Dynamics of Active Magnetic Bearings with Magnet Cores in the Shape of a Cone

S. FUKATA AND Y. KOUYA

ABSTRACT

An approximately decoupling control system is designed for an active magnetic bearing system whose electromagnet cores are in the shape of a cone to have both functions of radial and thrust bearings. The bearing system is symmetric in structure: the rotor is symmetric and supported by two bearings composed of similar electromagnets at symmetrical positions with respect to the center of gravity. The magnet cores are made of solid steel. Experimental results are given for frequency responses of the rotor motion to a disturbance input and for transient responses to impact given to the rotor. The former is compared with the numerical results. The results show the usefulness of the design method of the control system.

1. INTRODUCTION

Active magnetic bearings whose electromagnet cores are in the shape of a cone to have both functions of radial and thrust bearings might be useful to simplify the structure and so to reduce the components of the bearing systems, although the hardware of the bearings might be complicated compared with the single-functional separated bearing systems. In addition, since it is possible to make the magnet cores with laminated stacks, the characteristics of the thrust bearings can be improved. Also, the rotor might become shorter. For such bearing systems, however, the design of the control system is complicated because of a multi-input-output system. A method for designing a multivariable control system was presented in [1] with a dynamical model for a similar bearing system.

In this paper an approximately decoupling control system is designed for the bearing system composed of magnet cores in the shape of a cone. The bearing system is symmetric in structure: a symmetric rotor is supported at symmetric positions with respect to the center of gravity by two bearings with similar electromagnets. The magnet cores are made of solid steel

Satoru Fukata, Kyushu University, 6-10-1 Hakozaki, Higashiku, Fukuoka 812, Japan.
Yoshinori Kouya, Saga University, 1 Honjyo-machi, Saga 840, Japan.

for simplicity of manufacturing. Experimental results are given for frequency responses of the rotor motion to a disturbance input and for transient responses to impact given to the rotor. The former is compared with the numerical results.

2. LINEARIZED MODEL OF THE BEARING SYSTEM

Figure 1 shows a structure of the active magnetic bearing system which is symmetric in structure; the characteristics of the electromagnets are similar in the two bearings.

To describe the rotor motion, we set the x, y and z axes originated at the gravity center of the rotor in the steady state into the axial, horizontal and vertical directions, respectively. In the followings we omit the equations of the horizontal direction because they are similar to the vertical ones. The notations are summarized as follows:

Variables:

x : axial displacement of rotor
z : translatory displacement in the vertical direction
z_R : conical displacement in the vertical direction
e_{zij} : actual control inputs into electromagnets
q_i : variables equivalent to magnetic flux
d_i : disturbance forces

Data:

m : mass of rotor
m_R : $=J/L^2$, equivalent mass
m_P : $=J_P/L^2$, equivalent mass
J : moment of inertia about radial axis
J_P : moment of inertia about spin axis

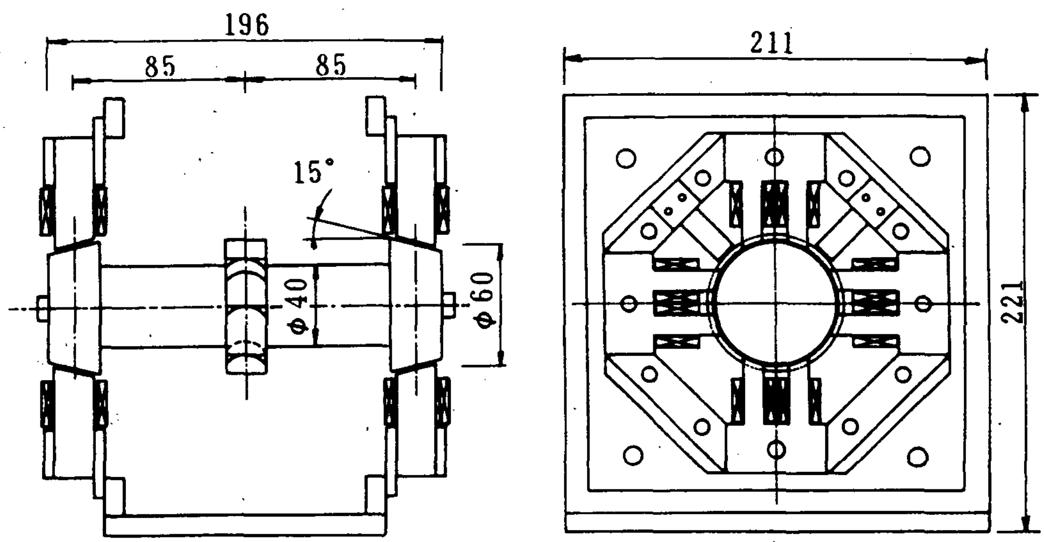

Figure 1 Magnetic bearings in the shape of a cone

L : distance between gravity center and supporting bearing
D : diameter of rotor at the center of bearing
ℓ_0 : air-gap length in the steady state
A : area of magnet core
N : turns of magnet coil
T_i : time constants of generating magnetic flux
T_e : time constant with the effects of eddy currents
b : gain of output coil current for input
α : half angle of cone
β : angle of stator from radial direction
μ_0 : permeability of air
ω : angular velocity of rotor spin
F_{z10} : bias forces
I_{z10} : bias currents

The linearized equations of rotor motion in the symmetrical bearing system are obtained as follows [2]:
Vertical direction:

$$m\ddot{z} = 2K_{Fz}q_z + d_z \tag{1a}$$

$$T_1T_2\ddot{q}_z + (T_1+T_2)\dot{q}_z + q_z - a_{zz}(T_e\dot{z} + z) = b(T_e\dot{u}_z + u_z) \tag{1b}$$

$$m_R\ddot{z}_R - m_P\omega\dot{y}_R = 2pK_{Fz}q_{zR} + d_{zR} \tag{2a}$$

$$T_1T_2\ddot{q}_{zR} + (T_1+T_2)\dot{q}_{zR} + q_{zR} - a_{zRR}(T_e\dot{z}_R + z_R)$$

$$- a_{zRx}(T_e\dot{x} + x) = b(T_e\dot{u}_{zR} + u_{zR}) \tag{2b}$$

Axial direction:

$$m\ddot{x} = 4(\tan\alpha/\cos\beta)K_{Fz}q_x + d_x \tag{3a}$$

$$T_1T_2\ddot{q}_x + (T_1+T_2)\dot{q}_x + q_x - a_{xx}(T_e\dot{x} + x)$$

$$- a_{xzR}(T_e\dot{z}_R + z_R) = b(T_e\dot{u}_x + u_x) \tag{3b}$$

where

$$a_{zz} = a_{0z}(1 + c_{Iz}^2)/2, \qquad a_{zRR} = p\,p_R\,a_{zz} \tag{4}$$

$$a_{zRx} = p\,a_{0z}(1 - c_{Iz}^2)\tan\alpha/2\cos\beta \tag{5}$$

$$a_{xx} = [a_{0z}(1 + c_{Iz}^2) + 2c_{Fzy}a_{0y}]\tan\alpha/4\cos\beta \tag{6}$$

$$a_{xzR} = p_R a_{0z}(1 - c_{Iz}^2)/4 \tag{7}$$

$$a_{0z} = (I_{z10}/\ell_0)\cos\alpha\cos\beta \tag{8}$$

$$p = 1 - \frac{D}{2L}\sin\alpha, \qquad p_R = 1 - \frac{D}{2L}\tan\alpha\cos\beta \tag{9}$$

$$F_{z10} = \frac{\mu_0 A\cos\alpha}{4}\left(\frac{N I_{z10}}{\ell_0}\right)^2\cos\beta, \qquad K_{Fz} = 4\frac{F_{z10}}{I_{z10}} \tag{10}$$

$$c_{Iz} = \frac{I_{z20}}{I_{z10}}, \qquad c_{Fzy} = \frac{K_{Fy}}{K_{Fz}} = \frac{I_{y10}}{I_{z10}} \tag{11}$$

In the above equations, the five control variables are related to the actual control signals input into the electromagnets as follows:

$$2u_z = e_{z1} + e_{z2}, \qquad 2u_{zR} = e_{z2} - e_{z1} \tag{12}$$

$$2u_x = e_{x2} - e_{x1} \tag{13}$$

where, for j=1, 2

$$2e_{zj} = e_{zj1} - c_{Iz}e_{zj2} \tag{14}$$

$$2e_{xzj} = e_{zj1} + c_{Iz}e_{zj2}, \qquad 2e_{xj} = e_{xzj} + c_{Fzy}e_{xyj} \tag{15}$$

The motion of the horizontal direction is described by equations similar to eqs. (1) and (2) with $c_{Iz}=1$. In the above model, only the conical motion in the vertical direction and the axial motion interact each other through their displacements, and the others are independent of each other. The dynamics of the variables q_i equivalent to the magnetic flux are derived from a model which considers the effects of eddy currents in the magnet cores by the first order and is the combination of the models in [3] and [4]. The dynamics is written with the two first-order time-lag systems: for example, in eq. (1) we have

$$q_z = q_{z1} + q_{z2},$$

$$T_j\dot{q}_{zj} + q_{zj} - k_j a_{zz} z = k_j b u_z \quad \text{for } j=1,\ 2 \tag{16}$$

where

$$k_1 = \frac{T_e - T_1}{T_2 - T_1}, \qquad k_2 = 1 - k_1$$

3. CONFIGURATION OF CONTROL SYSTEM

If the control inputs of the five control systems are given, we can derive the actual control inputs given to the electromagnets from the relations (12)-(15). Five of the 8 actual inputs are enough to control the rotor motion of five-degree-of-freedom. Here, to use all the electromagnets effectively for control, we construct the actual inputs as shown in **Fig. 2**.

As a control law, we apply an analog PID control of single loop given by the Laplace transform

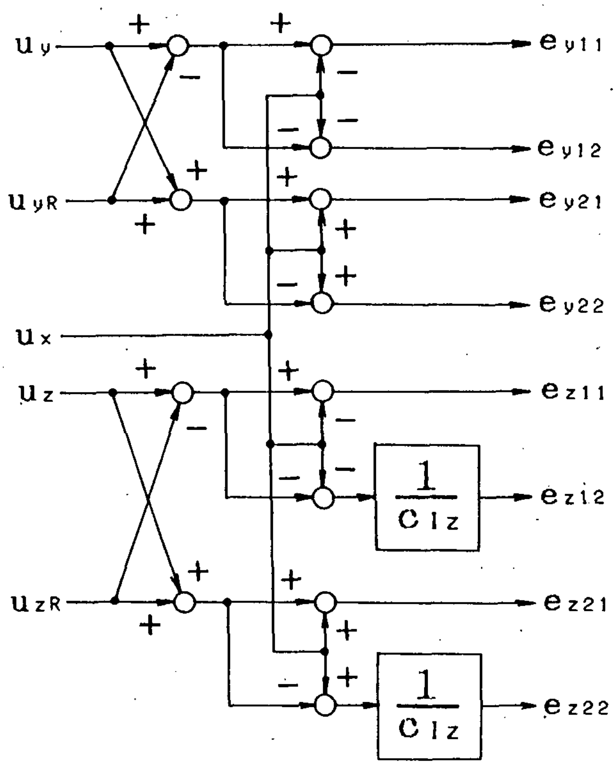

Figure 2 Construction of actual control inputs

$$U(s) = -G_C(s) Z_m(s) + U_O(s),$$

$$G_C(s) = \frac{1}{(1+T_{N1}s)(1+T_{N2}s)}\left[k_P + \frac{k_I}{1+T_I s} + \frac{k_D s}{(1+T_D s)(1+T_{DN}s)}\right] \quad (17)$$

where $Z_m(s)$ is the output of the displacement sensor, $U_O(s)$ is a disturbance input, k_P, k_I and k_D are gains, T_I and T_D are time constants of pseduo-action, and T_{DN}, T_{N1} and T_{N2} are time constants for noise cut.

4. EXPERIMENTS

The main part of the experimental setup is shown in Fig. 1. The magnet cores are made of solid steel for both stator and rotor. Two radial displacement sensors of eddy current type are located near the rotor at each of the symmetrical positions of 61mm from the center of the frame, and one axial sensor is located near the end of the rotor. The primary data are given in TABLE I. The control inputs, the outputs of the PID compensators, were limited to the absolute value of about 2V. The actual control inputs into the electromagnets were restricted within about -1V and about 3V to give the input into the power amplifier (the sum of the bias input and the

TABLE I DATA OF EXPERIMENTAL SETUP

Rotor: Mass	$m=2.75$	kg
Moment of inertia	$J=109\times10^{-4}$	kgm^2
Distance	$L=0.085$	m
Diameter	$D=0.060$	m
Electromagnet:		
Magnet coil: Turns	$N=200$	
Resistance (at 18℃)	$R=0.54$	Ω
Inductance	$L_c=11\times10^{-3}$	H
Area of magnet core	$A=2.28\times10^{-4}$	m^2
Air-gap length	$\ell_0=0.5\times10^{-3}$	m
Angle of cone	$\alpha=15$	deg.
Angle of stator	$\beta=20$	deg.
Gain of power amplifier	$b=1.15$	A/V
Displacement sensor:		
Distance	$L_s=0.061$	m
Gains: Translatory	$k_s=21.2\times10^3$	V/m
Conical	$k_R=15.2\times10^3$	V/m
Axial	$k_x=10.0\times10^3$	V/m
Bias coil currents:		
Horizontal	$I_{y10}=I_{y20}=1.0$	A
Vertical	$I_{z10}=1.0$, $I_{z20}=1.6$	A
Bias force: Theoretical	$F_{z10}=10.4$	N
Force constants: Theoretical	$K_{Fz}=41$	N/A
Measured	$K_{Fz}=37$	N/A
Dynamics of control force:		
$T_1=0.3\times10^{-3}$ s,	$k_1=0.40$	A/V
$T_2=2.5\times10^{-3}$ s,	$k_2=0.60$	A/V

TABLE II PARAMETERS OF COMPENSATORS

Gains (V/V)	Conical	Axial
k_P	0.732	1.08
k_I	40	40
$k_D(\times10^{-3}s)$	3.67	6.62

Time constants:

$T_I=0.44$	s
$T_D=0.10$, $T_{DN}=0.040$	$\times10^{-3}$s
$T_{N1}=0.066$	$\times10^{-3}$s
$T_{N2}=0.033$ (Conical)	$\times10^{-3}$s
$=0.094$ (Axial)	$\times10^{-3}$s

actual control input) by positive values smaller than 4V. The following data were obtained in non-rotation state of the rotor.

It was confirmed in the experiments that only the conical motion of the vertical direction and the axial motion interact each other, and the others are independent of each other. The results will be shown only for the interacting systems. With the control parameters given in TABLE II, the interaction was weak as shown later. Neglecting the weak interaction, from the frequency responses of the closed loop systems to the input amplitude of 0.5V, we have the phase and gain margins of the open-loop transfer characteristics as follows:

Conical control: Phase 27 deg. at 177Hz, Gain 9dB.
Axial control : 29 deg. at 100Hz, 11dB.

4.1 DYNAMICS OF ELECTROMAGNETS

To examine the dynamics of the generating magnetic flux, the frequency response to a sinusoidal input into the power amplifier with the bias input was measured by a search coil of two turns wound around the magnet core in the fixed gap of 0.5mm. **Figure 3** shows the result for the amplitude of 0.5V(58% of the bias input) by the solid line. This characteristics are approximated by the broken line with the sum of two first-order time-lag systems of eq.(16) whose parameters are given in TABLE I. The gain is shifted by 1dB below the theoretical value (about 90%) to fit to the experimental result. This shift estimates the force constant K_{Fz} as 37N/A.

4.2 FREQUENCY RESPONSES

Frequency responses of the rotor motion are examined for a sinusoidal disturbance input superimposed on the control input signal, $U_0(s)$ in eq.

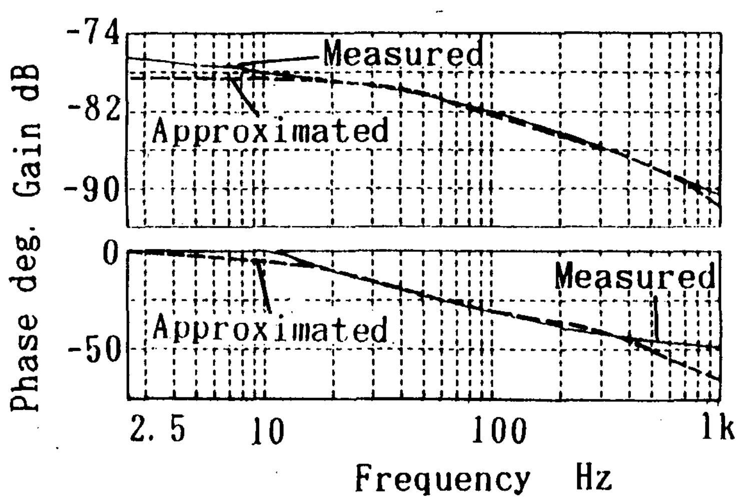

Figure 3 Frequency characteristics of magnetic flux

(17). The axial displacement are displayed by 1.5 times the sensor output to simplify the comparison with the conical displacement.

Figure 4 shows the response of the conical displacement to the input of amplitude 0.5V given to the conical control system of the vertical direction by the solid line. In the gain response, the curve marked "Axial" is the result of the axial motion. As we expected, the influence on the axial motion is small. The broken lines give the numerical results with the force constant K_{Fz}=37 N/A. The numerical results are in good agreement with the experimental results in low frequencies, but not in high frequencies. We have a good agreement over the frequencies for the conical motion if we adopt the lower force constant. Thus the dynamically generating magnetic

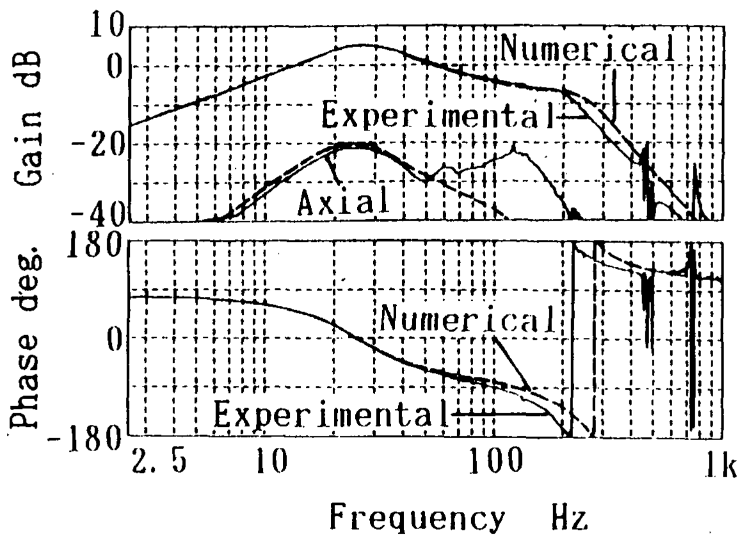

Figure 4 Frequency responses to conical disturbance

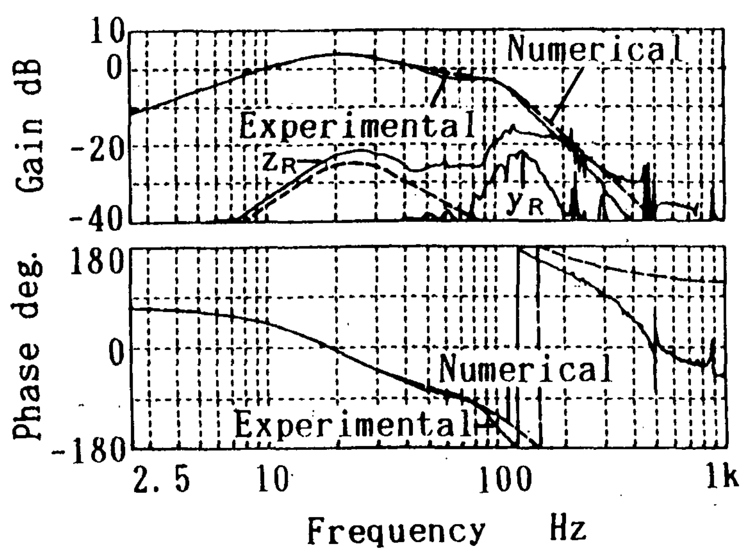

Figure 5 Frequency responses to axial disturbance

flux between the two magnet cores seems less than that obtained from Fig. 3.

Figure 5 gives the response of the axial displacement to the input into the axial control system. The curves marked "z_R" in the gain response are the results of the conical motion of the vertical direction. The comparison between the experimental and numerical results and the influence on the other motion are similar to the preceding results.

The experimental results give a rise in the gain, which is not given in the numerical results, for the interacting motion in the range of about 50Hz to 300Hz. Such a rise appears in the other motions; in the figure an example is shown with "y_R" for the conical motion of the horizontal direction. It may be concluded that these rises are caused by the insufficient stiffness of the stator(frame) and of the mount of the displacement sensors, and by the interaction of the control forces due to the asymmetry of the magnetic circuits.

4.3 IMPULSE RESPONSES

Figure 6 shows a conical transient response with the control signal to impact given at the end of the rotor into the vertical direction. The axial motion was negligibly small to be shown. Since the translatory motion is caused simultaneously, the maximal displacement in the bearing might be near twice the maximal conical displacement on the side of the impact. Hence the rotor seems to be moved close to the stator.

Responses to impact given to the axial direction are shown in **Fig. 7** with the axial control input. Other displacements were negligible. The maximal displacement of the conical motion is smaller than one-tenth of that of the axial motion. This interaction was observed also for impacts giving smaller axial motions. The maximal axial displacement corresponds to the decrease of the air gap by about 0.15mm, 30% of the steady air gap.

The interaction around the gain cross-over frequencies in the frequency responses does not clearly appear in the impulse responses. The impulse responses show that the decoupling control is attained practically even in a nonlinear range of the dynamics. It is interesting that the axial motion has a slight effect on, but is little affected by the conical motion of the vertical direction.

5. CONCLUSIONS

The approximately decoupling control system was designed for the symmetric active-magnetic bearing system whose magnet cores are in the shape of a cone. The dynamics were examined for the frequency characteristics and the impulse responses. The former was compared with the numerical results. The experimental results showed the usefulness of the method for designing the decoupling control system.

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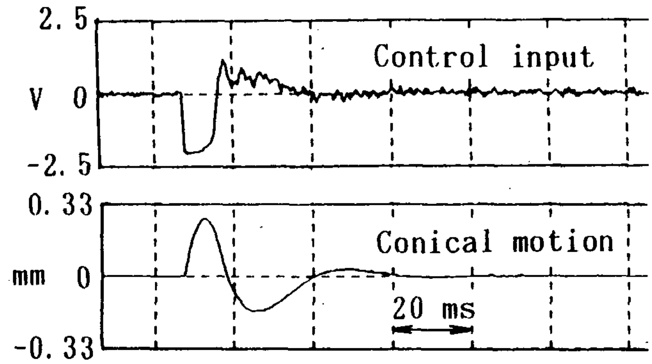

Figure 6 Response to conical impact

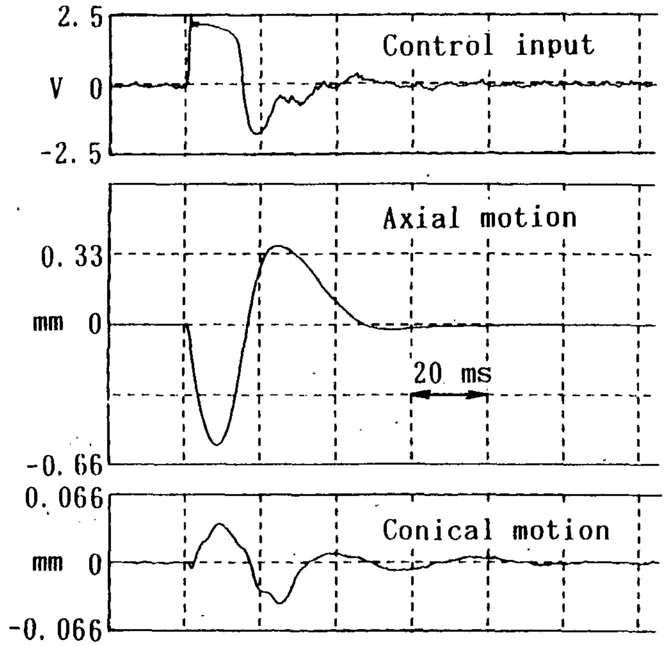

Figure 7 Responses to axial impact

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