A Comparison between "Passively" and "Actively" Controlled Magnetic Bearings

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ABSTRACT

We propose the following terminology: a magnetic bearing (MB) is called *"passive"* if it is, from the point of view of its dynamical interaction with the supported body, *"equivalent"* to an (arbitrarily complex) interconnection of *positive* masses, *positive* springs and *positive* dampers. There are several well-known advantages of passive control (simplicity, granted stability in presence of certain unmodelled dynamics etc.). Our final goal is to quantitatively describe some limitations caused by the passivity constraint. As a first step in this direction we investigate a benchmark problem where the MB-supported body is a two-mass oscillator. We consider the requirement of *asymptotic disturbance rejection* at some given frequency ω_0. Furthermore, the magnitude response around ω_0 should be sufficiently *"flat"*. This paper shows that there is a bound on this "flatness" which cannot be surpassed by any *passive* bearing. However, if *active* bearings are allowed, this bound cancels and magnitude response around ω_0 is allowed to be arbitrarily flat.

1 INTRODUCTION

The notion of *passivity* has been well established in electrical networks, control and system theory for a very long time, see references in [1]. Roughly speaking, passive systems are characterized by stability plus an energy flow condition (dissipation) related to the input/output behavior. Any (negative) feedback interconnection of passive systems is passive again, hence stable. This very important property of passive systems has many applications.

Unfortunately, the above notion of passivity does not match the terminology used by those involved with magnetic bearings. For example, an ideal PD controlled magnetic bearing with sensor/acuator collocation is called "active" even if it should be called passive in the above sense. But things get even worse: a permanent magnet bearing (which might be modeled by a *negative* spring) is called "passive" even if it ought to be called active in the above sense, since it might have a destabilizing property!

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2 MYSTERIOUS WATERBED EFFECTS

Any skilled control engineer is familiar with the following observation: suppose you wish to reduce the magnitude of some closed–loop transfer function in some frequency range 1 by tuning the controller. Suppose you succeed in doing so, it is very likely that the magnitude response will rise in some other frequency range 2, see figure 1. A Waterbed is a fine illustration for this. In particular, high peak values of the magnitude response may appear in the case of harmonic disturbance cancellation at some given frequencies, see figure 2. Our final goal is a deeper understanding of the "waterbed effect" in the case of passively controlled systems. This goal lies beyond the scope of this paper; however, we will take some steps towards it.

3 PURPOSE OF THIS PAPER

Consider a sinusoidal disturbance where the *nominal* value of the frequency is given, say ω_0, but where amplitude and phase are not available. The actual frequency is only known with some accuracy. Furthermore, let us require asymptotic disturbance rejection at ω_0 in some closed–loop transfer function $T(s)$, i.e. $T(i\,\omega_0) = 0$, see figure 2. Then, a sufficiently flat magnitude response $|T(i\,\omega)|$ around ω_0 is desirable because of small uncertainties of the disturbance frequency. In the sequel of this paper, we investigate the flatness described by slope $\kappa := |T'(i\,\omega_0)|$, i.e. the derivative modulus at $i\,\omega_0$. The purpose of this paper is to show that κ *cannot* be arbitrarily small in the case of passive control. However, in the case of active control, we will show that κ can be chosen to be arbitrarily small.

FIGURE 1: WATERBED EFFECT !

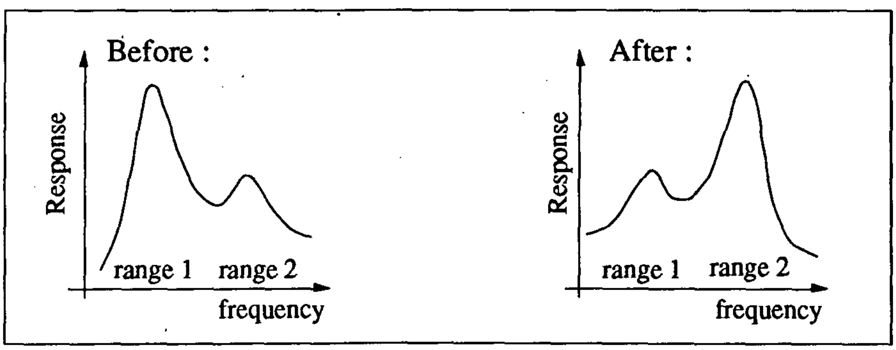
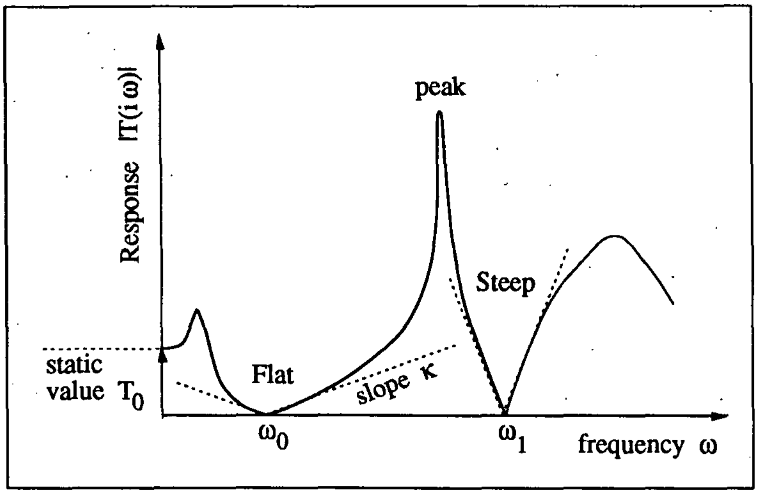


FIGURE 2: FLATNESS OF HARMONIC DISTURBANCE CANCELLATION.



4 EXAMPLE

The following example helps to explain the ideas. Consider a supported two–mass oscillator $m_1 = m_2 = c = 1$, see figure 3. Let $T(s)$ be the dynamic compliance of bottom mass m_1. We are seeking a *passive* bearing such that the following requirements hold:

- Disturbance cancellation at ω_0, i.e. $T(i\,\omega_0) = 0$.
- Low static compliance $T_0 := T(0)$.
- Flat magnitude response $|T(i\,\omega)|$ around ω_0, i.e. small slope $\kappa := |T'(i\,\omega_0)|$.

Let us describe the bearing dynamics by its transfer function $C(s)$ relating the bearing force to the velocity of top mass m_2. A passive bearing leads to a *positive real* transfer function $C(s)$. Recall:

Definition [1]. Let $C(s)$ be a rational transfer function with real coefficients. Then, $C(s)$ is termed *positive real* if $C(s)$ has *no* poles in the open right half-plane, and if $\Re e(C(i\,\omega)) \geq 0$, for all ω. Furhermore, any pole $p = i\,\omega_\star$ on the imaginary axis must be simple and its residuum has to satisfy $\lim_{s\to p} \{(s-p)\,C(s)\} > 0$.

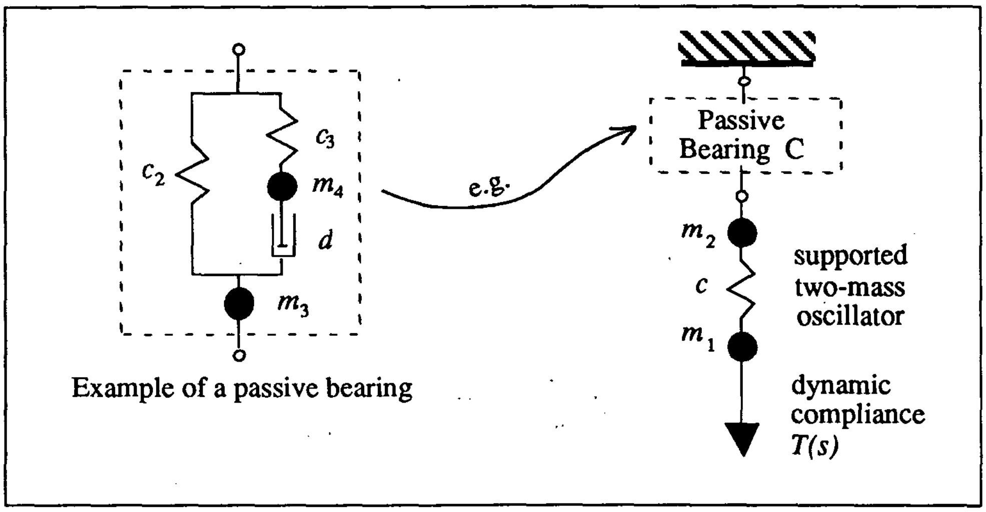
Bottom mass compliance $T(s)$ and bearing $C(s)$ are related by the following *linear fractional* map

$$T(s) \;=\; \frac{s\,C(s) \;+\; (s^2+1)}{s(s^2+1)\,C(s) \;+\; s^2(s^2+2)} \tag{1}$$

Inserting the requirement $T(\pm\,i\,\omega_0) = 0$ in (1), an interpolation condition for bearing $C(s)$ follows:

$$C(\pm\,i\,\omega_0) \;=\; \pm\,i\;\frac{1-\omega_0^2}{\omega_0} \tag{2}$$

FIGURE 3: TWO–MASS OSCILLATOR SUPPORTED BY A PASSIVE BEARING.



Note that the right hand side of (2) has zero real part. There is a simple physical interpretation to this: bearing C is *not dissipative* at frequency ω_0.

Bearing C also has to eliminate the rigid body mode of the unsuspended two–mass oscillator, i.e. $T_0 = T(0)$ has to be *finite*. This implies one more interpolation condition for $C(s)$:

$$C(0) \;=\; \infty \quad \text{resp.} \quad c_b \;:=\; \lim_{s\to 0}\{s\,C(s)\} \;\neq\; 0 \tag{3}$$

where c_b denotes the static bearing stiffness. Note that the static bearing stiffness c_b "combines" with stiffness c of the two–mass oscillator, i.e.

$$T_0 \;=\; c^{-1} + c_b^{-1} \;=\; 1 + c_b^{-1} \tag{4}$$

For passive bearings we have the obvious bound $T_0 > 1$ since $c_b > 0$.

Now we propose a "special type" of passive bearing for the two–mass oscillator. It consists of an arrangement of two masses m_3, m_4, two springs c_2, c_3, and one damper d, see left side of figure 3. The corresponding transfer function is

$$C(s) = \frac{n(s)}{d(s)} \quad \text{where} \quad d(s) \;=\; m_4\,s^3 + d\,s^2 + c_3\,s\,,$$

$$n(s) = m_3 m_4\,s^4 + d(m_3 + m_4)\,s^3 + (c_3 m_3 + c_2 m_4)\,s^2 + d(c_2 + c_3)\,s + c_2 c_3 \tag{5}$$

We claim that by an appropriate "tuning" of the bearing parameters we may achieve harmonic disturbance cancellation, i.e. $T(i\,\omega_0) = 0$, at one arbitrarily given frequency ω_0. In fact, we obtain the following two conditions on the bearing parameters by a straightforward calculation:

$$c_3 = m_4\,\omega_0^2 \tag{6}$$

$$c_2 = (1 + m_3)\,\omega_0^2 \;-\; 1 \;>\; 0 \tag{7}$$

Let us try to use the remaining freedom of $5 - 2 = 3$ parameters in order to achieve low values for *both* static compliance T_0 and slope $\kappa := |T'(i\,\omega_0)|$. Note that static bearing stiffness c_b equals c_2. Consequently, (4) leads to

$$T_0 \;=\; 1 + c_2^{-1} \tag{8}$$

A tedious manipulation (insert (6) & (7) in (5), and (5) in (1), then differentiate at $i\,\omega_0$) yields

$$\kappa \;=\; 2\,(1 + m_3 + m_4)\,\omega_0 \tag{9}$$

Obviously, we have the bound $\kappa > 2\,\omega_0$ which follows from (9) by letting $m_3, m_4 \to 0$. However, small values of m_3 could lead to negative values of spring c_2, see equation (7). By combining these two bounds it easily follows

$$\kappa \;>\; \max\,(\frac{2}{\omega_0},\; 2\,\omega_0) \tag{10}$$

Finally, by combining (8), (7) and (9), where $m_4 \to 0$, we obtain the following interesting trade–off relationship:

$$T_0 \;>\; \frac{\kappa\,\omega_0}{\kappa\,\omega_0 \;-\; 2} \tag{11}$$

Figure 4 displays equation (11) for a fixed value of $\omega_0 = 0.5$.

Conclusion: The above *passive* bearing setup does *not simultaneously* allow arbitrarily *small* values for T_0 and κ. Decreasing flatness slope κ implies increasing static compliance T_0 and vice versa.

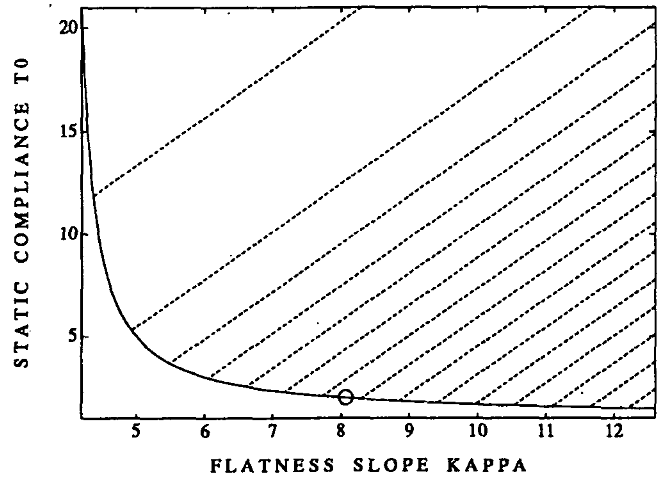
Example: The following example corresponds to the $\circ$ mark in figure 4. Let us fix ω_0 to $\omega_0 = 0.5$. Furthermore let us require a static compliance of $T_0 = 2$. From (11) we have the bound $\kappa > 8$. This corresponds to the flattest magnitude response around ω_0 for the above setup. Now, let us determine parameter values in order to come close to this bound. From (4) it follows that $c_2 = 1$, and (7) gives $m_3 = 7$. Choose m_4 small, say $m_4 = \frac{1}{16}$. Then, (6) yields $c_3 = \frac{1}{64}$. Choose $d = 1$. Finally, (5) gives the corresponding bearing transfer function $C(s) \;=\; (28s^4 + 452s^3 + 11s^2 + 65s + 1)\,/\,(4s^3 + 64s^2 + s)$. From (1) we obtain the dynamic compliance of the suspended two–mass oscillator

$$T(s) = \frac{(4s^2+1)\,(8s^2+129s+2)}{32s^6+516s^5+48s^4+645s^3+14s^2+65s+1}$$

Figure 5 shows the magnitude response $|T(i\,\omega)|$. We achieve a slope $\kappa := |T'(i\,\omega_0)| = \frac{129}{16} = 8.0625$ which comes very close to the theoretical a priori bound $\kappa > 8$.

Relation to the "waterbed effect": Figure 5 shows two high magnitude peaks corresponding with two weakly damped modes. In fact, if we wish to come close to bound (11), see figure 4, two complex conjugate pairs of poles of $T(s)$ approach the imaginary axis. In other words, low values of static compliance T_0 and flat magnitude response κ around the absorber frequency ω_0 imply a rise of magnitude response at some other frequencies. Thus, we are confronted with the waterbed effect.

FIGURE 4: ACHIEVABLE FLATNESS TRADE–OFFS.

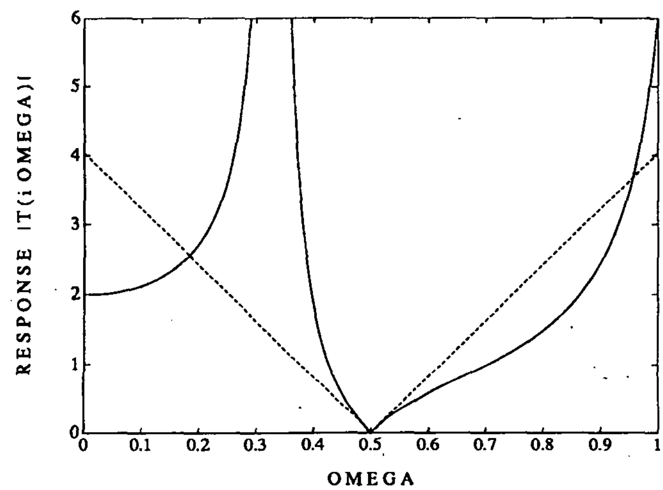


5 A DEEPER ANALYSIS

The previous section raised a number of questions:

- Bound (11) relates two quite different objects, namely static magnitude T_0 and slope κ of magnitude response around ω_0. What do T_0 and κ have in common?
- To this point, we only considered a very special setup of passive bearing C. Are there other passive bearings that meet the same requirements?
- Particularly: for which values of ω_0, T_0, κ does there exist a passive bearing? Can we improve bounds (10) & (11)?
- Figure 5 shows that peak value $\max_\omega |T(i\omega)|$ may be very important. Could we somehow use the "freedom" of all passive bearings in order to reduce this peak below a prescribed bound?

FIGURE 5: $\omega_0 = \frac{1}{2}, T_0 = 2, \kappa = \frac{129}{16}$, peak ≈ 5460 (!)



The last question will be left open as a challenge for future research. The remainder of this paper is devoted to answering the first three questions.

It is well–known that the following map

$$F(s) = \frac{1 - C(s)}{1 + C(s)} \tag{12}$$

gives a one–to–one correspondence between *positive real* functions $C(s)$ and *bounded real functions* $F(s)$, i.e. stable functions with gain less or equal 1, i.e. $|F(i\omega)| \leq 1$ for all ω. Now, use the mapping chain $T(s) \mapsto C(s) \mapsto F(s)$ in order to transform all requirements from T to F. From straightforward manipulations it follows:

$$\begin{cases} F(0) = f_1, & |F'(0)| = \varrho_1 \\ F(i\,\omega_0) = f_2, & |F'(i\,\omega_0)| = \varrho_2 \\ F(-i\,\omega_0) = -f_2, & |F'(-i\,\omega_0)| = \varrho_2 \end{cases} \tag{13}$$

$$\text{where} \quad f_1 = -1, \quad \varrho_1 = 2\,T_0 - 1, \quad f_2 = e^{i\,\varphi}$$

$$\tan\varphi = \frac{2\,\omega_0\,(1 - \omega_0^2)}{\omega_0^4 - 3\omega_0^2 + 1}$$

$$\text{and} \quad \varrho_2 = \frac{2\,(\omega_0\,\kappa - \omega_0^2 - 1)}{\omega_0^4 - \omega_0^2 + 1} \tag{14}$$

Equation (13) is known as *"Nevanlinna–Pick" boundary* interpolation problem. In the appendix we compile known basic facts [2] such as the condition for the existence of solutions.

Equation (14) shows what T_0 and κ have in common; they link with derivative moduli ϱ_1 resp. ϱ_2. Now, use (17) to build the corresponding (3×3) hermitian Pick matrix $\boldsymbol{\Lambda}$. $\boldsymbol{\Lambda}$ depends on ω_0, T_0 and κ. If $\boldsymbol{\Lambda}(\omega_0, T_0, \kappa) > 0$ there are passive bearings that achieve the requirements corresponding to ω_0, T_0 and κ. After some tedious manipulations we came up with a

big surprise: The definiteness analysis of $\boldsymbol{\Lambda}(\omega_0, T_0, \kappa)$ leads to the same bounds (10) & (11) !

In other words, the special passive bearing setup (5) investigated in the previous section plays an outstanding role among the set of all passive bearings because bound (11) cannot be improved.

6 A GLIMPSE AT THE ACTIVE CASE

In [5] *active* vibration absorber systems were investigated using observer theory. In [4] *active* control of the two mass oscillator example has been examined, where it turned out that *arbitrarily* low peak values $\max_\omega |T(i\,\omega)|$ can be achieved, provided the controller bandwidth is appropriately high. Therefore, it should not be surprising that if we require asymptotic disturbance rejection at ω_0 we can achieve arbitrarily low values of both T_0 and κ. In fact, the absorber requirements can easily be incorporated in the "model matching problem" [3]. The active case is much simpler than the passive one since the tough passivity constraint on C is replaced by the common internal stability requirement of feedback setup $[P, C]$.

7 CONCLUSIONS

Although *passive* control has some advantages such as robust stability in presence of certain modeling errors [6], some *inherent* limitations of passive control are shown in this paper. We considered the requirement of asymptotic disturbance rejection at some given frequency ω_0, where we examined the flatness κ of magnitude response around ω_0. In the case of passive control it was shown that there is a lower bound for flatness κ. This inherent limitation drops out in the case of active control. We presented a two-mass oscillator benchmark problem where we explicitly derived the limitations of passive control.

ACKNOWLEDGMENT

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APPENDIX: INTERPOLATION

The workhorse of this paper is "boundary Nevanlinna–Pick" interpolation. For the sake of completeness this appendix recalls basic facts on existence and parametrization of solutions. For proofs and further details refer to [2, pp. 462]. We are seeking *stable* transfer functions $F(s)$ which have gain less or equal 1, i.e. $|F(i\,\omega)| \leq 1$, for all ω, and which satisfy a set of given interpolation conditions on the boundary of the domain, i.e.

$$F(i\,\omega_k) = f_k, \quad (k = 1, \ldots, n) \tag{15}$$

$$|F'(i\,\omega_k)| = \varrho_k, \quad (k = 1, \ldots, n) \tag{16}$$

where $s_k := i\,\omega_k$ are n given points on the boundary, i.e. on the imaginary axis, and where f_k are given complex function values of modulus 1, i.e. $|f_k| = 1$. Furthermore, the moduli ϱ_k of derivative values $F'(i\,\omega_k)$ are prescribed. The existence of solutions relates on the definiteness properties of $n \times n$ "Pick–matrix" $\Lambda := [\Lambda_{\ell m}]$ defined by

$$\Lambda_{\ell m} := \begin{cases} \dfrac{1 - \bar{f}_\ell f_m}{\bar{s}_\ell + s_m}, & \ell \neq m \\ \varrho_\ell, & \ell = m. \end{cases} \tag{17}$$

Then a necessary condition for the above problem to have a solution is that Λ be positive semidefinite, and a sufficient condition is that Λ be positive definite. In particular, if no constraint is placed on the values $|F'(i\,\omega_k)|$, we may take ϱ_k to be arbitrarily large such that Pick matrix $\boldsymbol{\Lambda}$ is positive definite regardless of what the off–diagonal entries are. If $\boldsymbol{\Lambda} > 0$ the *set of all solutions* is parametrized by the following linear fractional map

$$F(s) \;=\; \frac{\Theta_{11}(s)\,Q(s) \;+\; \Theta_{12}(s)}{\Theta_{21}(s)\,Q(s) \;+\; \Theta_{22}(s)}$$

where $Q(s)$ is an arbitrary stable function with gain equal or less than 1, for which $\Theta_{21}(s)\,Q(s) \;+\; \Theta_{22}(s)$ has simple poles at the points $s_1, \ldots, s_n$. Here

$$\boldsymbol{\Theta}(s) \;=\; \begin{bmatrix} \Theta_{11}(s) & \Theta_{12}(s) \\ \Theta_{21}(s) & \Theta_{22}(s) \end{bmatrix} \quad \text{is given by}$$

$$\boldsymbol{\Theta}(s) \;=\; \boldsymbol{I} \;-\; \boldsymbol{C}_0\,(s\boldsymbol{I} \;-\; \boldsymbol{A}_0)^{-1}\,\boldsymbol{\Lambda}^{-1}\,\boldsymbol{C}_0^*\,\boldsymbol{J}\,, \quad \text{where}$$

$$\boldsymbol{C}_0 \;=\; \begin{bmatrix} f_1 \ldots f_n \\ 1 \ldots 1 \end{bmatrix}, \; \boldsymbol{A}_0 \;=\; \begin{bmatrix} s_1 & & 0 \\ & \ddots & \\ 0 & & s_n \end{bmatrix},$$

and $\quad \boldsymbol{J} \;=\; \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$

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