Stability Analysis of Rotor Supported by Anisotropic Active Magnetic Bearings

B. NAGAI AND Y. OKADA

ABSTRACT

Magnetic bearing has some stability problems at high speed rotation such as gyroscopic and inductive coupling effect. In this paper, high speed rotor supported by anisotropically controlled magnetic bearings is analized, and damping property and stable region are calculated by using root locus diagram. From the analysis, anisotropical controller stabilizes the inductive coupling effect drastically. It sometimes affects adversely to the gyroscopic effect.

This idea is applied to a 3 mass flexible rotor supported by 2 magnetic journal bearing systems, and its capability is tested.

INTRODUCTION

The electromagnetic bearings have attractive characteristics such as cleanness and low drag torque, and their application has been gradually increasing. In order to wide application, however, high speed stability should be established.

Analog controlled magnetic bearings have been widely used to support the rigid rotor [1],[2]. Recently, digital controller has been investigated because of their flexibility and capability of high level control. Hisatani reported 1 DOF digital magnetic bearing [3]. The authors also reported a stabilizing technique of high speed journal magnetic bearings [4],[5].

High speed instability of the rotor are induced by two kinds of coupling effects: gyroscopic effect and inductive force. They sometimes affect adversely to the system stability when rotor runs at high speed. Usually, gyroscopic effect never makes rotor unstable. Increasing the rotating speed, however, it changes the duplicated poles, one to high and the other to low frequency. If the controller has an integral (I) operation, the low frequency pole becomes unstable. More harmful effect is the inductive coupling. One of the duplicated poles moves to the right in s-plane, which means unstable. To overcome this difficulty, cross feedback technique has been introduced [4],[5]. However, the controller

Bunshu Nagai, Yohji Okada, School of Engineering, Ibaraki University, 4-12-1 Nakanarusawa, Hitachi, JAPAN 316

becomes very complicated.

This paper describes the damping property of rotor supported by anistropically controlled magnetic bearings. Oil film journal bearing is usually made anisotropic in order to avoid oil whips. Anisotropically controlled magnetic bearing can be realized simply by changing proportional feedback gains for each direction.

In this paper, high speed rotor supported by anisotropically controlled magnetic bearing is analized, and damping property and stable region are calculated by using root locus diagram. From the analysis, anisotropical controller stabilizes the inductive coupling effect drastically. However, it affects adversely to the gyroscopic effect. Hence it is recommended that the feedback gains are adjusted up to 50% difference. This idea is applied to a 3 mass 2 journal bearing rotor, and its capability is tested.

THEORETICAL CONSIDERATION

In order to analize the stability of rotor supported by anisotropic magnetic bearings, let us consider a simple rigid rotor. Then this idea is extended to 3 mass flexible rotor. They are assumed to have the gyroscopic and inductive coupling effects.

Inclined Mode of Rigid Rotor

A rigid rotor and its coordinate system are shown in Fig. 1. For simplicity, the rotor

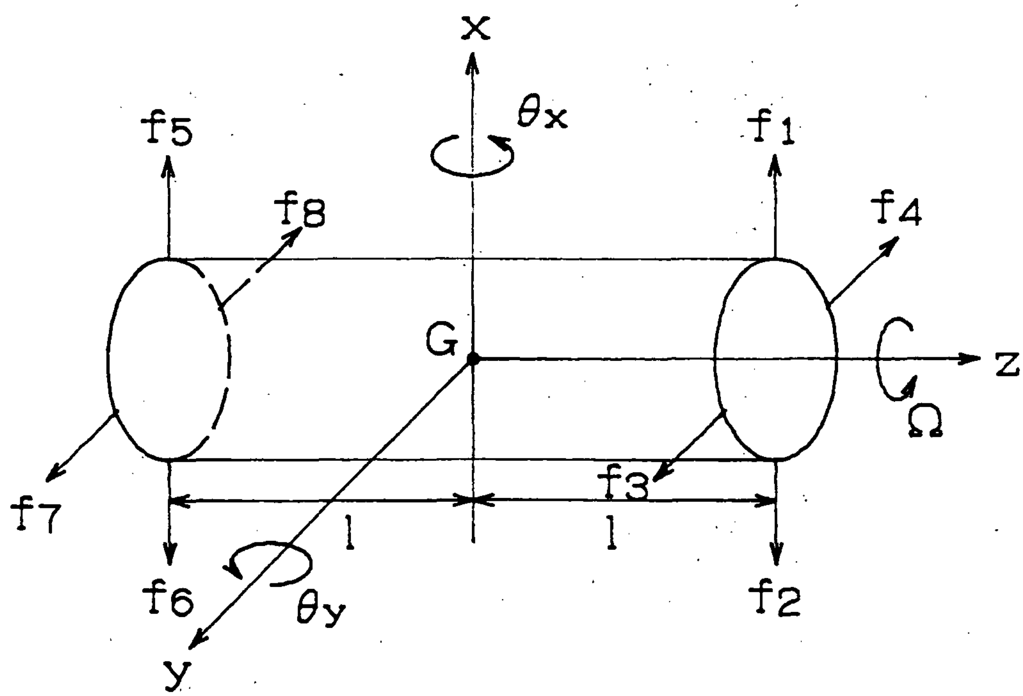

Fig.1 Configuration of Rigid Rotor

is assumed not to move in z-direction. It rotates at a constant speed Ω around z-axis. The translational and the rotational motions of the rotor in x, y, θ_x and θ_y directions are written in the following equations.

$$m\ddot{x} + K_E\Omega y = f_1 - f_2 + f_5 - f_6 = f_x \tag{1}$$

$$m\ddot{y} - K_E\Omega x = f_3 - f_4 + f_7 - f_8 = f_y \tag{2}$$

$$J\ddot{\theta}_x + J_P\Omega\dot{\theta}_y + l^2K_E\Omega\theta_y = lf_4 - lf_3 + lf_7 - lf_8 = \tau_x \tag{3}$$

$$J\ddot{\theta}_y - J_P\Omega\dot{\theta}_x - l^2K_E\Omega\theta_x = lf_1 - lf_2 - lf_5 + lf_6 = \tau_y \tag{4}$$

where m is the mass, J is the moment of inertia and J_P is the polar moment of inertia of the rotor. $J_P\Omega$ is so called gyroscopic moment and K_E is the inductive force coefficient. This inductive coupling affects adversely to the rotor stability. In order to avoid this effect, it is recommended to use the laminated plates. However, in such a machine in which the induction motor is installed in the shaft, this inductive coupling force is generated by the motor.

The equations of the translational motion of the rotor (1), (2) are the special case of the equations of the rotational motion (3) and (4) by neglecting the gyroscopic moment term $J_P\Omega$. Therefore, the equations (1) and (2) are involved in the equations (3) and (4).

Suppose that the magnetic bearing forces τ_x and τ_y are generated from the proportional and derivative (PD) feedback,

$$\tau_x = -K_{px}\theta_x - K_{Dx}\dot{\theta}_x \tag{5}$$

$$\tau_y = -K_{py}\theta_y - K_{Dy}\dot{\theta}_y \tag{6}$$

where K_{px} and K_{py} are the proportional feedback gains for x and y directions, and K_{Dx} and K_{Dy} are the derivative feedback gains for x and y directions, respectively. Let us transform these equations using Laplace operator and $K_e = l^2K_E$, then we have

$$(Js^2 + K_{Dx}s + K_{px})\theta_x + (J_Ps + K_e)\Omega\theta_y = 0 \tag{7}$$

$$-(J_Ps + K_e)\Omega\theta_x + (Js^2 + K_{Dy}s + K_{py})\theta_y = 0 \tag{8}$$

From these equations, the characteristic equation of this system can be written by

$$(Js^2 + K_{Dx}s + K_{px})(Js^2 + K_{Dy}s + K_{py}) + (J_Ps + K_e)^2\Omega^2 = 0 \tag{9}$$

By drawing the root locus diagram of this characteristic equation, let us analyze the system stability. Fig.2.(a) shows the root locus with equal positional feedback gains ($K_{px} = K_{py}$). Without velocity feedback ($K_{Dx} = K_{Dy} = 0$), double roots for θ_x and θ_y directions are located at the cross mark on the imaginary axis. By increasing the gyroscopic effects, one of these roots travels to high frequency (foward precision) along the imaginary axis and another to low frequency (backward precision), respectively. This system, however, becomes unstable only when gyroscopic effects and integral feedback are associated together [2], [4].

On the other hand, the inductive effect $K_e\Omega$ moves one of these double roots to the left (stable region) along the dashed line and another to the right (unstable region) of the s-plane. Then the inductive effects are very harmful for the magnetic bearings-rotor system. Therefore, it is recommended to use the laminated plates [3].

Next, let us consider the case in which the positional feedback gains of each directions of the bearing are different ($K_{px} \neq K_{py}$). The root locus is shown by the solid line in Fig.2.(b). Because of the different stiffness, roots on the imaginary axis are located at the different frequency. By increasing the gyroscopic effect, the higher root moves toward higher, and the lower root travels toward lower frequency, respectively.

On the other hand, two different roots on the imaginary axis travel closer each other by the inductive effect. Then they coincide and one of these roots moves to the left and another to the right of the s-plane. Therefore, it is respected that the system with different position feedback gains is more stable than one with the same gains.

If velocity feedbacks (K_{Dx}, K_{Dy}) are added, the roots, both in Fig.2.(a) and (b), travel along the dash-dot lines, then move along the thin dotted lines. They travel similar but include the damping effect.

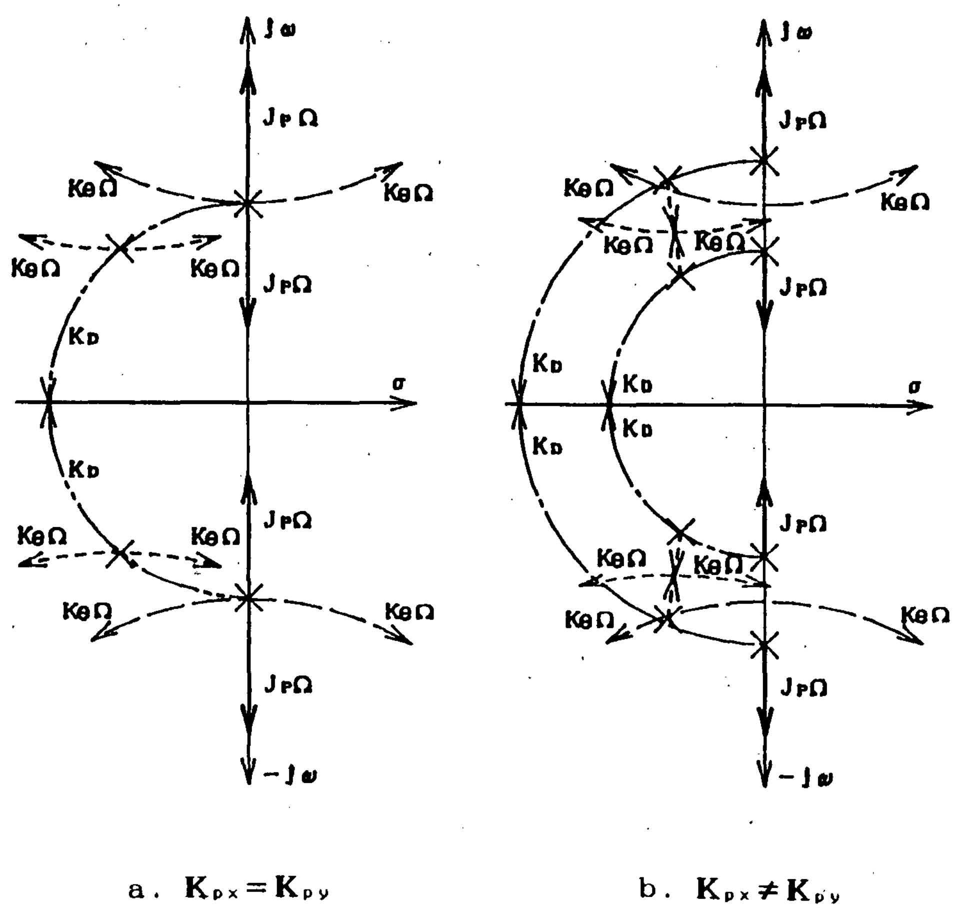

Fig.2 Root locus of Rigid Rotor

Flexible Mode of 3 disc–2 bearing System

Next, let us consider the analysis of the flexible rotor. The system is consisted of flexible shaft with three discs and two magnetic bearings. The schematic diagram is shown in Fig.3. The equations of motion of PID controlled system are written in equations (10) and (11).

$$\mathbf{M\ddot{X}} + \Omega \mathbf{J_p \dot{Y}} + \mathbf{KX} = -\mathbf{K_{px}X} - \mathbf{K_{Ix}}\int \mathbf{X}dt - \mathbf{K_{Dx}}\dot{X} - \mathbf{K_E}\Omega\mathbf{Y} \qquad (10)$$

$$\mathbf{M\ddot{Y}} - \Omega \mathbf{J_p \dot{X}} + \mathbf{KY} = -\mathbf{K_{px}Y} - \mathbf{K_{Iy}}\int \mathbf{Y}dt - \mathbf{K_{Dy}}\dot{Y} + \mathbf{K_E}\Omega\mathbf{X} \qquad (11)$$

The second terms of the left hand side of the above equations denote the gyroscopic effects. The first, second and third terms of the right hand side mean the P, I and D controlled forces, respectively; where K_p, K_I, K_D are the proportional, integral and velocity feedback gains, respectively. The fourth terms are the inductive effects, where K_E is the inductive coefficient.

Now let us consider the root locus of the PD controlled flexible rotor-bearing system which are affected by the gyroscopic and inductive effects. The system is analyzed by a numerical eigenvalue subroutine and the resulting root locus of the lower 3 modes of the PD controlled flexible rotor is shown in Fig.4.

Fig.4.(a) is the root locus of the system when the feedback gains are the same (K_{px} =

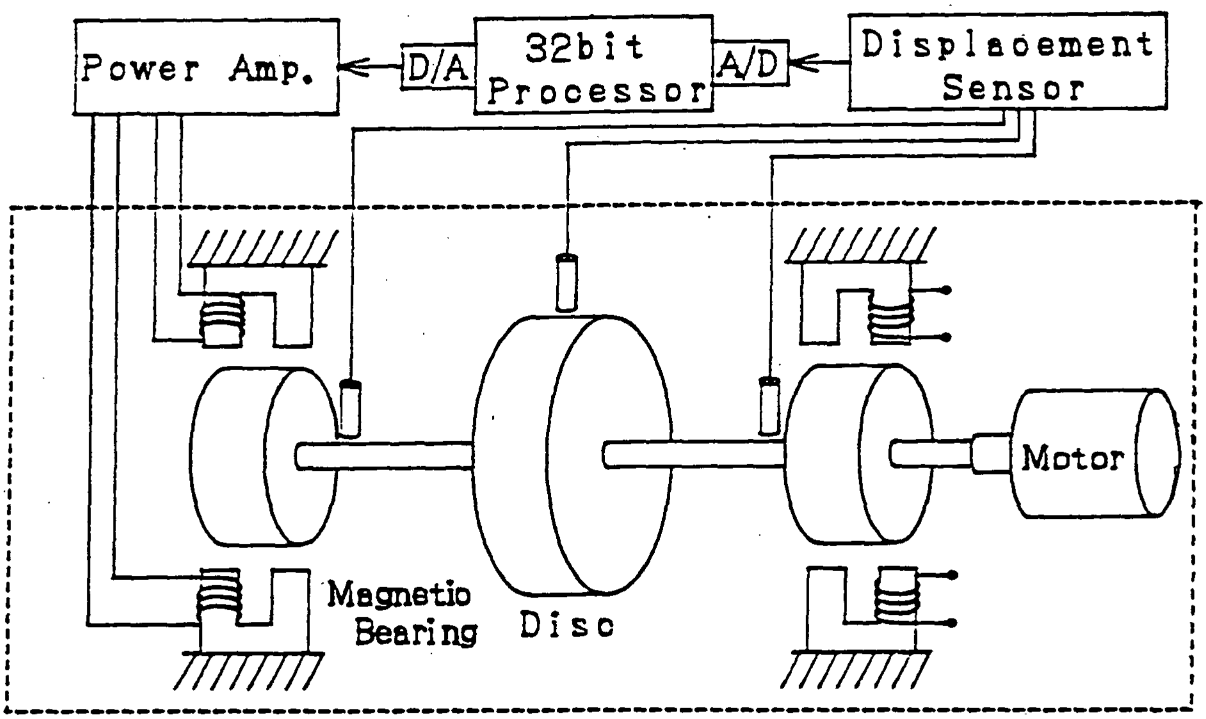

Fig.3 Configuration of Flexible Rotor

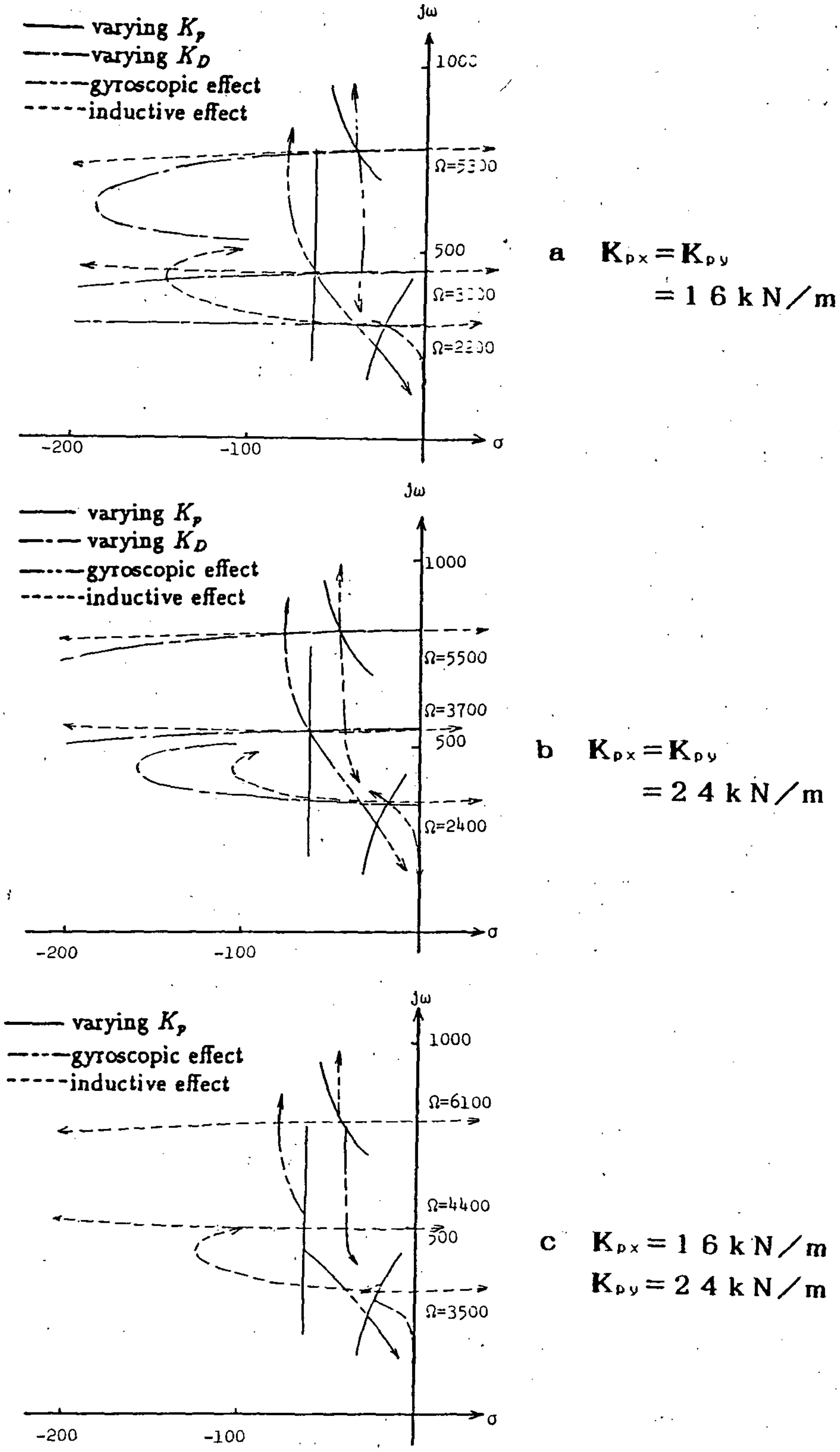

Fig.4 Root locus of PD Controlled Flexible Rotor

$K_{py} = 16kN/m$, $K_D = 10Ns/m$). In the diagram, the roots travel along the solid line when K_p increase, while the roots travel along dash-dot line with increasing K_D. When the rotating speed of the rotor is zero, the roots of the first, second and third mode are all double roots for x and y directions, respectively, because the stiffness is same in horizontal and vertical directions. As the rotating speed increases these double roots are separated into two roots. In case of PD control, these roots do not move into unstable region with only the gyroscopic effects as shown by the dash-dot-dot line. But the inductive coupling effect moves one of the double roots of each mode to the right as shown by the dotted line. The stability limits of rotational speed are 2200 rad/s at the first mode, 3000 rad/s at the second mode and 5300 rad/s at the third mode.

Fig.4.(b) shows the root locus when the positional feedback gains are increased ($K_{px} = K_{py} = 24kN/m$), and the velocity feedback gains are the same ($K_D = 10Ns/m$). The curves of the root locus are similar to those of Fig.4.(a). In this case, the stability limits of rotational speed are increased to 2400 rad/s at the first, 3700 rad/s at the second and 5500 rad/s at the third mode, respectively. The root locus shows that increasing stiffness expands the stable regions slightly.

The root locus of the flexible rotor supported by the anisotropic bearings is shown in Fig.4.(c), where the positional feedback gains in x and y directions are different ($K_{px} = 16kN/m$, $K_{py} = 24kN/m$, $K_D = 10Ns/m$). Because of the different stiffness, the roots of each mode locate first at the different position. These different roots of each mode approach each other until they coincide, then one of the roots of each mode moves to the left (stable) and another to the right (unstable) in the s-plane. Therefore, the roots travel longer than thos in the case of same positional feedback gains in Fig.4.(a). The stability limits of rotational speed are 3500 rad/s at the first, 4400 rad/s at the second and 6100 rad/s at the third mode, respectively.

By comparing Fig.4.(c) with Figs.4.(a) and 4.(b), the stable regions can be expanded by increasing the bearing stiffness. However, changing the positional feedback gains slightly expands the stable region more drastically than increasing the stiffness. From the analysis, anisotropic bearing has a great capability of stabilizing the instability of the flexible rotor caused by the inductive coupling effects, especially when the damping of the system is low.

Next, let us show the root locus of the PID controlled flexible rotor as in Fig.5. Because of the phase lag of I operation, the lower roots of the duplicated roots of each mode move to the unstable region with not only the inductive effects but also the gyroscopic effects, when the rotating speed increases. All of these roots are located more closely to the imaginary axis than the case of PD controller, hence these roots seem to be easily destabilized.

Fig.5.(a) shows the root locus when the positional feedback gains are equal ($K_{px} = K_{py} = 16kN/m$, $K_D = 10Ns/m$, $K_I = 10N/sm$). The stability limits of rotational speed by the gyroscopic effects are 7800 rad/s at the first and 10000 rad/s at the second modes, respectively. In this case, the roots at the third mode does not move to the unstable region. On the other hand, the stability limits of the inductive effects are 400 rad/s at the first, 1900 rad/s at the second and 4600 rad/s at the third mode, respectively. Compared with the case of the PD control, the stable region is largely reduced.

Fig.5.(b) shows the root locus when the position feedback gains are $K_{px} = K_{py} = 24kN/m$ and other parameters are the same in case of (a). This also shows the same

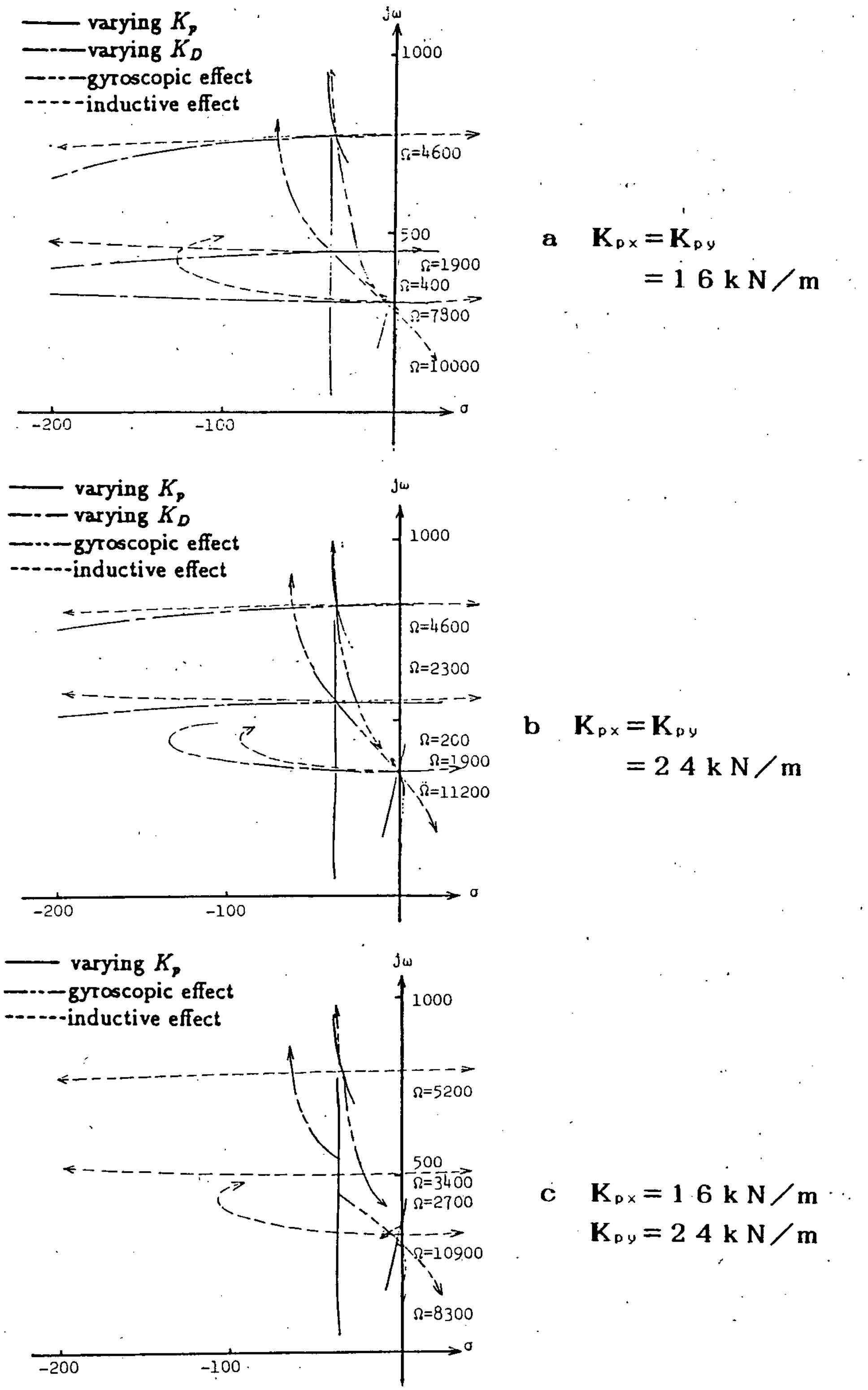

Fig.5 Root locus of *PID* Controlled Flexible Rotor

tendency as the case of (a). The stability limits of rotational speed by the gyroscopic effects are 1900 rad/s at the first and 11200 rad/s at the second modes, respectively. And the stability limits of the inductive effects are 200 rad/s at the first, 2300 rad/s at the second and 4600 rad/s at the third modes, respectively. By comparing with the case of (a), the instability caused by the gyroscopic effects is not stabilized, but is stabilized caused by the inductive effects, by increasing the stiffness.

Fig.5.(c) shows the root locus when the position feedback gains are different ($K_{px} = 16kN/m$, $K_{py} = 24kN/m$) and the other parameters are the same. The duplicated poles are separated by the different stiffness. The stability limits by the gyroscopic effects are 8300 rad/s at the first and 10900 rad/s at the second modes, respectively. But the stability limits by the inductive effects are largely expanded to 2700 rad/s at the first, 3400 rad/s at the second and 5200 rad/s at the third modes, respectively.

By comparing the case (c) with the case (a) and (b), the anisotropic bearing which can be easily realized by only changing the positional feedback gains in x and y directions improves the inductive effects, drastically.

CONCLUSIONS

Dynamics of the rigid rotor and the flexible rotor supported by the magnetic bearings are analized. The following results are obtained.

Gyroscopic effect never destabilizes the rotor supported by PD controlled magnetic bearings. However, it destabilizes the rotor when I operation is added to the PD controller.

Inductive coupling effect is very harmful to the stability of the rotor. Even magnetic bearings are made of laminated sheet, this inductive coupling effect may be generated by the induction motor.

The isotropic bearing system, which can easily be realized only by changing the positional feedback gains, has a great capability of expanding the stable region, especcially when the damping of the rotor-bearing system is low.

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