Analysis, Modelling and Simulation of the Large-Angle Magnetic Suspension Test Fixture

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ABSTRACT

As part of a NASA program to demonstrate the magnetic suspension of objects over wide ranges of attitudes, a laboratory-scale research project has been undertaken, the Large-Angle Magnetic Suspension Test Fixture (LAMSTF). A cylindrical element containing a permanent magnet core is levitated above a planar array of electromagnets, permitting demonstration of stability and control in five degrees-of-freedom, and of controlled rotation in one degree-of-freedom over a range of 360 degrees. The LAMSTF features a large air-gap (around 0.1 meters), and hence differs fundamentally from typical bearing configurations. Following a brief review of the hardware and the objectives of the research effort, this paper concentrates on analysis, modelling and simulation efforts, considered to be directly relevant to magnetic bearing systems.

Prediction of magnetic fields and forces is essential in magnetic suspension system design. This is particularly difficult in the presence of iron cores or yokes, unless the magnetic circuit approximation is used. A sophisticated finite element computer program, VF/GFUN, is being used to calculate magnetic fields for LAMSTF. Selected field calculations are presented with comparison to actual measurements. The design of LAMSTF deliberately includes eddy current paths. The effect of eddy currents on system dynamics is being studied with a view to incorporating modifications into the system dynamic model and digital controller. The latest results are presented. Linearized equations of motion have been developed for this class of configuration, in order to permit system modelling and the design and analysis of controllers. The development of these equations are reviewed briefly. A full, non-linear simulation is also being developed using $\mathrm{MATRIX}_\mathrm{X}$. The status of this nonlinear model, and difficulties encountered in its development, are discussed.

INTRODUCTION

The objectives of this research effort are to suspend a cylindrical element

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containing a permanent magnet core, to demonstrate stability and control in five degrees-of-freedom, and to permit controlled rotation of the model in one degree-of-freedom over the full range of 360 degrees. A design constraint is that all suspension and control electromagnets are to be behind a flat plane, located some distance from the suspended element. Since this is a ground-based experiment and in order to maintain generality, the plane is chosen to be horizontal, with the model levitated above the plane by repulsive forces.

Potential applications for this type of magnetic suspension system, (large-gap, capable of controlled excursions through large angular ranges) include the manipulation and pointing of space payloads, microgravity vibration isolation and the suspension of models in wind tunnels. The principal uncertainty is thought to be the control of the suspended element, particularly over a wide range of orientation. A suspension system of this particular type has not been attempted before, various design features being quite unusual, such as "levitation" entirely by repulsive forces (attractive forces usually employed) and no "decoupling" of electromagnets (normally the electromagnet configuration is devised so as to provide some natural decoupling between degrees-of-freedom).

It was therefore decided to proceed with design and construction of a small-scale proof-of-concept demonstration system, in order to verify the feasibility of this design approach and to facilitate control system development [1].

HARDWARE DESCRIPTION

The general configuration is illustrated in Figure 1. An array of five, room temperature, copper electromagnets are distributed evenly on a 13.77 cm radius. The coils are wound with 509 turns of AWG 10 enamelled copper wire on bakelite forms, with iron cores. There is no provision for active cooling, it being felt that operation of the LAMSTF would be rather intermittent in nature. The design maximum steady-state current is around 15 Amperes (Coil 1), limited by temperature rise. Temperature sensors with over-temperature alarms and power cut-outs are also fitted. The electromagnets are mounted on a heavy aluminum plate 1.27 cm thick. Each electromagnet is driven by a transistor switching power amplifier, rated at ±150V and ±30A continuous, with the capability of full four-quadrant operation. The switching frequency is 22kHz.

The suspended element consists of 16 wafers of Neodymium-Iron-Boron permanent magnet material, each approximately 0.8 cm diameter and 0.3175 cm thick epoxied into an aluminum tube, 5.32 cm long and 0.9525 cm outside diameter. The total model mass is 22.5 grams and the moment of inertia about tranverse axes is 5.5×10^{-6} kg.m^2. The assumed magnetization is 954930 A/m (1.2 Tesla).

The position sensing system follows a traditional approach of multiple light beams partially interrupted by the model. The beams are arranged in two orthogonal planes (vertical and horizontal) in this case. The light sources are miniature infra-red light-emitting diodes, intended for use with fiber-optics. The light receivers are matching infra-red phototransistors. Due to beam dispersion, collimating lenses were added to both transmitter and receiver. The complete sensor system is mounted on a framework which is initially fixed in one orientation relative to the suspension electromagnets. However, the design permits rotation of the framework about a vertical axis, either manually, or by some form of motor drive to be added later. A schematic diagram of the assembly and the installation of the sensors on the electromagnet array is shown in Figure 2.

To facilitate early commissioning of LAMSTF, an analog controller was constructed, again following traditional practice. Position sensor outputs are summed-and-differenced, where appropriate, to derive motion signals in suspended element axes, that is: axial, lateral, and vertical translations and pitch and yaw rotations. Each signal passes through conventional, dual, series phase-advance compensators. A rather unusual feature is the incorporation of comprehensive decoupling at the output of the controller. A block diagram of the controller is shown in Figure 3. The controller must stabilize the three unstable modes of motion found in this type of suspension system [1,2], illustrated in Figure 4. The most important are the so-called "compass needle" modes, where the suspended element is attempting to reverse its direction in the axial component of applied field. The natural frequency of these modes is around 10 Hz in this case.

MAGNETIC FIELD AND RELATED ANALYSIS

Each electromagnet has a resistance of around 0.74 Ω, not including connecting leads, and an inductance of around 27.5 mH, with iron cores present. There is some uncertainty and ambiguity in the inductance values, due to eddy current effects. A table of inductance values of the coil plus iron core, measured with three different standard instruments is given below.

TABLE 1 - Inductance Measurements

Instrument	Frequency (Hz)	Inductance (mH)	Q factor
LEADER LCR-745	f=120/1000	L=35.0/25.2	Q=5.1/3.8
GENRAD 1650A	1000 (hiQ/loQ)	L=27.4/25.5	Q=3.7/3.8
HP 4261A	120/1000	L=35.1/25.3	Q=4.7/3.7

Eddy currents arise whenever time-varying flux from an electromagnet penetrates a conducting medium. Of course, in magnetic suspension applications the control of the suspended object is usually maintained by constant adjustment of the electromagnet currents. If eddy currents are generated during these adjustments, then the rate of change of field at the model (corresponding to the rate of application of forces and moments) might be reduced, or subject to a phase lag. The mounting of the five LAMSTF electromagnets on a heavy aluminum plate was intended to permit a full assessment of the effects of eddy currents on suspension stability.

Following a simplified approach, wherein the eddy current circuit is supposed to be invariant with excitation frequency, it can be shown that the terminal characteristics of the electromagnet will be as follows :

$$\frac{I}{V} = \frac{1}{(R + Ls) - \frac{(L_{m1}s)^2}{R_e + L_{e1}s} - \cdots} \tag{1}$$

- where R_{en}, L_{en} are the resistance and inductance of the n'th eddy current circuit and L_{mn} is the mutual inductance between primary (electromagnet coil) and secondary. Continuing, the field components generated at the centroid of the suspended object can be expressed as :

$$B = I\left(k + \frac{k_e\, L_m\, s}{R_e + L_e s} + \cdots \right) \tag{2}$$

- where k, k_e are constants representing the field generated at the suspension location by the electromagnet and the eddy current respectively. The break frequency is clearly the inverse of the time constant of the eddy current circuit. It is argued that the factors k_e, L_e/R_e and L_m can be estimated by geometrical analysis and by careful measurements of electromagnet terminal characteristics. An obvious objection to this representation is the fact that the penetration depth (skin depth) of eddy currents will decrease with increasing frequency, leading to changes in eddy current circuit resistance and so forth. However, the frequencies of interest in LAMSTF are quite low, so that this effect will be weak.

Some measurements of the terminal characteristics of a LAMSTF coil are summarized in Figure 5, with results of calculations based on Equation (1). Two separate defects from ideal behaviour are observed, related to currents in the alloy plate and also the (unlaminated) iron electromagnet core.

The magnetic field generated by each electromagnet has been calculated using the computer program VF/GFUN. This uses an integral equation formulation, only requiring discretization of the iron regions in a problem. Some field measurements have been made to validate these computations. The axial field on the axis of one electromagnet, carrying a current of 10 amps, is shown in Figure 6. Agreement is excellent. The vertical field on an axis passing through the centroid of the suspended element is also shown in Figure 6. The field distribution appears to be in good agreement, but with a systematic discrepancy in magnitude. It is noted that the fields in this location are very weak and that the gradients in all directions are strong, thus presenting a difficult measurement challenge. Calculated field and field gradient components are used for all further design and analysis work.

LINEAR SIMULATION

Following the approach detailed in [3,5], linearized equations representing the dynamics of the suspended element in the quasi-static suspension fields have been derived. These are perturbation equations from equilibrium. Also, state-space representations of the electromagnet-power supply combinations, including the effects of mutual inductance between adjacent electromagnets, and of the dual phase-advance controllers, have been developed. These equations are implemented as a system simulation using MATLAB. The objective here is to permit relatively straightforward qualitative and quantitative studies of the effects of various additional terms on the dynamics of this complex multi-degree-of-freedom system. These terms include eddy currents, as discussed above, mutual inductance, sensor break frequencies, digitized controller sample rates and so forth. The baseline system simulation is functional, and some preliminary results are shown in Figure 7. The predicted coupling between various degrees of freedom, and the qualitative nature of system responses agrees with experimental observations.

SIMULATION of the NONLINEAR SYSTEM

A nonlinear model of LAMSTF has been developed using MATRIX_X/System-Build software. The development of this model is a continuing process, whereby additional effects can be incorporated into the model as they are encountered, such as eddy currents. The implementation of the nonlinear model is based upon the complete equations of motion of the magnetic suspension system as presented in [3]. These equations include the effects of the magnetic forces and torques, along with the equations of motion of the suspended element. The resulting equations of motion are written in suspended element coordinates, in terms of the magnetic fields ({B}), the first-order gradients ([∂B]) and core properties, as shown as equations 3 and 4. Note that a bar over a variable indicates that it is referenced to suspended element coordinates instead of inertial coordinates.

$$\{\dot{\bar{\Omega}}\} = \left(\frac{1}{I_c}\right)(\mathrm{Vol}\ ([\bar{M}]\ [T_m]\ \{B\}) + \{\bar{T}_d\}) \tag{3}$$

$$\{\dot{\bar{V}}\} = \left(\frac{1}{m_c}\right)(\mathrm{Vol}\ ([T_m]\ [\partial B]\ [T_m]^{-1}\{\bar{M}\}) + \{\bar{F}_d\}) \tag{4}$$

- where $[\bar{M}]$ is the cross product matrix, such that $\{\bar{M}\} \times \{B\} = [\bar{M}]\ \{B\}$

$[T_m]$ is the transformation from inertial to suspended element coordinates

$\{\dot{\bar{\Omega}}\}$ is the angular acceleration of the suspended element

$\{\dot{\bar{V}}\}$ is the translational acceleration of the suspended element

I_c , m_c are the moment of inertia and the mass of the suspended element

By modelling the system in MATRIX$_X$/SystemBuild, the equations governing the nonlinear model are implemented using various blocks to represent individual functions and operations that must be performed, such as cross-products, gain matrices, integration and so on. The full nonlinear MATRIX$_X$/SystemBuild model of the system is shown in Figure 8.

A recent addition to the nonlinear model is the modification of the magnetic fields with respect to the translation of the suspended element. This is implemented by a second-order taylor series expansion of the magnetic fields with respect to position. This requires the addition of second-order gradients of the magnetic fields to the nonlinear model. The Taylor series expansion is implemented in the simulation by using an algebraic expression block.

The implementation of the nonlinear model has been verified by several independent efforts. The results obtained closely match results obtained from an analytical approach used in [5]. The analytical approach used linearized equations of motion, with small angle assumptions. A state-space representation of the system is then obtained from the linearized equations. Calculated eigenvalues, shown in Table 2, and eigenvectors agree approximately with values obtained from the MATRIX$_X$/SystemBuild nonlinear implementation of the system. Successful representation of the nonlinear model allows a digital controller to be designed and modelled using MATRIX$_X$/SystemBuild.

TABLE 2 - Comparison of Eigenvalues from Linear and Nonlinear Models

Mode	Linear Model (rad/s)	Nonlinear Model (rad/s)
1	±59.2596	±59.2597
2	±7.9717j	±7.9717j
3	±58.2943	±58.2944
4	±0.9556j	±0.9556j
5	±9.7762	±9.7762

ACKNOWLEDGEMENTS

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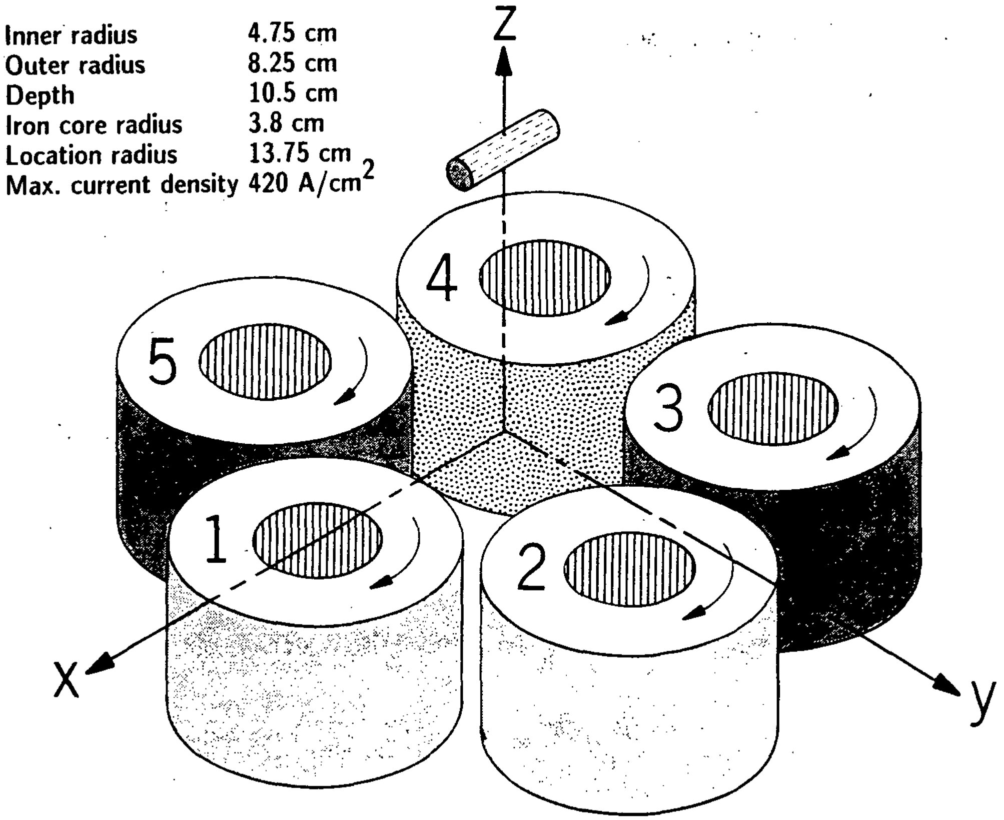

Figure 1 - Large-Angle Magnetic Suspension Test Fixture (LAMSTF) Configuration

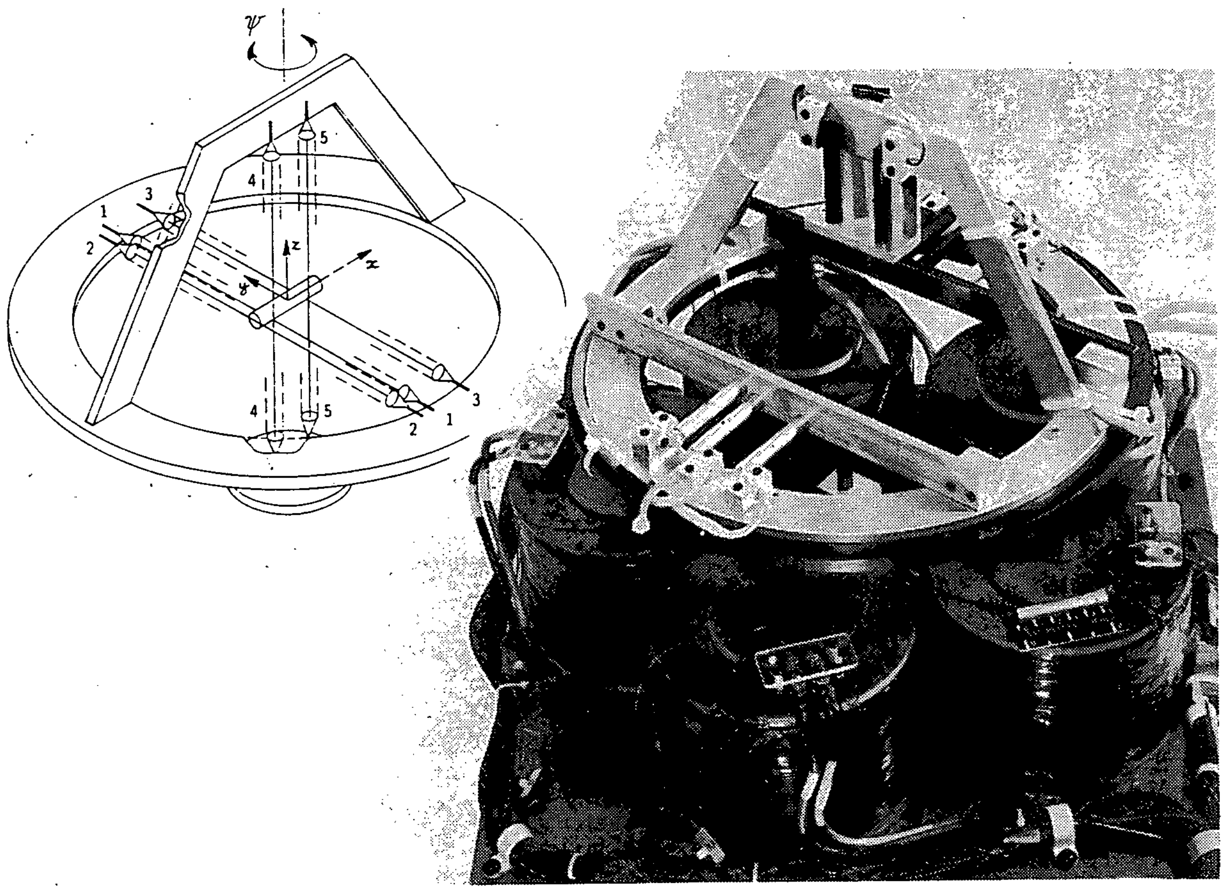

Figure 2 - Sensing System Schematic and Installed on Electromagnet Assembly

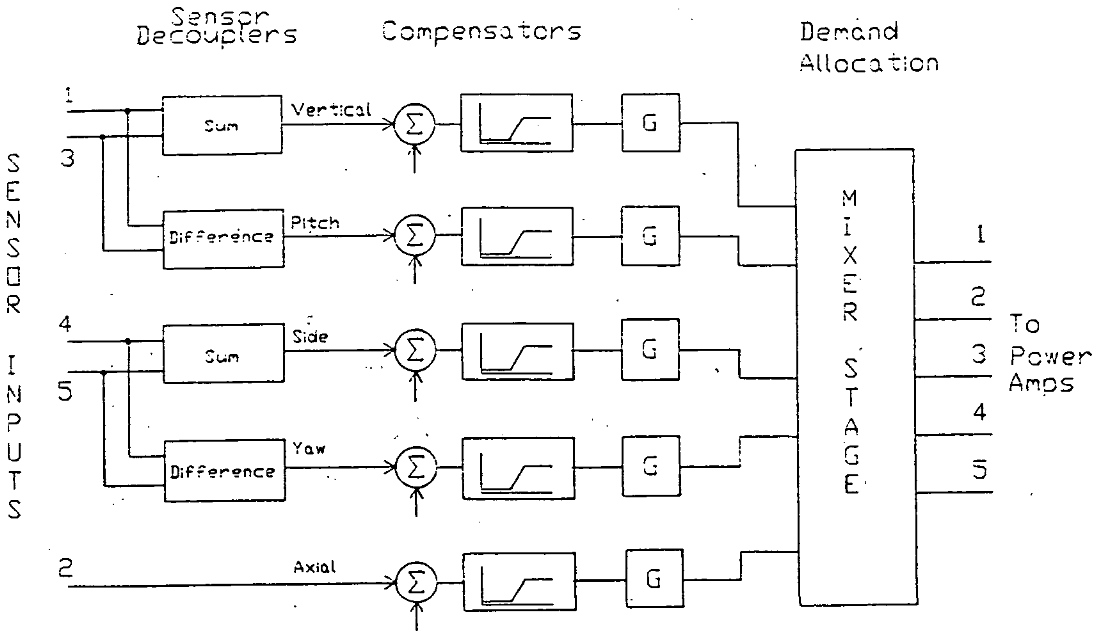

Figure 3 - Analog Controller Block Diagram

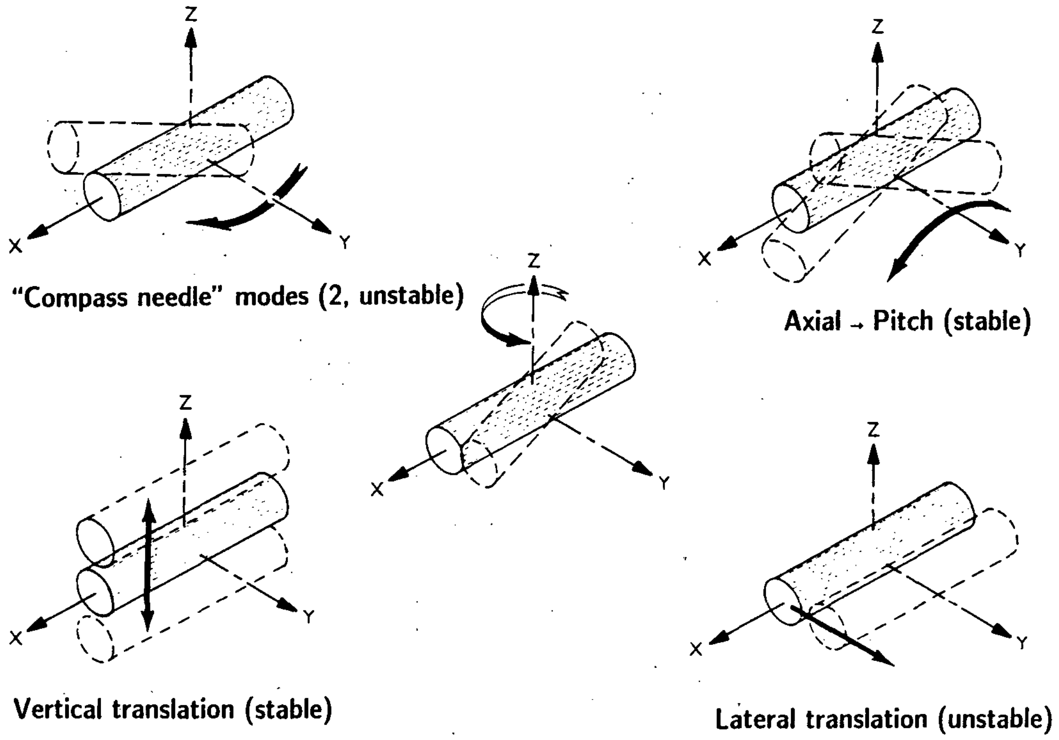

Figure 4 - Natural Modes of Suspended Element

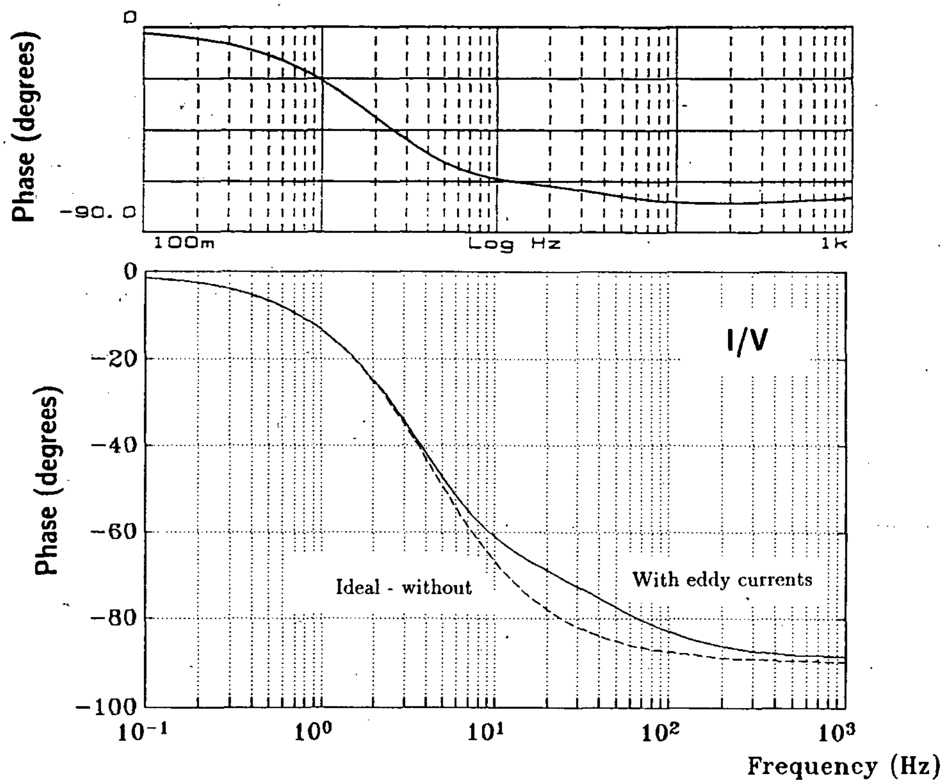

Figure 5 - Coil Terminal Characteristics

LAMSTF coil (top) and single time-constant model (bottom, using Equation 1)

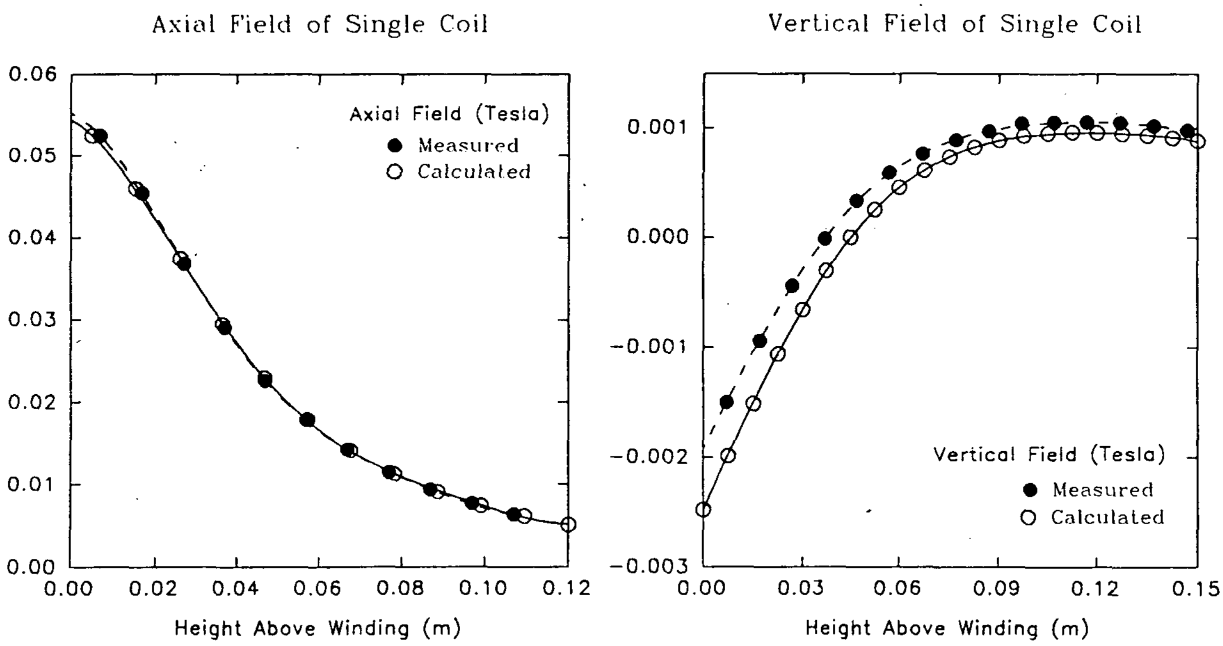

Figure 6 - Typical Comparisons of Computed and Measured Fields in LAMSTF

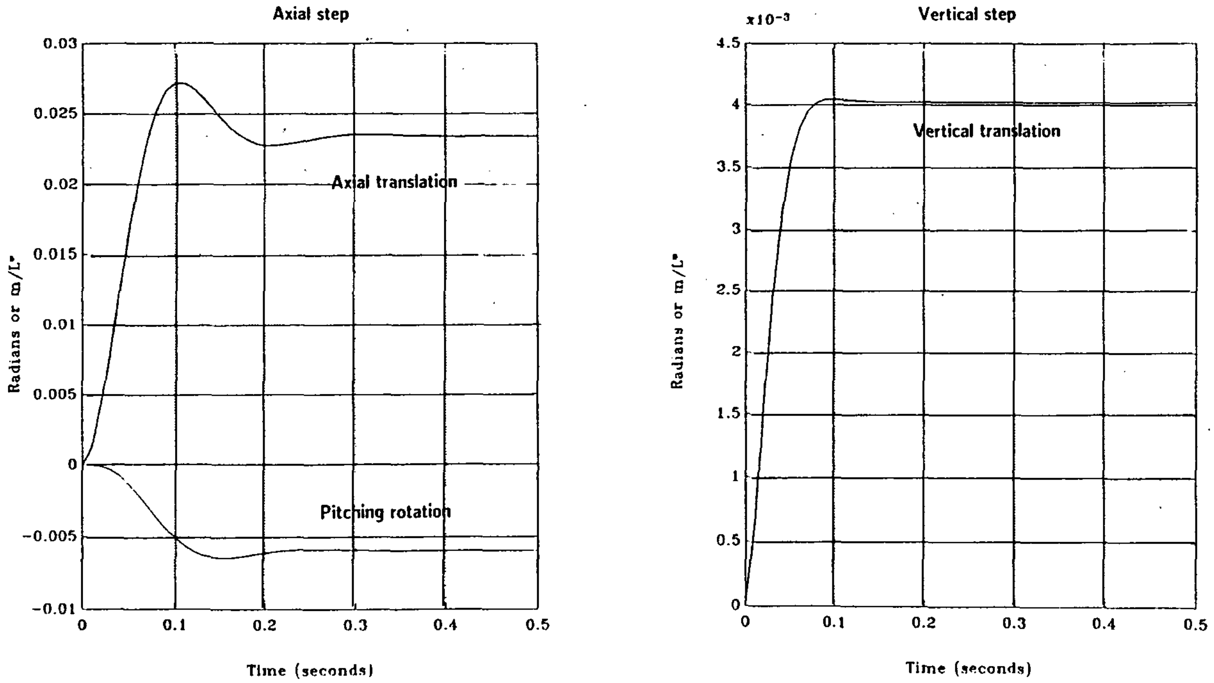

Figure 7 - Typical Simulated Responses from Linearized Simulation

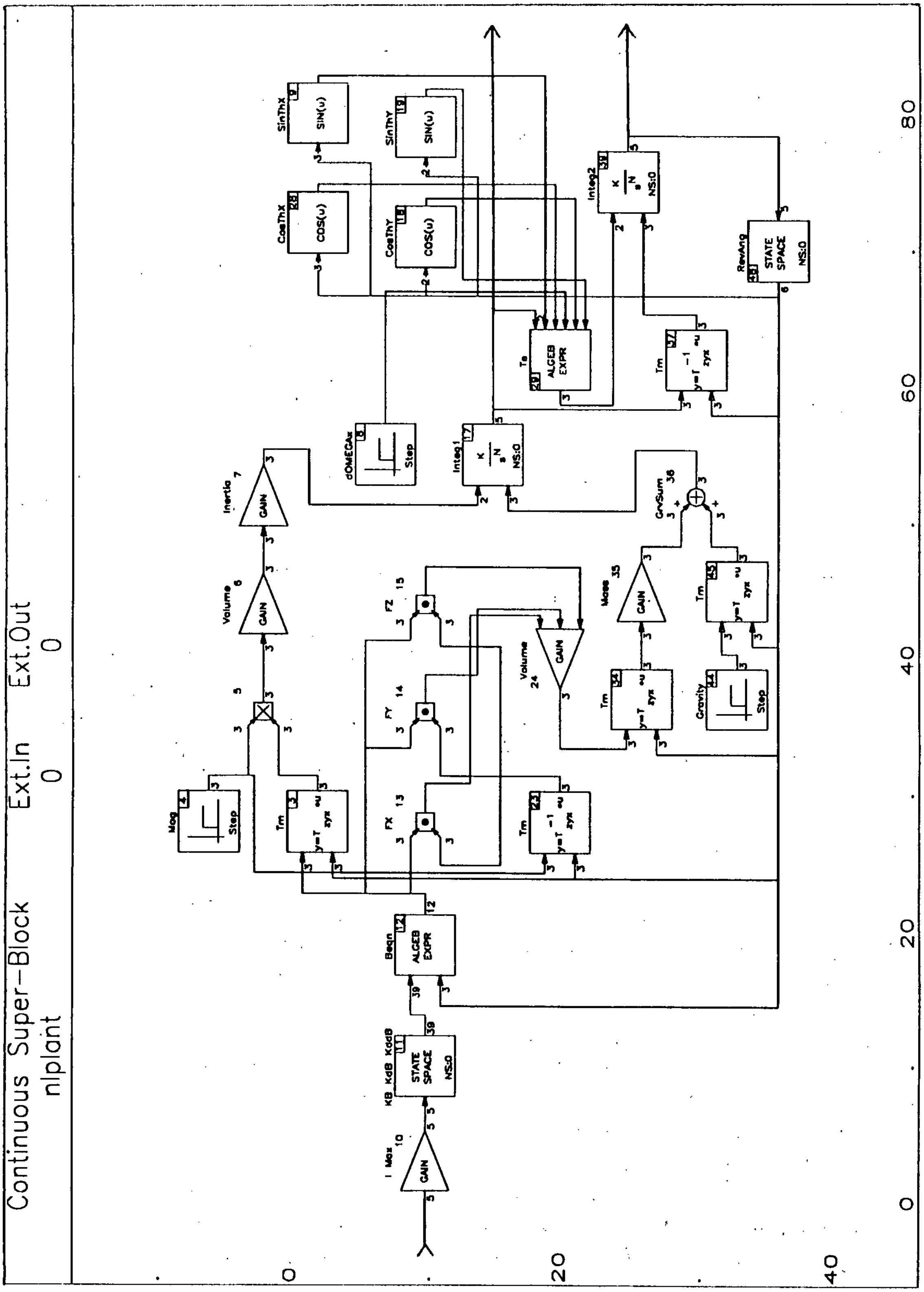

Figure 8 - MATRIX$_X$/SystemBuild Implementation of Nonlinear Model