Digital Control of Magnetic Bearings: Inversion of Actuator Model

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ABSTRACT

In this paper we introduce a combination of linear multivariable system control law synthesis together with actuator inversion to deal with non linear relations that appear in modelling the active magnetic suspension of a horizontal shaft.
Approximations are introduced in order to reduce computations involved in digital control scheme. Simulation results show a good agreement between the use of this control scheme and the behavior of the system with perfect actuators.

INTRODUCTION

This paper is devoted to the control structure of the electromagnets involved in active magnetic bearing of a horizontal shaft.
Lot of work has been published on that subject. Since non linear relations appear in the model of magnetic forces, approximation is usually made by linearizing the system around the nominal position. Furthermore, the voltage input is supposed continuous.
In our case, we have considered a chopper to feed the electromagnet, so that the manipulated variable is the cyclic ratio of the chopper, which induces supplementary non linear relations to the system. The whole control scheme can be represented as follows :

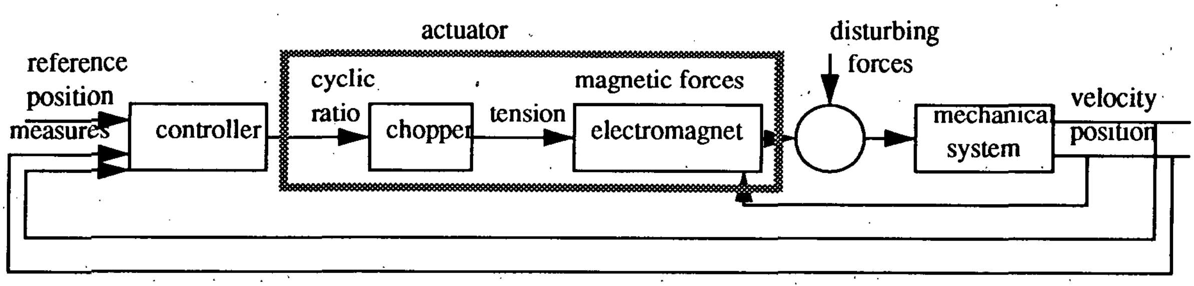

Figure 1 : Control scheme

We want to build a digital control of the magnetic bearing. So we have to find a compromise between complexity of control scheme taking into account non linear features and the amount of calculations involved in the control algorithm.

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We adopted a two step procedure to design that control scheme :
- In the first step, the actuator is supposed perfect, which means that it is possible to find the adequate cyclic ratio to get the required force. Thus we are interested in the computation of the forces that maintain the shaft in a good position despite of disturbing forces.
It becomes a multidimensionnal linear problem whose complexity comes from variable coupling terms due to gyroscopic effects depending on rotation speed.
- The second step consists in the realization of the magnetic forces that are required at each sampling interval by the above mentionned control algorithm. An inverse model of the actuator is used to determine the good cyclic ratio. Due to the complexity of the non linear expressions involved in the actuator model, an exact inversion is not feasible especially in real time. In fact this problem is reduced to the calculation of one particular root of a third order polynomial. This one is derived from the approximation of the mean value of the magnetic force during the sampling interval, and yields an accurate solution in the working domain of the actuator.

The paper is organized as follows. We briefly set the mechanical model of the system. Then we solve the problem of digital control assuming that actuators are perfect. The next section deals with inversion of actuators, that is the relation between the cyclic ratio and the magnetic force. Finally we present some results with comments showing that the approximations introduced in actuator inversion are valid.

MECHANICAL MODEL

This model has been exposed in many papers, so that we briefly recall its main features.
We consider a horizontal shaft and our interest is focused on the control of the position of the main axis of inertia. So we don't consider the translation or rotation, along or about the main axis of inertia. Futhermore, the rotor is supposed to be rigid and the main axis of inertia coincide with the geometric revolution axis.

Let us denote (figure 2) :

- F_{li}, F_{ri} $i \in \{1,4\}$ magnetic forces produced by electromagnets
- F_y, F_Z disturbing forces due to the reaction beetwen tool and worked piece
- $y_0, z_0, \dot{y}_0, \dot{z}_0$ motions of the center of mass G and their derivatives
- $\theta, \psi, \dot{\theta}, \dot{\psi}$ rotations of the body around G_Y and G_Z and their derivatives
- l_1, l_2, l_b distances between G and application of magnetic forces F_{li} and F_{ri} and disturbing forces respectively
- j_x, j_y, m moments of inertia about G_X and G_Y and mass of the body
- p rotation speed around G_X axis

The two principles of linear and angular momentum yield the following equations of motions :

$$\ddot{y}_0 = \frac{F_{l3} - F_{l4} + F_{r3} - F_{r4} + F_y}{m} \qquad \ddot{z}_0 = \frac{F_{l2} - F_{l1} + F_{r2} - F_{r1} + m \cdot g + F_z}{m} \tag{1}$$

$$\begin{bmatrix} \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \frac{p \cdot j_x}{j_y} \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta} \\ \dot{\psi} \end{bmatrix} + \frac{1}{j_y} \cdot \begin{bmatrix} -F_z \cdot l_b + (F_{l1} - F_{l2}) \cdot l_1 + (F_{r2} - F_{r1}) \cdot l_2 \\ F_y \cdot l_b + (F_{l3} - F_{l4}) \cdot l_1 + (F_{r4} - F_{r3}) \cdot l_2 \end{bmatrix}$$

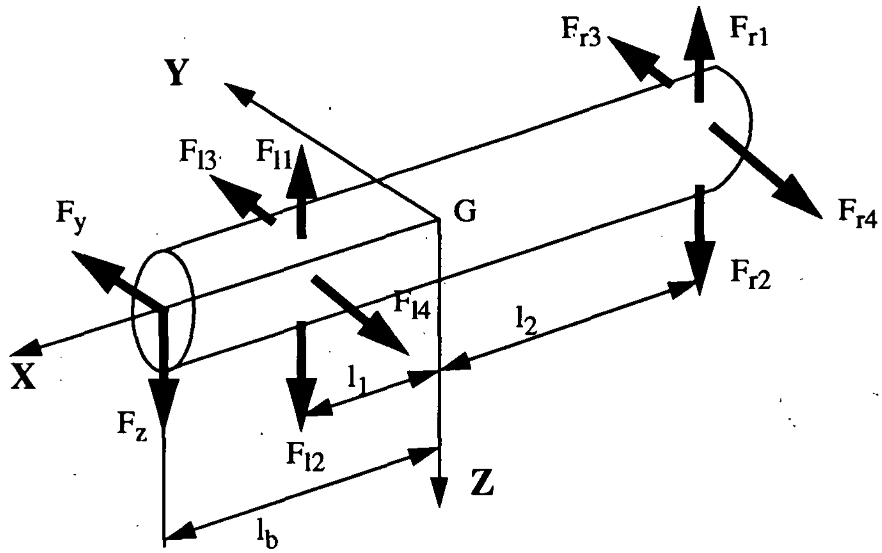


Figure 2 : Description of the mechanical model

They can be put in a continuous state space representation, as follows :

$$
\begin{bmatrix} \dot{y}_0 \\ \dot{z}_0 \\ \dot{\theta} \\ \dot{\psi} \\ \ddot{y}_0 \\ \ddot{z}_0 \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix}
=
\begin{bmatrix}
0&0&0&0&1&0&0&0\\
0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&-\frac{p\cdot j_x}{j_y}\\
0&0&0&0&0&0&\frac{p\cdot j_x}{j_y}&0
\end{bmatrix}
\cdot
\begin{bmatrix} y_0 \\ z_0 \\ \theta \\ \psi \\ \dot{y}_0 \\ \dot{z}_0 \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}
+
\begin{bmatrix}
0&0&0&0\\
0&0&0&0\\
0&0&0&0\\
0&0&0&0\\
0&0&\frac{1}{m}&\frac{1}{m}\\
-\frac{1}{m}&-\frac{1}{m}&0&0\\
\frac{l_1}{j_y}&-\frac{l_2}{j_y}&0&0\\
0&0&\frac{l_1}{j_y}&-\frac{l_2}{j_y}
\end{bmatrix}
\cdot
\begin{bmatrix}
F_{l1}-F_{l2}-\frac{l_2\cdot m\cdot g}{l_1+l_2}\\
F_{r1}-F_{r2}-\frac{l_1\cdot m\cdot g}{l_1+l_2}\\
F_{l3}-F_{l4}\\
F_{r3}-F_{r4}
\end{bmatrix}
+
\begin{bmatrix}
0&0\\
0&0\\
0&0\\
0&0\\
\frac{1}{m}&0\\
0&\frac{1}{m}\\
0&-\frac{l_b}{j_y}\\
\frac{l_b}{j_y}&0
\end{bmatrix}
\cdot
\begin{bmatrix} F_y \\ F_z \end{bmatrix}
\qquad (2)
$$

$\dot{\underline{X}} = A\cdot\underline{X}+B\cdot\underline{U}+B'\cdot\underline{U}'$

Let w be the nominal gap. Since measures g_{li} and g_{ri} are supposed to be made in the magnetic bearing planes, we have the following relations :

$$
\begin{aligned}
g_{l1} &= w+z_0-(l_1\cdot\theta) & \qquad g_{r1} &= w+z_0+l_2\cdot\theta\\
g_{l2} &= w-z_0+l_1\cdot\theta & g_{r2} &= w-z_0-(l_2\cdot\theta)\\
g_{l3} &= w-y_0-(l_1\cdot\psi) & g_{r3} &= w-y_0+l_2\cdot\psi\\
g_{l4} &= w+y_0+l_1\cdot\psi & g_{r4} &= w+y_0-(l_2\cdot\psi)
\end{aligned}
\qquad (3)
$$

which lead to the output equation : $\underline{Y} = \begin{bmatrix} g_{l1}-w \\ g_{r1}-w \\ g_{l3}-w \\ g_{r3}-w \end{bmatrix} = \begin{bmatrix} 0&1&-l_1&0&0&0&0&0\\ 0&1&l_2&0&0&0&0&0\\ -1&0&0&-l_1&0&0&0&0\\ -1&0&0&l_2&0&0&0&0 \end{bmatrix}\cdot\begin{bmatrix} y_0\\ z_0\\ \theta\\ \psi\\ \dot{y}_0\\ \dot{z}_0\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix}$ (4)

In the state matrix, a variable term $p \cdot \frac{j_x}{j_y}$ appears. It indicates the coupling term due to the gyroscopic effects, depending on rotation speed p about G_X axis.

CONTROL STRUCTURE DESIGN

Let us recall that we search a digital control scheme, and that at this stage actuators are supposed perfect, that is, are able to yield the required magnetic forces.
In these conditions, the problem is a linear time varying multidimensionnal one, and we need a discretised model of the mechanical system, expressed in terms of rotation speed about G_X axis, since it induces gyroscopic coupling terms as it was shown in previous section.
Such a discretised model is available and has the following expression :

$$\underline{X}(k+1) = F \cdot \underline{X}(k) + G \cdot \underline{U}(k) + G' \cdot \underline{U}'(k)$$

$$\text{where } F = \begin{bmatrix} 1 & 0 & 0 & 0 & T_e & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & T_e & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{\alpha} \cdot \sin(\alpha \cdot T_e) & \frac{1}{\alpha} \cdot (1 - \cos(\alpha \cdot T_e)) \\ 0 & 0 & 0 & 1 & 0 & 0 & \frac{1}{\alpha} \cdot (\cos(\alpha \cdot T_e) - 1) & \frac{1}{\alpha} \cdot \sin(\alpha \cdot T_e) \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cos(\alpha \cdot T_e) & \sin(\alpha \cdot T_e) \\ 0 & 0 & 0 & 0 & 0 & 0 & -\sin(\alpha \cdot T_e) & \cos(\alpha \cdot T_e) \end{bmatrix} \quad \text{with } \alpha = -\frac{p \cdot j_x}{j_y} \tag{5}$$

T_e sampling interval

and G matrix has the following structure :

$$\begin{bmatrix} 0 & 0 & \frac{a \cdot Te^2}{2} & \frac{a \cdot Te^2}{2} \\ \frac{-a \cdot Te^2}{2} & \frac{-a \cdot Te^2}{2} & 0 & 0 \\ \frac{b}{\alpha^2} \cdot (1 - \cos(\alpha \cdot Te)) & \frac{c}{\alpha^2} \cdot (1 - \cos(\alpha \cdot Te)) & \frac{b}{\alpha} \cdot (Te - (\frac{1}{\alpha} \cdot \sin(\alpha \cdot Te))) & \frac{c}{\alpha} \cdot (Te - (\frac{1}{\alpha} \cdot \sin(\alpha \cdot Te))) \\ \frac{-b}{\alpha} \cdot (Te - (\frac{1}{\alpha} \cdot \sin(\alpha \cdot Te))) & \frac{-c}{\alpha} \cdot (Te - (\frac{1}{\alpha} \cdot \sin(\alpha \cdot Te))) & \frac{b}{\alpha^2} \cdot (1 - \cos(\alpha \cdot Te)) & \frac{c}{\alpha^2} \cdot (1 - \cos(\alpha \cdot Te)) \\ 0 & 0 & a \cdot Te & a \cdot Te \\ -(a \cdot Te) & -(a \cdot Te) & 0 & 0 \\ \frac{b}{\alpha} \cdot \sin(\alpha \cdot Te) & \frac{c}{\alpha} \cdot \sin(\alpha \cdot Te) & \frac{b}{\alpha} \cdot (1 - \cos(\alpha \cdot Te)) & \frac{c}{\alpha} \cdot (1 - \cos(\alpha \cdot Te)) \\ \frac{-b}{\alpha} \cdot (1 - \cos(\alpha \cdot Te)) & \frac{-c}{\alpha} \cdot (1 - \cos(\alpha \cdot Te)) & \frac{b}{\alpha} \cdot \sin(\alpha \cdot Te) & \frac{c}{\alpha} \cdot \sin(\alpha \cdot Te) \end{bmatrix}$$

$$G' = \begin{bmatrix} \frac{Te^2}{2 \cdot m} & 0 \\ 0 & \frac{Te^2}{2 \cdot m} \\ \frac{l_b}{J_y \cdot \alpha} \cdot (Te - (\frac{1}{\alpha} \cdot \sin(\alpha \cdot Te))) & \frac{-l_b}{\alpha^2 \cdot J_y} \cdot (1 - \cos(\alpha \cdot Te)) \\ \frac{l_b}{\alpha^2 \cdot J_y} \cdot (1 - \cos(\alpha \cdot Te)) & \frac{l_b}{J_y \cdot \alpha} \cdot (Te - (\frac{1}{\alpha} \cdot \sin(\alpha \cdot Te))) \\ \frac{Te}{m} & 0 \\ 0 & \frac{Te}{m} \\ \frac{l_b}{\alpha \cdot J_y} \cdot (1 - \cos(\alpha \cdot Te)) & -\left(\frac{l_b}{J_y \cdot \alpha} \cdot \sin(\alpha \cdot Te)\right) \\ \frac{l_b}{J_y \cdot \alpha} \cdot \sin(\alpha \cdot Te) & \frac{l_b}{\alpha \cdot J_y} \cdot (1 - \cos(\alpha \cdot Te)) \end{bmatrix} \tag{6}$$

where $a = \frac{1}{m}$ $b = \frac{l_1}{j_y}$ $c = -\frac{l_2}{j_y}$.

To get this model, we suppose that the magnetic forces as well as the disturbing forces are constant during sampling interval. Concerning the second ones, we assume that they have small variations during the sampling period.

This model is somewhat complex, and it is a priori difficult to build a state feedback law to control the system.

Fortunately, there exist appropriate linear combinations of force inputs that lead to the decomposition of the whole system into 3 subsystems of dimension 2, 2 and 4 respectively. It is then easier to determine control law for each one.

$$V_1(k) = U_3(k) + U_4(k) \qquad V_2(k) = U_1(k) + U_2(k)$$
$$V_3(k) = b \cdot U_1(k) + c \cdot U_2(k) \qquad V_4(k) = b \cdot U_3(k) + c \cdot U_4(k)$$

Two subsystems deal with translations of the center of gravity and are almost identical and entirely uncoupled. We used pole placement to derive feedback gain matrix.

$$\begin{bmatrix} X_1(k+1) \\ X_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & Te \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X_1(k) \\ X_2(k) \end{bmatrix} + \begin{bmatrix} \frac{a \cdot Te^2}{2} \\ a \cdot Te \end{bmatrix} \cdot V_1(k)$$

$$V_1(k) = e_1(k) + K \cdot \underline{X}(k) \quad \text{with} \quad K = \begin{bmatrix} K_1 & K_2 \end{bmatrix} .$$

The third one concerns rotations about G_Y and G_Z that cannot be decoupled without introducing cancellation of poles-zeros that lie on the unit circle. The control of this subsystem requires coefficients that are dependant of the speed rotation p in the feedback gain matrix in order to get performances that remain almost invariant with respect to speed variations.

Since the linear combinations are invertible it is possible to come back to real force inputs.

$$\begin{bmatrix} X_5(k+1) \\ X_6(k+1) \\ X_7(k+1) \\ X_8(k+1) \end{bmatrix} = \begin{bmatrix} 1 & f_2 & 0 & f_1 \\ 0 & \cos(\alpha \cdot Te) & 0 & \sin(\alpha \cdot Te) \\ 0 & -f_1 & 1 & f_2 \\ 0 & -\sin(\alpha \cdot Te) & 0 & \cos(\alpha \cdot Te) \end{bmatrix} \cdot \begin{bmatrix} X_5(k) \\ X_6(k) \\ X_7(k) \\ X_8(k) \end{bmatrix} + \begin{bmatrix} \frac{f_1}{\alpha} & f_3 \\ f_2 & f_1 \\ -f_3 & \frac{f_1}{\alpha} \\ -f_1 & f_2 \end{bmatrix} \cdot \begin{bmatrix} V_3(k) \\ V_4(k) \end{bmatrix} \qquad (7)$$

$$f_1 = \frac{1}{\alpha} \cdot (1 - \cos(\alpha \cdot Te)) \qquad f_2 = \frac{1}{\alpha} \cdot \sin(\alpha \cdot Te)$$

$$f_3 = \frac{1}{\alpha} \cdot \left(Te - \left(\frac{1}{\alpha} \cdot \sin(\alpha \cdot Te)\right)\right)$$

$$\underline{V}(k) = \underline{e}(k) + K \cdot \underline{X}(k) \qquad K = \begin{bmatrix} 0 & K_{12} & 0 & -K_{22} \\ 0 & K_{22} & 0 & K_{12} \end{bmatrix}$$

ACTUATOR INVERSION

The previous control scheme leads to a control algorithm that computes periodically the required forces to be applied to the shaft, under the assumption that these forces are constant during the sampling interval .
The magnetic force is produced by means of an electromagnet whose coil is fed by the following device :

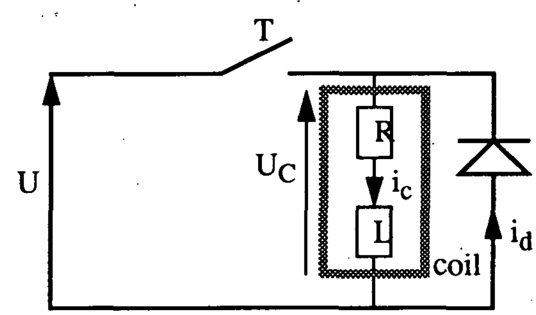


Figure 3 : coil feeding

The caracteristics of the coil (resistance and inductance) are supposed constant, so voltage and current evolve as indicated in figure 4.

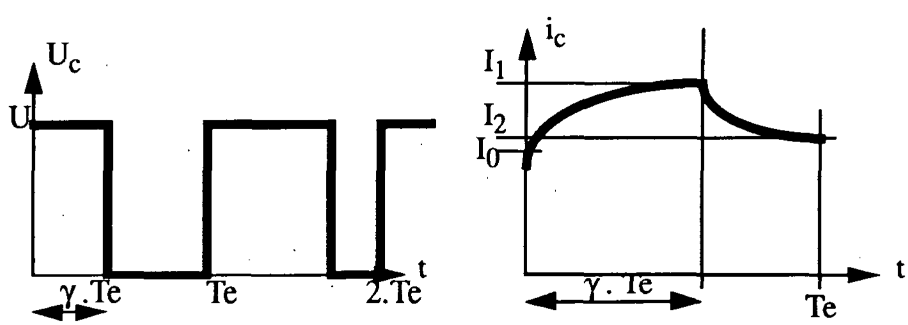


Figure 4 : current and tension evolution

Now the problem we have to solve is the calculation of the cyclic ratio γ at time k.Te, knowing the force F to be applied at that time, considering the commonly used non linear relation

$$F(t) = D \cdot \frac{i_c^2(t)}{g^2(t)} \tag{8}$$

where $i_c(t)$ is the current in the coil
$g(t)$ is the gap between electromagnet ant the shaft
D is a constant parameter
F is the magnetic force

A first method to derive $\gamma(k)$ consists in the inversion of the relation linking the final state at time (k+1).Te to the variable $\gamma(k)$. It involves double integration and is too much complex to be applied here.
A second procedure consists in the search of $\gamma(k)$ such that the mean value $\bar{F}(k)$ of F(t) for $t \in [k \cdot T_e, (k+1) \cdot T_e]$ is equal to the constant value F(k) required by the control algorithm.
In order to yield the problem easily tractable we make the following assumptions :
- the switching period is equal to the sampling interval.
- the gap is constant during the sampling interval.
- two electromagnets are concerned by the production of a force F(k). But only one works at once, according to the sign of F(k).

Since relation between $\bar{F}(k)$ and $\gamma(k)$ contains terms in γ and $exp(\gamma)$, an exact inversion would be difficult and unusable in practice (computational burden would be too heavy and not compatible with a small sampling period). It is why affine approximations of exponential arcs of the curve $i_c(t)$ for $t \in [k \cdot T_e, (k+1) \cdot T_e]$ have been used, which leads to a polynomial expression of $\bar{F}(k)$ of order five in γ, which can be reduced to a polynomial of order three in the working range of the electromagnet.
Various affine relations can be candidate to approximate $\bar{F}(k)$.
We have chosen the one described on figure 5, on the basis of two criteria :
- the relative error $\frac{i_c(T_e) - \hat{i}_c(T_e)}{i_c(T_e)}$ to keep a good model of the behavior of the coil.
- the relative error $\frac{D(i_c) - \hat{D}(\hat{i}_c)}{D(i_c)}$ where $D(i_c) = \int_0^{Te} \left[\int_0^t (i_c^2(\sigma) \cdot d\sigma) \right] dt$, which represents the effect on the shaft positionning (double integration of a force).

These two expressions have been evaluated for various values of $i_c(0)$ ranging from 0 to $I_{c\,max}$ and various values of γ ranging from 0 to *one*. The approximation is suitable for a ratio $\frac{Te}{\tau}$ less or equal to 0.1, where τ is the time constant of the electromagnet.
Furthermore it can be shown that only one root of the third order polynomial has to be computed by means of an analytic expression, which reduces significantly the computational effort.

The linear approximation is defined by the following equations :

for $0 \le t \le \gamma \cdot T_e$: (D1) => $\hat{i}_c(t) = -\frac{R}{L} \cdot (I_0 - \frac{U}{R}) \cdot t + I_0$

for $\gamma \cdot T_e \le t \le T_e$: (D2) => $\hat{i}_c(t) = c \cdot t + d$ with $c = -\frac{R}{L} \cdot (-\frac{R}{L} \cdot (I_0 - \frac{U}{R}) \cdot \gamma \cdot T_e + I_0)$ (9)

and $d = \frac{R}{L} \cdot (-\frac{R}{L} \cdot (I_0 - \frac{U}{R}) \cdot \gamma \cdot T_e + I_0) \cdot (\gamma \cdot T_e + \frac{L}{R})$

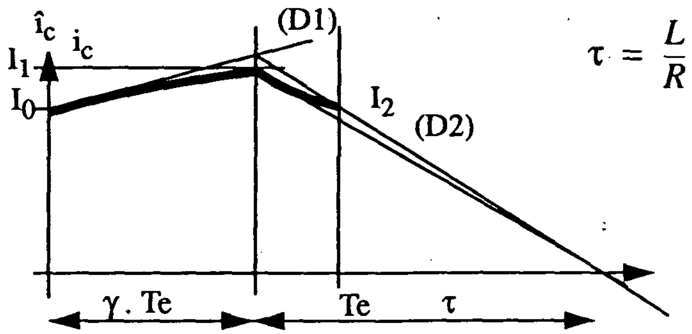


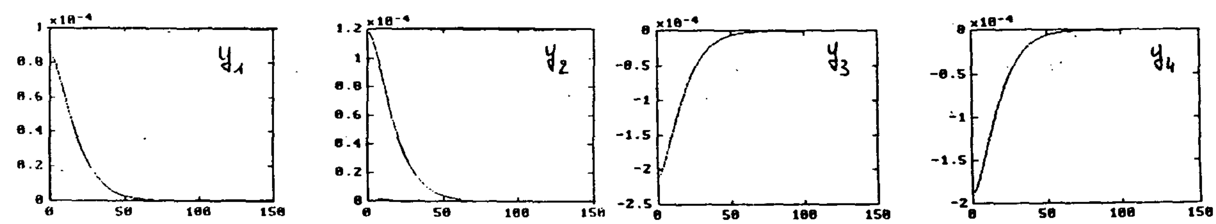
Figure 5 : linear approximation of the current curve

SIMULATION RESULTS

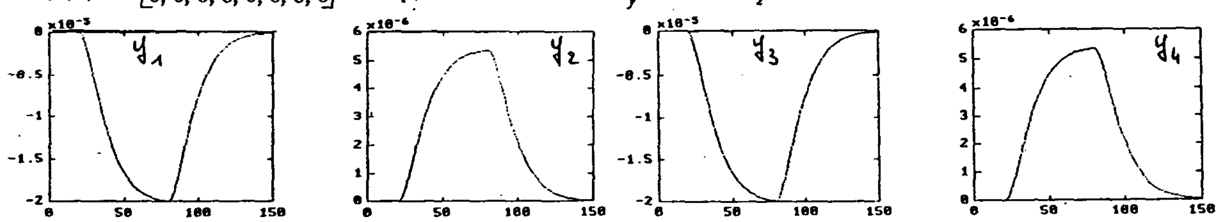
Results presented below were obtained with a simulation program using Matlab language. Work is currently in progress for implementing these algorithms on a Personnal Computer with transputer architecture for the control of a true process.

We first did tests to get information about the behavior of the shaft in case of perfect actuator. Figures 6 show the evolution of shaft positionning starting with given initial conditions or under the influence of disturbing forces applied at the end of the shaft.

$X(0) = \left[2\cdot 10^{-4}, 1\cdot 10^{-4}, 3\cdot 10^{-4}, 2\cdot 10^{-4}, 0, 0, 0, 0\right]^T \quad p = 60\cdot rd\cdot s^{-1} \quad F_y = 0 \quad F_z = 0$



$X(0) = \left[0, 0, 0, 0, 0, 0, 0, 0\right]^T \quad p = 60\cdot rd\cdot s^{-1} \quad F_y = 10 \quad F_z = -10$



$X(0) = \left[0, 0, 0, 0, 0, 0, 0, 0\right]^T \quad p = 60\cdot rd\cdot s^{-1} \quad F_y = 20 \quad F_z = -20$

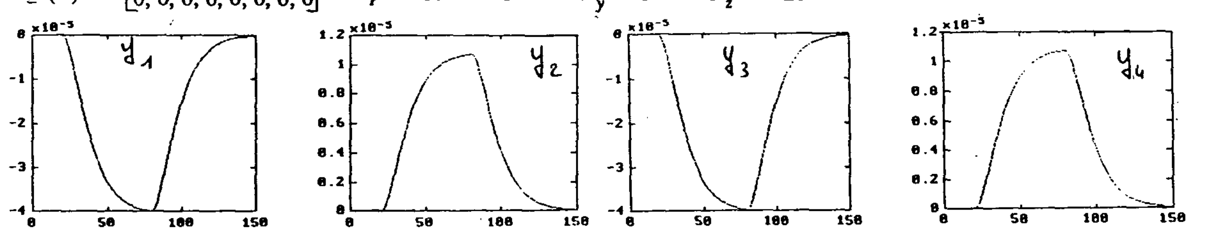


Figure 6 : Shaft positionning assuming perfect actuators

After that, we introduce the non linear model of the actuator together with the inversion algorithm presented in section 4 . Saturations are treated as follows :

- at time k.Te, from the measure of i_c(k.Te) it is possible to derive average maximum force (F_{max}(k) for γ=1) and minimum force (F_{min}(k) for γ=0), that can be produced by the electromagnet .

- Let $\overline{F}(k)$ be the force required by control algorithm ; then

if $\overline{F}(k) \geq \overline{F}_{max}(k)$ then $\gamma(k)=1$

else if $\overline{F}(k) \leq \overline{F}_{min}(k)$ then $\gamma(k)=0$

    else compute $\gamma(k)$

    end

end

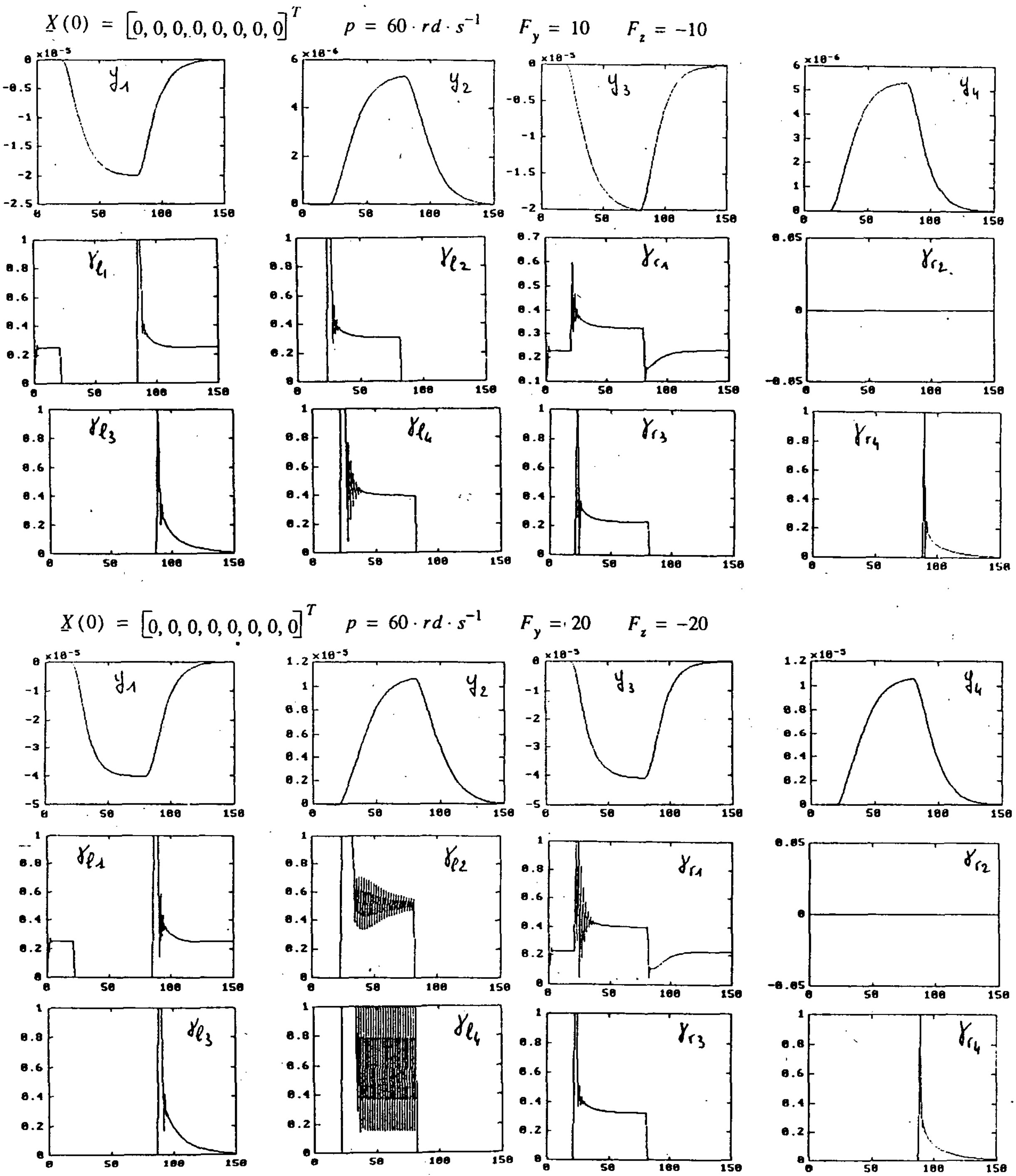


Figure 7 : Shaft positionning with real actuators

At the sight of the curves given in figures 7, some comments can be made :
- the actuator inversion does not affect the behavior of the whole system . Assumptions that were made in section 4 are acceptable.
- for some values of the required forces, some oscillations of the value of γ are noted. This is due to the features of the chopper, and a more precise analysis of the behavior of that device shows that because of constraints on input voltage, and cyclic ratio, it is sometimes impossible to produce the required average magnetic force at each sampling interval, but averaging is made over several periods. In this case, it doesn't affect the behavior of the process.

CONCLUSION

In this paper we have presented a method for designing the control structure of a non linear multivariable system. It combines conventionnal control law synthesis for linear system and inversion methods to deal with non linear features of some parts of the system.
Approximations have been made in order to simplify calculations that must be done in real time since we have considered a digital control scheme.
Simulation results show a good behavior of the whole system. Work is currently in progress to implement these algorithms in a PC computer including a transputer architecture to test this control scheme on a real process.

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