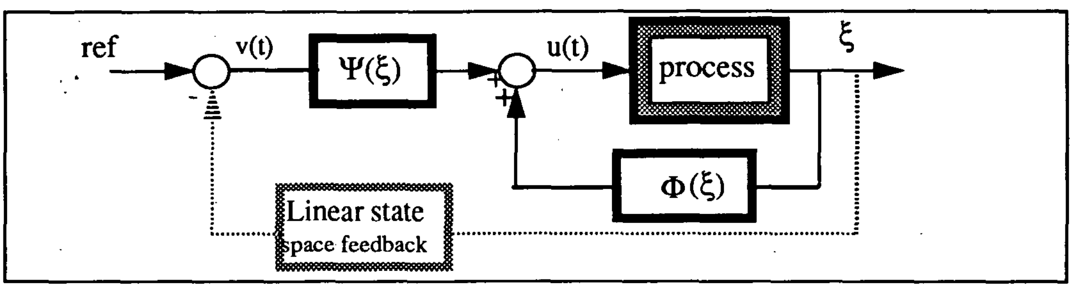

Figure 3: Bloc-diagram of the nonlinear immersion control

Before giving the complete control law, we will explain the method on a single-input/single-output simple example.

PEDAGOGIC EXAMPLE

Let equations (5) be the pedagogic system:

$$\begin{cases} \dot{\xi}_1 = \xi_2 \\ \dot{\xi}_2 = \xi_1^2 + \xi_2 u \\ \Gamma = \xi_1 \end{cases} \quad (5)$$

with $[\xi_1 \ \xi_2]^T$ the state vector, u the control and Γ the output.

If we calculate the successive derivatives of the output (ξ_1) in order to get the control u appeared in the expression, we obtain:

$$\dot{\Gamma} = \dot{\xi}_1 = \xi_2$$

$$\ddot{\Gamma} = \xi_1^2 + \xi_2 u \qquad (6)$$

To the considered output (ξ_1), we associate a characteristic number δ equal to the order of the last derivative minus one. In the example we obtain $\delta = 2 - 1 = 1$. Using a static feedback (4), we want that (5) to have the same behavior as (7):

$$\frac{\Gamma(s)}{v(s)} = \frac{k}{s^2 - \sigma_1 s - \sigma_0} \qquad (7)$$

We calculate the control law by equalizing the second derivative of Γ in (6) and in (7):

$$\xi_1^2 + \xi_2 u = \sigma_1 \dot{\Gamma} + \sigma_0 \Gamma + kv$$

then (if $\xi_2 \neq 0$):
$$u(t) = \frac{\sigma_1 \dot{\Gamma} + \sigma_0 \Gamma - \xi_1^2}{\xi_2} + \frac{k}{\xi_2} v(t) = \Phi(\xi) + \Psi(\xi) v(t)$$

The closed loop system is then equivalent to the following linear system:

$$\frac{d}{dt}\begin{bmatrix}\Gamma \\ \dot{\Gamma}\end{bmatrix} = \begin{bmatrix}0 & 1 \\ \sigma_0 & \sigma_1\end{bmatrix}\begin{bmatrix}\Gamma \\ \dot{\Gamma}\end{bmatrix} + \begin{bmatrix}0 \\ k\end{bmatrix} v \qquad \eta = \begin{bmatrix}\Gamma \\ \dot{\Gamma}\end{bmatrix}$$

In the multivariable case, after feedback, the system is decoupled, that is we obtain m linear systems of order $\delta_i + 1$ for $i = 1, \ldots, m$, where each input acts only on one output.

APPLICATION TO THE AMB

The state vector is given by:

$$\xi^T = [x_g \ y_g \ \alpha \ \beta \ \dot{x}_g \ \dot{y}_g \ \dot{\alpha} \ \dot{\beta} \ i_x \ i_y]$$

the control laws are given by: $u_1(t) = E_x$, $u_2(t) = E_y$; and we choose $h_1(t) = x_g$, $h_2(t) = y_g$: the outputs are the positions of the center of the rotor.

Building the model (1) into the form (2) and after applying the previous method for multivariable systems [5], we obtain:

$$u_1(t) = Ri_x + \frac{m\lambda}{2F_1 e_0^2} \cdot \frac{[e_0 - q(i_x)\Delta e_x]}{q(i_x)\, i_x} \cdot \tilde{v}_1 \tag{8-1}$$

$$u_2(t) = Ri_y + \frac{m\lambda}{2F_1 e_0^2} \cdot \frac{[e_0 - q(i_y)\Delta e_y]}{q(i_y)\, i_y} \cdot \tilde{v}_2 \tag{8-2}$$

with:

$$\tilde{v}_1(t) = \sigma_2 \ddot{x}_g + \sigma_1 \dot{x}_g + \sigma_0 x_g + kv_1 \tag{9-1}$$

$$\tilde{v}_2(t) = \sigma_2 \ddot{y}_g + \sigma_1 \dot{y}_g + \sigma_0 y_g + kv_2 \tag{9-2}$$

Equations (8) transform the system given in (1) into two triple integrators. Equation (9) are used to stabilized these integrators with coefficients σ_i.

Remark:

The control method needs an analytic model [4]. But the model (1) of the AMB is nonanalytic due to the brutal variation of the air-gap when the sign of the current changes. In order to obtain an analytic model we have approximated the sign function by:

$$q(i) = \frac{2}{\pi}\operatorname{atan}(ai) \qquad \text{(with } a \text{ a constant)}$$

After the linearization, we obtain two linear systems of order 3 with the following input/output behavior (where $\Gamma = x_g$ or y_g ; the position of the rotor):

$$\dddot{\Gamma} = \sigma_2 \ddot{\Gamma} + \sigma_1 \dot{\Gamma} + \sigma_0 \Gamma + kv \tag{10}$$

In order to compensate the unbalance (the center of mass of the rotor does not coincide with the geometric center) we are using previous works [6] about the compensation of this effect in linear functioning (with a constant current in the coils). In these studies, the author proposes a control where the current is a function of the estimates of the first and second derivatives of the position Γ :

$$i = -C_d \Gamma - C_v \dot{\hat{\Gamma}} - C_a \ddot{\hat{\Gamma}} \tag{11}$$

With: $\dot{\hat{\Gamma}}$ velocity observer; $\ddot{\hat{\Gamma}}$ acceleration observer; C_d : displacement feedback gain; C_v: velocity feedback gain; C_a : acceleration feedback gain.

In order to compare this relation with (10), we suppose to apply a voltage control (E) using the following relation (L , the self-inductance of the coil, is supposed to be constant):

$$E = L\frac{di}{dt} + Ri \tag{12}$$

Taking the derivative of (11) and replacing it into (12), we obtain (supposing: $\ddot{\hat{\Gamma}} = \ddot{\Gamma}$ *and* $\dot{\hat{\Gamma}} = \dot{\Gamma}$):

$$\dddot{\Gamma} = -\left(\frac{C_v}{C_a} + \frac{R}{L}\right)\ddot{\Gamma} - \left(\frac{C_d}{C_a} + \frac{RC_v}{LC_a}\right)\dot{\Gamma} - \frac{RC_d}{LC_a}\Gamma - \frac{1}{LC_a}E \tag{13}$$

we identify (10) and (13) and finally we obtain:

$$\sigma_2 = -\left(\frac{C_v}{C_a} + \frac{R}{L}\right);\ \sigma_1 = -\left(\frac{C_d}{C_a} + \frac{RC_v}{LC_a}\right);\ \sigma_0 = -\frac{RC_d}{LC_a};\ k = -\frac{1}{LC_a} \tag{14}$$

The nonlinear system linearized and decoupled by a static feedback and stabilized by a linear state feedback with v as the input and Γ as the output is equivalent to the system in linear functioning with E as the input and Γ as the output. Then, if we know the C_d, C_v gains in the linear case, we can obtain the coefficients σ_i in the nonlinear one.

we choose $\sigma_0 = -9.10^7$, $\sigma_1 = -3120000$, $\sigma_2 = -1200$; figure 4 shows the comparison, in simulation, between a linear lead controller calculated with the D.F.A method and the nonlinear control. The figure 4a (displacement of the rotor) verifies the good compensation of the unbalance by the nonlinear control, the figure 4b shows the good decoupling when using the nonlinear control.

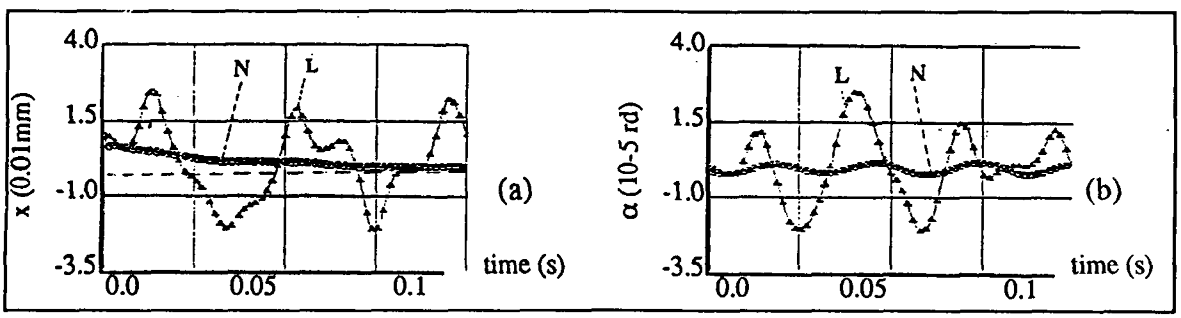

Figure 4: simulation: a) displacement according to x, b) variation of α ; (L) : D.F.A., (N) : NonLinear

LINEAR FUNCTIONING

We introduce a constant current I_0 in the coils. This current is chosen in order to obtain a linear functioning (see Figure 1). The model (1) can then be linearized around an operating point. The figure 5 presents the bloc diagram in linear functioning (the self-inductance is supposed to be constant). The controller is a PID where two dominant poles are imposed, then:

$$C(s) = 2\frac{(1+0.045s)^2}{s(1+0.005s)} \tag{15}$$

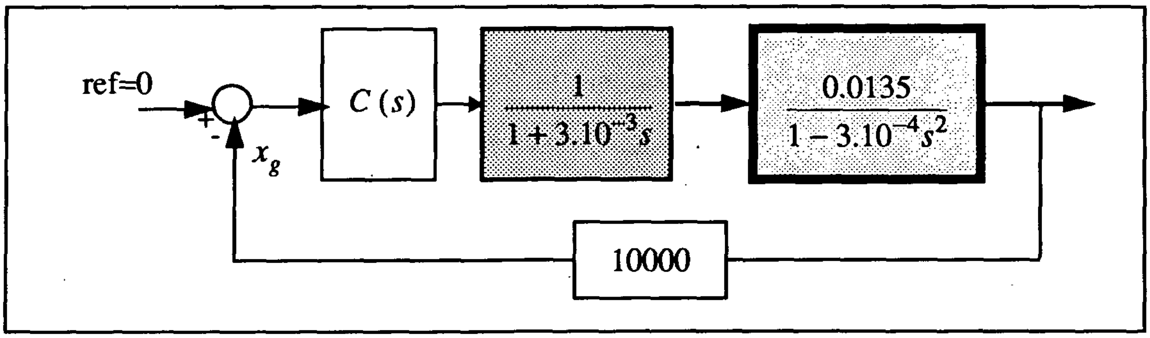

Figure 5: bloc-diagram in linear functioning

APPLICATIONS

In order to apply the control laws on a computer, we have to obtain a discretization representation of the control law. In the case of a nonlinear control we use the works of Monaco and Normand-Cyrot [7] by stopping the development at order zero. The control law is then given by:

$$u_1(k) = Ri_x(k) + \frac{m\lambda}{2F_1 e_0^2} \cdot \frac{[e_0 - q(i_x)x_g(k)]}{0.001 + q(i_x)i_x(k)} \cdot \tilde{v}_1(k) \tag{16}$$

(we proposed to modify the reference in order to avoid the singularity of the control law, the least-significant bit of our 12-bits digital input corresponds to 5mA. So, the denominator of (16) is always different from zero). In the applications it is not possible to stabilize the triple integrator with a complete state space feedback due to the lack of time (estimation of the first and second derivatives of the output). The triple integrator is then stabilized using a double lead controller given by:

$$\frac{\tilde{v}(s)}{v(s)} = 10\frac{1+0.068s+0.0011s^2}{1+0.006s+9.10^{-6}s^2} \tag{17}$$

The computation time of algorithm (16) and (17) on the axes x and y is about equal to $710\mu s$. We choose a sampling period of $850\mu s$. The figure 6 shows the variation of the displacement according to axis x and the corresponding control (the voltage in the coils), for a frequency of rotation equal to 25hz, in three cases: linear functioning with PID, nonlinear functioning with lead controller (D.F.A. method) and non linear functioning with nonlinear control
Using TABLE I (corresponding to trials of Figure 6) we can see the energy savings in nonlinear functioning and the good rejection of the unbalance with the nonlinear control.
The Figure 7 shows the influence of a step response for the three presented controls at a rotation speed of 25hz. We can see a better transient response in the case of the nonlinear control.

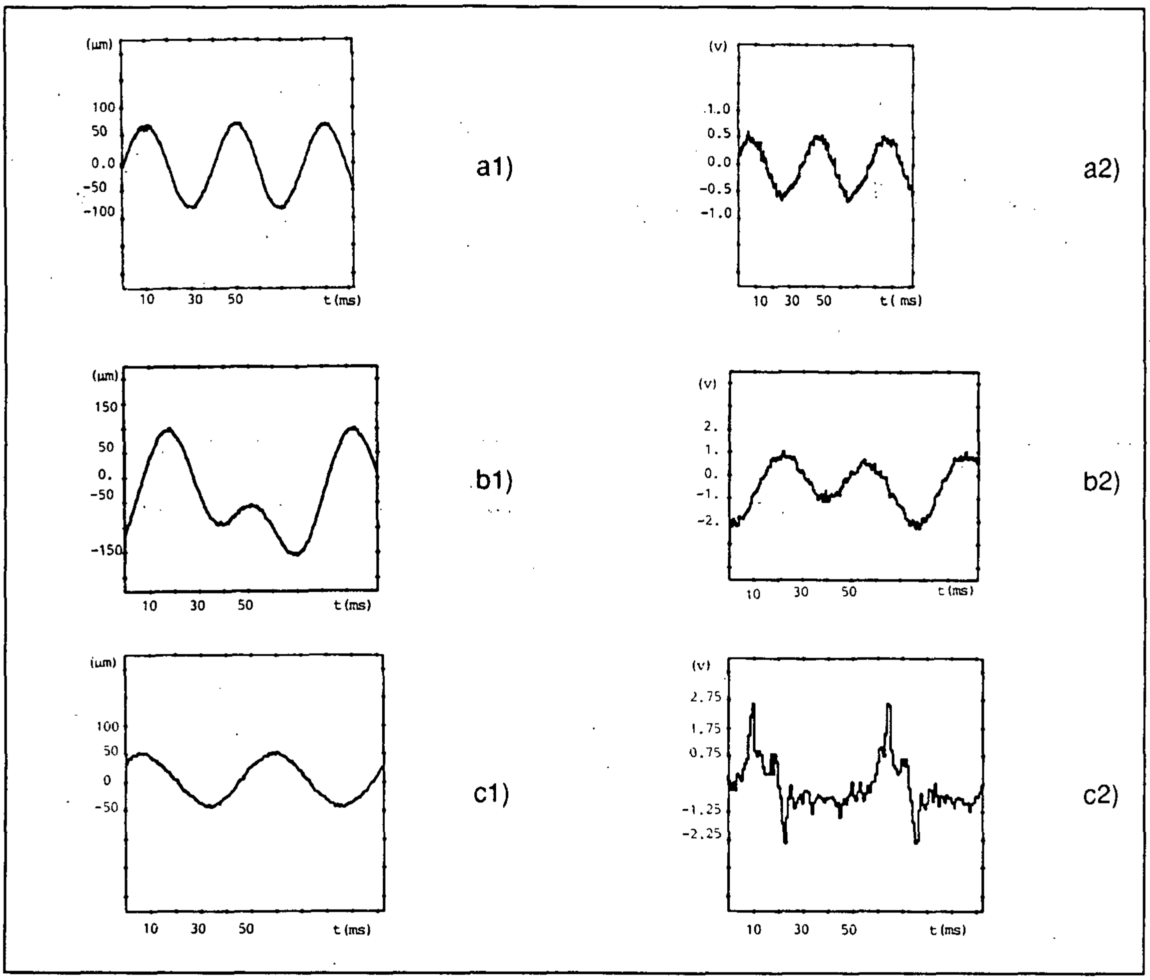

Figure 6: a) linear functioning, b) non linear functioning (D.F.A. method), c) non linear control a1), b1), c1): displacement according to x; a2), b2), c2) variation of the command (f=25hz)

TABLE-I COMPARISON BETWEEN LINEAR AND NONLINEAR FUNCTIONING

axis x	non linear D.F.A.(Fig.6b)	nonlinear control (Fig. 6c)	linear (Figure 6a)
displacement μm	$-150 \le x \le 100$	$-50 \le x \le 50$	$-75 \le x \le 75$
command (V)	$-2. \le u \le 1.$	$-2.5 \le u \le 2.5$	$-0.6 \le u \le 0.6$
power (W) (for 1 turn of roto)	12.8	21	96

CONCLUSION

The model of an AMB is nonlinear when we do not use a constant current in the coils. In order to take into account all the nonlinearities of the model we have used a nonlinear control

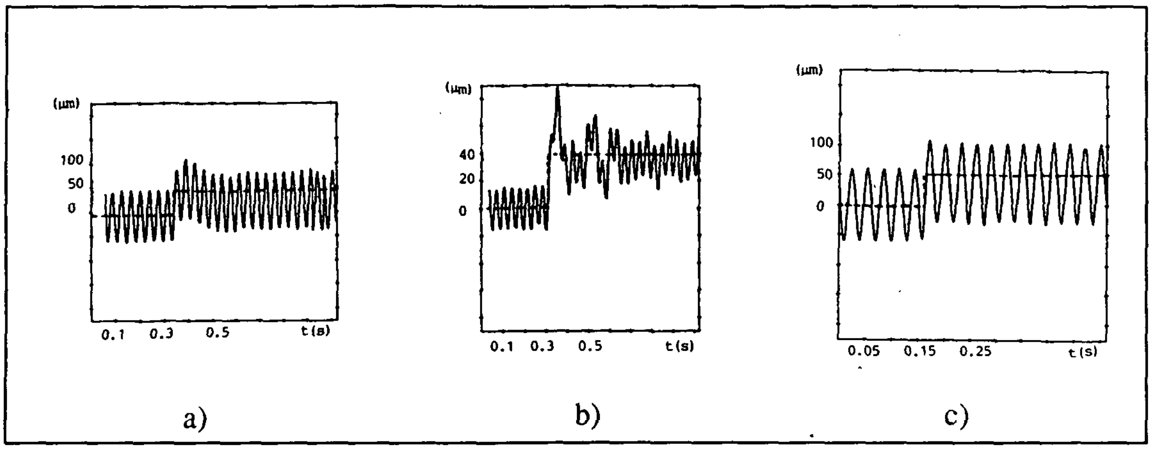

Figure 7: Step response, displacement according to x: a) linear functioning b) nonlinear functioning (D.F.A. method), c) nonlinear functioning (nonlinear control)

based on differential geometry. This law has given good results in simulation or in application compared with controls based on the linear model of an AMB (using constant currents in the coils) or based on the nonlinear functioning with a linear controller (using the D.F.A method). The compensation of the unbalance obtained in the case of a nonlinear control is quite the same as in the case of the linear functioning, but we have shown that the energy savings are significant in the nonlinear functioning.

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