Vibration of a Viscous Liquid-Filled Rotor Supported by Magnetic Bearings

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ABSTRACT

This paper describes the stability study of a partially liquid-filled rotor system supported by magnetic bearings. The liquid is assumed to be viscous and incompressible. In addition to the translational mode, the tilting mode of the rotor system is also taken into account here. In order to investigate the stability of the rotor system, complex eigenvalues are calculated numerically by using Newton-Raphson's method.

1. INTRODUCTION

A rotor partially filled with liquid is widely used in rotating machines like centrifuges. The instability called asynchronous whirling motion has been observed over a range of rotating speed when such kinds of rotating machines are working[1-2]. Since this instability may result in a large vibrational amplitude in rotating machines and lower down their performance, it is necessary to control the instability.

On the other hand, magnetic bearings have a lot of advantages such as elimination of lubrication system, ability to control vibrations, etc., and therefore have been studied by many researchers in recent years[3-5]. In our previous study[6], we made an attempt to apply a pair of magnetic bearings to a liquid-filled rotor system in order to support it and to control the fluid-induced unstable vibration. Our results have shown it is practicable to control the unstable vibration by means of magnetic bearings, but for designing a more effective controller, detailed information about the stability of a liquid-filled rotor is needed.

The stability of a rotor which is partially filled with liquid has been investigated for many years. For example, Wolf[1] studied the stability of a rotor which is partially filled with an invicid incompressible liquid and also performed an experiment. Saito[2] has found a stable bound diagram for a liquid-filled hollow axis which is under the parallel motion, but in his study he considered that the depth of the liquid was small enough compared with the diameter of the hollow axis.

However, those results are not sufficient to be used when we design a liquid-filled rotor supported by magnetic bearings, not only because the supporting stiffness and damping of a magnetic bearing vary with the external load but because it is necessary to find the relations

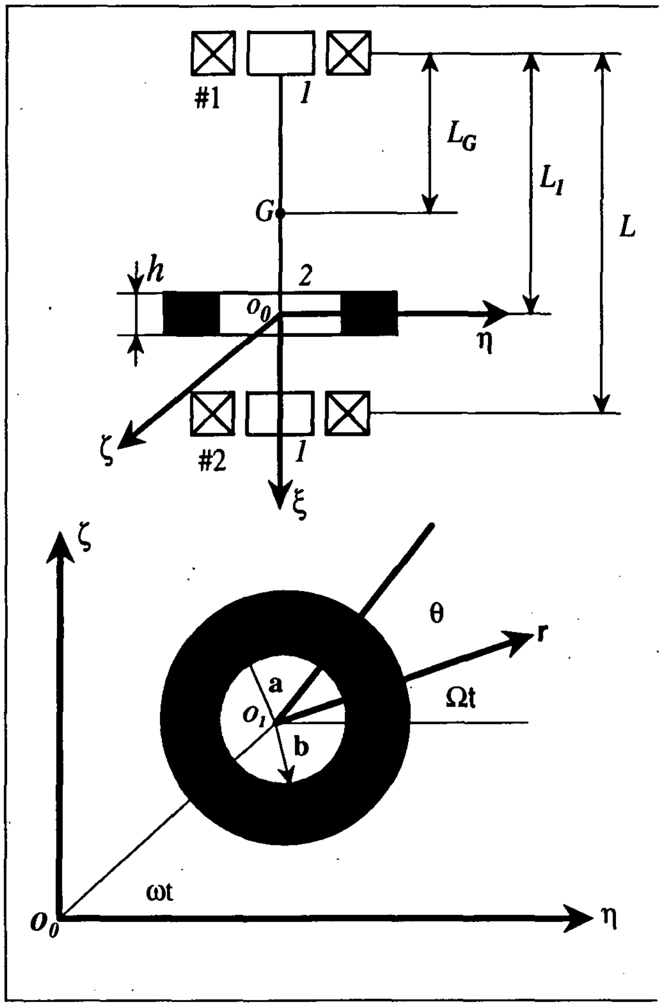


Figure 1. Schematic diagram of the system and the coordinates

between the stability of a rotor system and control parameters to develope the control algorithm for the fluid-induced instability as well.

This paper presents a study on the stability analysis of a partially liquid-filled rotor system which is supported by a pair of magnetic bearings. The liquid is regarded as viscous and incompressible. The unbalance of the rotor is assumed to be negligible. The description of the rotor system consists of three parts: the motion of the liquid, the motion of the rotor system, and the control system of magnetic bearings. The effects on the stability of various parameters, such as, the viscosity of the liquid, the liquid fill ratio, the mass of the rotor, the control parameters of the PD controller, etc., are investigated theoretically.

2. GOVERNING EQUATIONS

2.1 PROBLEM DEFINITION AND COORDINATES

The vibration of the rotor system including liquid is treated as a coupled problem here. The schematic diagram of the system under consideration is shown in Figure 1. A rotor partially filled with viscous liquid is placed vertically. Two active radial magnetic bearings #1 and #2 are used at each end of the shaft connected to the rotor to support the rotor system. In the vertical direction, the system is kept stable by another kind of bearing with which is not dealt in this paper. In addition, rotors 1 and 2 indicate the rotor of the magnetic bearing and the rotor including liquid respectively.

2.2 EQUATIONS OF THE ROTOR SYSTEM

Let L be the total length of the rotor system, h the length of rotor 2 , L_1 the position of the rotor 2 from the upper edge of the system, and L_G the position of the mass center from the upper edge of the system. The basic equations of the rotor system with respect to the coordinate system $O_0 - \xi\eta\zeta$ fixed on space are

$$\left.\begin{aligned} &(2m_1 + m_2)\frac{d^2z}{dt^2} = F_1 + F_2 + P_f \\ &\{2J_{d1} + J_{d2} + m_1L_G^2 + m_1(L-L_G)^2 + m_2(L_1 - L_G)^2\}\frac{d^2\phi}{dt^2} \\ &\quad -j(2J_{p1} + J_{p2})\Omega\frac{d\phi}{dt} = F_2(L - L_G) - F_1L_G + P_f(L_1 - L_G) \end{aligned}\right\}, \tag{1}$$

where m_1 and m_2 are the masses of rotors 1 and 2 respectively, J_{d1}, J_{d2} the inertia moments, and J_{p1}, J_{p2} the pole inertia moments. In addition, F_1 and F_2 are the control inputs of magnetic bearings #1 and #2 respectively, while P_f is the general liquid force acting on the rotor system. Furthermore, z is the complex indication of the displacement z_η and z_ζ in $\eta-$ and $\zeta-$directions, and ϕ is complex indication of the rotation angle ϕ_ζ and ϕ_η around $\zeta-$ and $\eta-$axes, that is

$$\left.\begin{aligned} z &= z_\eta + jz_\zeta \\ \phi &= \phi_\zeta + j\phi_\eta \end{aligned}\right\}. \tag{2}$$

2.3 EQUATIONS OF THE LIQUID

Let a be the radius of the thin rotor including liquid, ρ the density of the liquid and ν the viscosity of the liquid. Only considering the first mode of the circumferential surface waves, the basic equations of the liquid with respect to the coordinates $O_1 - r\theta$ fixed to the rotor can be expressed as

$$\left.\begin{aligned} &\frac{\partial u}{\partial t} - 2\Omega v - \nu(\nabla^2 u - \frac{u}{r^2} - \frac{2}{r^2}\frac{\partial v}{\partial\theta}) = -\frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{d^2\bar{z}}{dt^2}e^{-j(\Omega t+\theta)} \\ &\frac{\partial v}{\partial t} + 2\Omega u - \nu(\nabla^2 v - \frac{v}{r^2} + \frac{2}{r^2}\frac{\partial u}{\partial\theta}) = -\frac{1}{\rho r}\frac{\partial p}{\partial\theta} + j\frac{d^2\bar{z}}{dt^2}e^{-j(\Omega t+\theta)} \\ &\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial v}{r\partial\theta} = 0 \end{aligned}\right\}, \tag{3}$$

where $\nabla^2 = \partial^2/\partial r^2 + \partial/(r\partial r) + \partial^2/(r^2\partial\theta^2)$, $\bar{z} = z + (L_1 - L_G)\phi$, while u and v are the fluid velocity in $r-$ and $\theta-$ directions respectively, and b is the radius of the liquid free surface.

Assuming w is the vibrational displacement of the free surface of the liquid, the boundary conditions may be given as

$$\left.\begin{aligned} u &= 0 \\ v &= 0 \end{aligned}\right\} \quad \text{at} \quad r = a, \tag{4}$$

$$\left.\begin{aligned} &u = \frac{\partial w}{\partial t} \\ &-p - \rho\Omega^2 bw + 2\rho\nu\frac{\partial u}{\partial r} = 0 \\ &\rho\nu(\frac{\partial v}{\partial r} - \frac{v}{r} + \frac{\partial u}{r\partial\theta}) = 0 \end{aligned}\right\} \quad \text{at} \quad r = b, \tag{5}$$

where equation (4) indicates that the relative velocity between the liquid-filled rotor and the liquid at $r = a$ must be 0 and equation (5) indicates the equilibrium condition of pressure at liquid free surface $r = b$.

2.4 CONTROLLER OF MAGNETIC BEARINGS

Letting g_0, i_0 and i_1 be the gap between the rotor and the electromagnets, the bias current in coil and the control input current respectively, we can linearized the attractive force acting on the rotor system from magnetic bearings as

$$\left.\begin{aligned} F_1 &= K_i i_1 + K_n(z - L_G\phi) \\ F_2 &= K_i i_2 + K_n\{z + (L - L_G)\phi\} \end{aligned}\right\}, \tag{6}$$

where K_i and K_n are parameters depending on i_0, g_0 and the physical constants of the magnetic bearings. In addition, the control current i_1 and i_2 will be decided according to the conventional PD control law with a first-order delay filter.

$$\left.\begin{aligned} i_1 &= -K_p(z - L_G\phi) - K_d\frac{d\alpha}{dt} + K_d L_G\frac{d\beta}{dt} \\ i_2 &= -K_p\{z + (L - L_G)\phi\} - K_d\frac{d\alpha}{dt} - K_d(L - L_G)\frac{d\beta}{dt} \end{aligned}\right\}, \tag{7}$$

$$\left.\begin{aligned} T_d\frac{d\alpha}{dt} + \alpha &= z \\ T_d\frac{d\beta}{dt} + \beta &= \phi \end{aligned}\right\}, \tag{8}$$

where T_d is the time constant of the filter, α, β the output of the filter and K_p, K_d is the proportional gain, differential gain respectively.

2.5 ANALYSIS OF COMPLEX EIGENVALUES

In order to investigate the stability of the rotor system, we will calculate the complex eigenvalues of the rotor system. Defining $u, v, p, w, \alpha, \beta, z$ and ϕ as

$$\left.\begin{aligned} u &= u_1(r)e^{j\{(\omega-\Omega)t-\theta\}} \\ v &= v_1(r)e^{j\{(\omega-\Omega)t-\theta\}} \\ p &= p_1(r)e^{j\{(\omega-\Omega)t-\theta\}} \end{aligned}\right\}, \tag{9}$$

$$w = w_1 e^{j\{(\omega-\Omega)t-\theta\}} \ , \tag{10}$$

$$\left.\begin{aligned} \alpha &= \alpha_1 e^{j\omega t} \\ \beta &= \beta_1 e^{j\omega t} \end{aligned}\right\} , \tag{11}$$

$$\left.\begin{aligned} z &= z_1 e^{j\omega t} \\ \phi &= \phi_1 e^{j\omega t} \end{aligned}\right\} , \tag{12}$$

and substituting equation (9) into equation (3) yield

$$\nu r^2 \frac{du_1}{dr^4} + 6\nu r \frac{d^3u_1}{dr^3} + \{3\nu - j(\omega-\Omega)r^2\}\frac{d^2u_1}{dr^2} - \{\frac{3\nu}{r} - 3j(\omega-\Omega)r\}\frac{du_1}{dr} = 0 \ , \tag{13}$$

$$v_1 = -j(u_1 + r\frac{du_1}{dr}) \ , \tag{14}$$

$$p_1 = \rho(\omega-\Omega)rv_1 - 2j\rho\Omega r u_1 + j\rho\nu r(\frac{d^2v_1}{dr^2} + \frac{dv_1}{rdr} - \frac{2v_1}{r^2} - \frac{2ju_1}{r^2}) + \rho\omega^2 r\bar{z}_1 \ . \tag{15}$$

Where $\bar{z}_1 = z_1 + (L_1 - L_G)\phi_1$. From the above three equations, u_1, v_1 and p_1 can be solved as

$$\left.\begin{aligned} u_1 &= \delta_1 c_1 + \delta_2 c_2 + \delta_3 e^{Im(-\lambda r+\lambda a)} c_3 + \delta_4 e^{Im(\lambda r-\lambda b)} c_4 \\ v_1 &= \delta_5 c_1 + \delta_6 c_2 + \delta_7 e^{Im(-\lambda r+\lambda a)} c_3 + \delta_8 e^{Im(\lambda r-\lambda b)} c_4 \\ p_1 &= \delta_9 c_1 + \delta_{10} c_2 + \delta_{11} e^{Im(-\lambda r+\lambda a)} c_3 + \delta_{12} e^{Im(\lambda r-\lambda b)} c_4 + \rho\omega^2 r\bar{z}_1 \end{aligned}\right\} , \tag{16}$$

where

$$\left.\begin{aligned} &\delta_1 = 1 \ , \quad \delta_2 = \frac{1}{r^2} \ , \quad \delta_3 = \frac{\overline{\mathcal{H}}_1^1(\lambda r)}{\lambda^2 r} \\ &\delta_4 = \frac{\overline{\mathcal{H}}_1^2(\lambda r)}{\lambda^2 r} \ , \quad \delta_5 = -j \ , \quad \delta_6 = \frac{j}{r^2} \\ &\delta_7 = -\frac{j\overline{\mathcal{H}}_0^1(\lambda r)}{\lambda} + \frac{j\overline{\mathcal{H}}_1^1(\lambda r)}{\lambda^2 r} \ , \quad \delta_8 = -\frac{j\overline{\mathcal{H}}_0^2(\lambda r)}{\lambda} + \frac{j\overline{\mathcal{H}}_1^2(\lambda r)}{\lambda^2 r} \\ &\delta_9 = \frac{\rho\nu}{r} - j\rho r(\Omega+\omega) \ , \quad \delta_{10} = -\frac{\rho\nu}{r^3} - \frac{j\rho(3\Omega-\omega)}{r} \\ &\delta_{11} = \frac{\rho\nu}{\lambda r}\overline{\mathcal{H}}_0^1(\lambda r) - \frac{\rho\nu + 2j\rho\Omega r^2}{\lambda^2 r^2}\overline{\mathcal{H}}_1^1(\lambda r) \\ &\delta_{12} = \frac{\rho\nu}{\lambda r}\overline{\mathcal{H}}_0^2(\lambda r) - \frac{\rho\nu + 2j\rho\Omega r^2}{\lambda^2 r^2}\overline{\mathcal{H}}_1^2(\lambda r) \end{aligned}\right\} . \tag{17}$$

and $\overline{\mathcal{H}}_0^1(\lambda r), \overline{\mathcal{H}}_1^1(\lambda r), \overline{\mathcal{H}}_0^2(\lambda r)$ and $\overline{\mathcal{H}}_1^2(\lambda r)$ are defined in equation (18).

$$\left.\begin{aligned} \overline{\mathcal{H}}_0^1(\lambda r) &= \mathcal{H}_0^1(\lambda r)e^{Im(\lambda r)} \\ \overline{\mathcal{H}}_1^1(\lambda r) &= \mathcal{H}_1^1(\lambda r)e^{Im(\lambda r)} \\ \overline{\mathcal{H}}_0^2(\lambda r) &= \mathcal{H}_0^2(\lambda r)e^{-Im(\lambda r)} \\ \overline{\mathcal{H}}_1^2(\lambda r) &= \mathcal{H}_1^2(\lambda r)e^{-Im(\lambda r)} \end{aligned}\right\} , \tag{18}$$

here $\lambda^2 = j(\Omega - \omega)/\nu$ and $Im(\lambda r) < 0$. As a result, we can avoid the overflow problem when calculating Hankel functions $\mathcal{H}_0^1(\lambda r), \mathcal{H}_1^1(\lambda r)$, $\mathcal{H}_0^2(\lambda r), \mathcal{H}_1^2(\lambda r)$ numerically even ν is very small.

Furthermore, substituting equations (9) and (16) into boundary conditions (4), (5), we can solve the liquid pressure p and then integrating p over the inner surface of the rotor gives the general liquid force P_f as

$$P_f = \Delta \bar{z}_1 e^{j\omega t} = \Delta\{z + (L_1 - L_G)\phi\} \ , \tag{19}$$

where, Δ is a function of $\rho, \nu, \Omega, \omega, a, b, h$ and can be calculated numerically. Hence, solving the relation equations between F_1, F_2 and z, ϕ, α, β from equations (6), (7) , then substituting those and equation (19) into equation (1) will result in two differential equations with respect to variables z, ϕ, α, β, they are

$$\left.\begin{aligned}
&(2m_1 + m_2)\frac{d^2z}{dt^2} + \{2(K_iK_p - K_n) - \Delta\}z \\
&\quad -\{(K_iK_p - K_n)(2L_G - L) + \Delta(L_1 - L_G)\}\phi \\
&\quad +2K_iK_d\frac{d\alpha}{dt} - K_iK_d(2L_G - L)\frac{d\beta}{dt} = 0 \\
&-\{(K_iK_p - K_n)(2L_G - L) + \Delta(L_1 - L_G)\}z \\
&\quad +\{2J_{d1} + J_{d2} + m_1L_G^2 + m_1(L - L_G)^2 + m_2(L_1 - L_G)^2\}\frac{d^2\phi}{dt^2} \\
&\quad -j(2J_{p1} + J_{p2})\Omega\frac{d\phi}{dt} \\
&\quad +[(K_iK_p - K_n)\{L_G^2 + (L - L_G)^2\} - \Delta(L_1 - L_G)^2]\phi \\
&\quad -K_iK_d(2L_G - L)\frac{d\alpha}{dt} + K_iK_d\{L_G^2 + (L - L_G)^2\}\frac{d\beta}{dt} = 0
\end{aligned}\right\} . \tag{20}$$

Combining equation (20) with (8) and introducing equations (11), (12) into them give

$$[\boldsymbol{D}]\{\boldsymbol{x}\} = \{\mathrm{o}\} \ , \tag{21}$$

where, $\{\boldsymbol{x}\} = \{z_1, \phi_1, \alpha_1, \beta_1\}^T$ and $[\boldsymbol{D}]$ is a 4×4 matrix,which can be expressed as a function of the following dimensionless variables.

$$\begin{array}{llllll}
\omega_0 = \sqrt{\dfrac{K_{p0}}{(2m_1 + m_2)}} & , & \bar{\nu} = \dfrac{\nu}{\omega_0 a^2} & , & \bar{\Omega} = \dfrac{\Omega}{\omega_0} \\
\bar{\omega} = \dfrac{\omega}{\omega_0} & , & \bar{K}_p = \dfrac{K_iK_p - K_n}{K_{p0}} & , & \bar{K}_d = \dfrac{K_iK_d}{(2m_1 + m_2)\omega_0} \\
\bar{T}_d = T_d\omega_0 & , & \bar{M}_1 = \dfrac{m_1}{2m_1 + m_2} & , & \bar{M}_2 = \dfrac{m_2}{2m_1 + m_2} \\
\bar{M}_L = \dfrac{\rho\pi a^2 h}{2m_1 + m_2} & , & \bar{J}_{p1} = \dfrac{J_{p1}}{(2m_1 + m_2)L^2} & , & \bar{J}_{p2} = \dfrac{J_{p2}}{(2m_1 + m_2)L^2} \\
\bar{J}_{d1} = \dfrac{J_{d1}}{(2m_1 + m_2)L^2} & , & \bar{J}_{d2} = \dfrac{J_{d2}}{(2m_1 + m_2)L^2} & &
\end{array}$$

In order to get nontrivial solutions of z_1, ϕ_1, α_1 and β_1 from equation (21), it is obvious that the following equation must be satisfied.

$$|D| = 0. \tag{22}$$

Therefore, from equation (22) dimensionless eigenvalue $\bar{\omega}$ can be calculated by using Newton-Raphson's method.

3. NUMERICAL RESULTS

A numerical calculation was conducted on basis of the above theory.

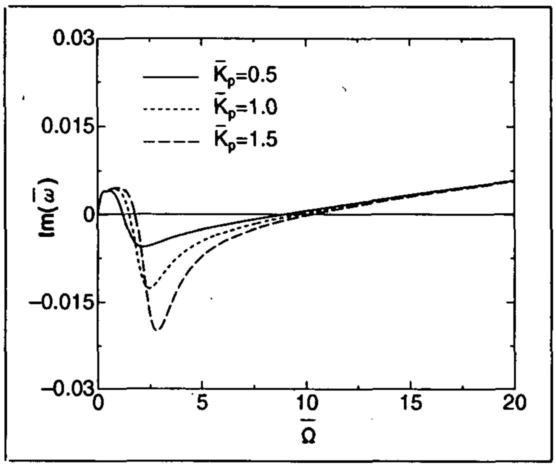


Figure 2. Effects of the proportional gain $\bar{K}_p$ on the stability

From equation (22), many complex eigenvalues can be calculated, but we will only investigate those which are responding to the unstable vibrational mode. The complex eigenvalue $\bar{\omega}$ has a real part and an imaginary one. The rotor system is stable if the imaginary part is positive and vice versa. The zero imaginary part indicates the critical point between the unstable and the stable states.

Figure 2 shows the effects of the proportional gain $\bar{K}_p$ on the stability of the system. It is seen that the lower critical speed is approximately equal to 1.22, 1.42, 1.79, while the upper critical speed is about 9.00, 9.60, 10.20 respectively when $\bar{K}_p = 0.5, 1.0, 1.5$. the unstable region expands a little by little with increase in $\bar{K}_p$. Furthermore, the maximum of the imaginary part of $\bar{\omega}$, which measures the unstable degree of the system, becomes larger with increasing $\bar{K}_p$.

Figure 3 shows the effects of the differential gain $\bar{K}_d$ on the stability of the rotor system. It indicates that, with increasing $\bar{K}_d$ from 0.3 to 0.9, the lower critical speed has little change, while the upper critical speed grows largely, in other words, the unstable region expands a lot. In addition, it can also be seen that the unstable degree increases with the decrease in $\bar{K}_d$.

Effects of the liquid fill ratio b/a on the stability of the system are depicted in Figures 4, where $b/a = 0$ indicates that the rotor is completely filled with liquid. It is seen that the unstable region can not be observed when $b/a = 0.2$, while when b/a grows from 0.4 to 0.6, the unstable region gets wider first and then conversely gets narrower from a certain point.

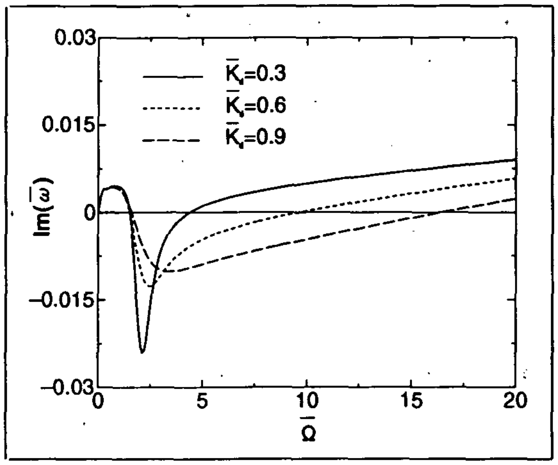


Figure 3. Effects of the differential gain $\bar{K}_d$ on the stability

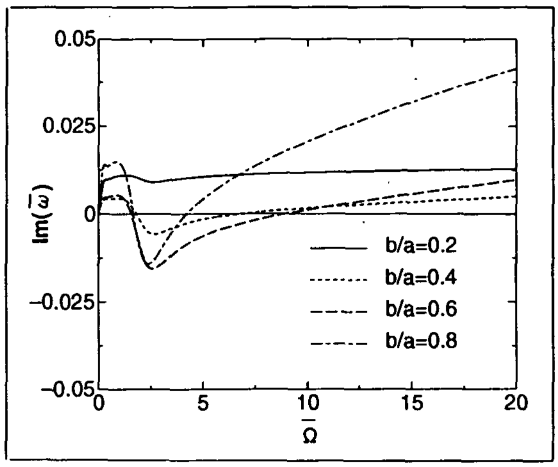
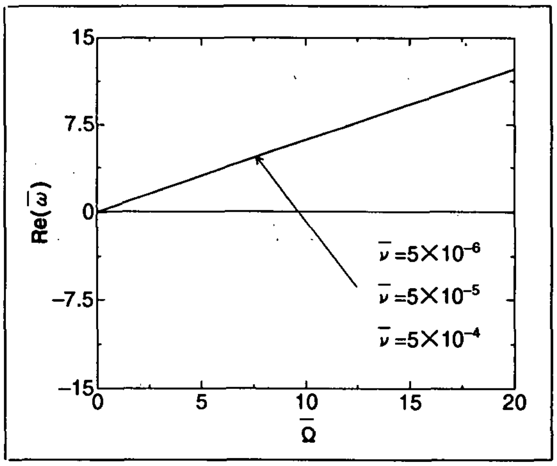


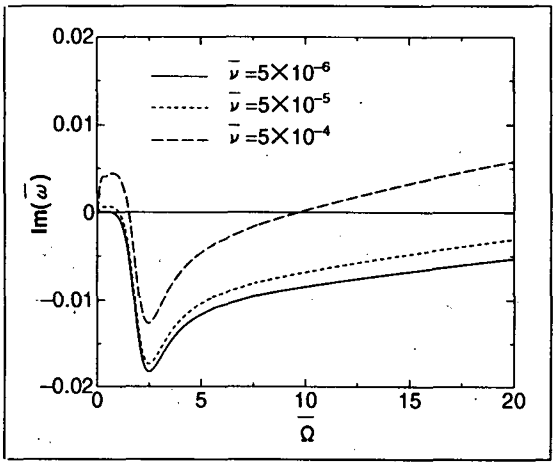
Figure 4. Effects of the liquid fill ratio b/a on the stability

The relations between the dimensionless rotating speed $\bar{\Omega}$ and $\bar{\omega}$ for $\bar{\nu} = 5 \times 10^{-6}, 5 \times 10^{-5}, 5 \times 10^{-4}$ is presented in Figures 5(a),(b). Figure 5(a) shows that the ratio of real part of $\bar{\omega}$ to $\bar{\Omega}$ is almost constant with increasing ν. On the other hand, Figure 5(b) shows that, as $\bar{\nu}$ grows from 5×10^{-6} to 5×10^{-4}, the lower critical speed increases, while the upper critical speed tends to decrease. Thus, the unstable region gets narrower and narrower as the viscosity is enlarged. Moreover, the unstable degree also becomes lower with increasing viscosity.

Figure 6 shows the effects of the mass ratio $\bar{M}_1/\bar{M}_2$ on the stability of the system. As $\bar{M}_1/\bar{M}_2$ grows, the lower critical speed almost keeps constant, the upper one, however, increases gradually. This is perhaps influenced by the second mode(tilting mode) of the rotor system, since the first mode(translational mode) is considered to be affected only by the total mass.



(a). The real part of $\bar{\omega}$



(b). The imaginary part of $\bar{\omega}$

Figure 5. Effects of the viscosity of liquid on the stability of the rotor system

4. CONCLUSIONS

The stability of a partially liquid-filled rotor system supported by magnetic bearings is investigated. It has been found that the viscosity influences the stability of the rotor system greatly. The unstable region and the unstable degree decrease with increasing viscosity. Moreover, the unstable degree increases with increasing the proportional gain of the PD controller, but decreases with increasing the differential gain. As the liquid fill ratio grows, the stability gets worse first and tends to be stable from a certain point. The ratio of the mass of the rotor of magnetic bearing to the mass of the rotor including liquid affects the instability of the system when the tilting mode is taken into account. The unstable region gets narrower gradually with decrease in the mass ratio. On basis of these results, we can design an controller, for example, a controller with a scheduled proportional gain and differential gain, so that the unstable vibrations may be reduced.

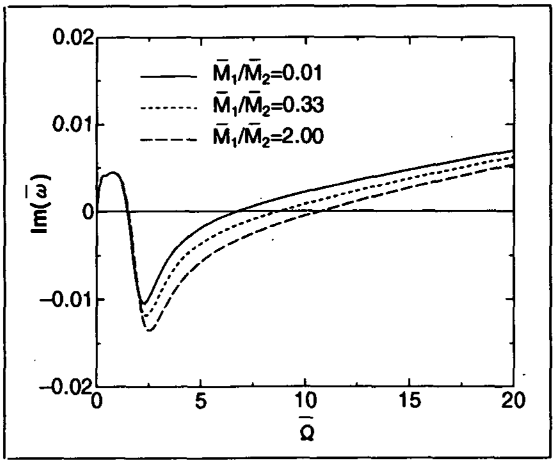


Figure 6. Effects of the rotor mass ratio $\bar{M}_1/\bar{M}_2$ on the stability

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