An Efficient Method for Stability Analysis for Active Magnetic Bearing Systems

C.-P. R. KU AND H. M. CHEN

ABSTRACT

The dynamic properties of active magnetic bearings are functions of rotor excitation frequencies, not rotor speed. Existing rotordynamic analysis requires trial-and-error iterations to find rotor-bearing system damped natural frequencies. A two-level, finite-element-based rotordynamic computer program with an efficient root-searching algorithm for stability analysis has been developed. Given an initial estimate of the first natural frequency, the program is able to automatically search for all the natural frequencies. A numerical example, which has several very close natural frequencies, is presented to demonstrate the program's capability.

INTRODUCTION

Recent advances in active magnetic bearing (AMB) technology present a new challenge to rotordynamicists. However, many rotordynamicists are not familiar with mathematical modeling of AMBs, which is traditionally performed in electrical engineering and control languages. Chen [1] has suggested that AMBs be treated as locally controlled devices similar to other types of bearings that provide load capacity, stiffness, and damping for rotor support. AMB stiffness and damping coefficients have been defined in a closed-form solution in the frequency domain [2]. Unfortunately, rotordynamic programs developed during the last two decades for conventional bearings are awkward to use with AMBs since: 1) the stiffness and damping coefficients of AMBs are a function of rotor whirl frequency, not rotor speed; and 2) AMB reacting forces are proportional to displacements measured at sensor locations offset axially from the bearings [3].

One approach to performing rotordynamic analysis of AMBs is a state-space method that combines the differential equations representing the bearing controllers with the rotor equations of motions [2, 4]. Since this method usually produces mixed-up eigenvalues of the rotor and the controller, efforts are required for sorting out the rotor modes. Also, coding a general computer program to accommodate all possible controller variations is difficult.

C.-P. Roger Ku and H. Ming Chen, Mechanical Technology Incorporated, 968 Albany-Shaker Road, Latham, New York 12110

In another approach, Chen et al. [5] developed an algorithm in which a magnetic bearing subroutine for all AMBs in a rotor system was written according to the AMB control scheme. The AMB controller is represented by its component transfer functions. This subroutine, which provides the AMB stiffness and damping coefficients, is compiled separately and called by a main rotordynamic program. Although the algorithm has been successfully applied to an AMB system with a submerged rotor [6, 7], the damped natural frequency searching scheme was based on trial and error. Finding all potentially unstable modes in this manner is time consuming, especially when a system is implemented with notch filters and has several modes that are closely spaced at the notch [8].

To eliminate the trial-and-error process, an efficient algorithm for calculating damped natural frequencies of a rotor supported by AMBs has been developed and is presented in this paper. In this algorithm, the Secant method is used with a two-level rotordynamic computer program. The program automatically searches for the damped natural frequencies in AMB systems. A numerical example using an industrial pump supported by AMBs is presented to demonstrate the effectiveness of the algorithm.

FORMULATION OF SYSTEM DYNAMICS

A rotor-bearing system is modeled to consist of four parts: shaft, casing, disks, and bearings. The shaft is considered to be a flexible beam and is divided into several elements that have distributed mass and elasticity. Each shaft element consists of two stations. Each station has four degrees of freedom (DOF) of motion - two lateral and two angular. By using a finite-element method [9], each element is represented by an 8×8 mass matrix and an 8×8 stiffness matrix.

Machine casings are generally much stiffer than their rotors. The predominant flexible natural frequencies of a casing are usually well above the range of rotordynamic interest. Casing mass affects system rotordynamics, especially when the casing is light and is not rigidly tied to a large foundation. To account for the mass effect, modeling the casing as a rigid body with six DOF at its center of gravity is adequate [6, 7]. The casing is assumed to be mounted on flexible supports, e.g., elastomer mounts, tied to the ground.

The AMBs are modeled as force elements acting on the shaft at bearing locations. Each AMB has two independent control axes. The electromagnetic force exerted on the rotor along a control axis in the X-direction, F_x, is defined as:

$$F_x = K_i I + K_m (X_{rb} - X_{sb}) \tag{1}$$

where

K_i = current stiffness
I = dynamic control current
K_m = magnetic stiffness
X_{rb} = rotor displacement at bearing center
X_{sb} = stator displacement at bearing center

Both K_i and K_m are functions of bearing size, pole configuration, and bias (steady-state) currents in the magnetizing coils. The control current, I, which modulates the bias, can be related to a displacement feedback by:

$$I = G(s)\,(X_{rs} - X_{ss}) \tag{2}$$

where

$G(s)$ = a series of component transfer functions multiplied together
X_{rs} = rotor displacement at sensor location
X_{ss} = stator displacement at sensor location

Note that G(s) is dependent on rotor excitation frequency and is programmed in a user-supplied AMB subroutine.

Combining the rotor [9] and casing [6, 7] formulas with the AMB control represented by the above equations, the system dynamics can be written as:

$$[M_s]\{\ddot{U}\} + ([C_s] + [C_b])\{\dot{U}\} + ([K_s] + [K_b])\{U\} = \{F\} \tag{3}$$

where

$[M_s]$ = rotor mass matrix
$\{U\}$ = system state vector
$[C_s]$ = rotor damping matrix, including gyroscopic effect
$[C_b]$ = bearing damping matrix
$[K_s]$ = rotor stiffness matrix
$[K_b]$ = bearing stiffness matrix
$\{F\}$ = system forcing vector

Matrices $[K_b]$ and $[C_b]$ contain the magnetic bearing properties that are calculated by the AMB subroutine for given excitation frequencies. Matrix $[C_s]$ usually contains a gyroscopic effect and is a function of rotor rotating speed.

STABILITY ANALYSIS METHOD

To perform a stability analysis of a rotor-bearing system, an eigen solution to Equation 3 must be found. For a system consisting of only conventional bearings, calculating the natural frequencies is straightforward since $[K_b]$ and $[C_b]$ are matrices with constant elements. However, if any matrix in Equation 3 depends on excitation frequencies, such as AMB stiffness and damping coefficients, a numerical iteration scheme is required. As illustrated in Figure 1*, the results of a sample calculation show that the natural frequencies coincide with the AMB excitation frequencies. This calculation is equivalent to finding the roots of a nonlinear matrix-algebra equation, A, as follows:

$$A(\Omega) = \Omega - \mathrm{Im}(\lambda(\Omega)) = 0 \tag{4}$$

where

Ω = natural frequency
Im = imaginary part
λ = system eigenvalues, complex numbers

Several numerical algorithms can be employed to solve the above equation. The most popular algorithm is the Newton-Raphson method. However, it requires evaluating the

---

*For ease of readability, all figures and tables are included at the end of this paper.

differential of Equation 4, i.e., $dA(\Omega)/d\Omega$. For most rotor-bearing systems, obtaining a closed-form formula for $dA(\Omega)/d\Omega$ is difficult. If $dA(\Omega)/d\Omega = 0$, convergence will be slow. To reduce computation time, the Secant method is employed instead of the Newton-Raphson method. In the Secant method, the evaluation of the differential is not required. The following equation presents the algorithm:

$$\Omega_n = \Omega_{n-1} - f\, A(\Omega_{n-1})\,(\Omega_{n-1} - \Omega_{n-2})/[A(\Omega_{n-1}) - A(\Omega_{n-2})] \tag{5}$$

where

n = number of iterations
f = a relaxation factor, usually equal to one

To use Equation 5, the user must provide a starting estimate for Ω_1. If the desired natural frequency, e.g., the first or second frequency, is not equal to the value of Ω_1, i.e., $A(\Omega_1) = 0$, the program 1) automatically assigns the value of Ω_2 as $\Omega_2 = \mathrm{Im}\,(\lambda(\Omega_1))$ and 2) proceeds to the next natural frequency calculation. After these two steps, the program uses Equation 4 to calculate Ω_n for all $n > 2$.

During the iterations, the rotor rotating speed is kept constant. To reduce computation time, the computer program saves the matrices that are not a function of excitation frequency, e.g., $[K_s]$, $[C_s]$, and $[M_s]$. For each iteration, the program only calculates the bearing dynamic properties $[K_b]$ and $[C_b]$, and performs the matrix manipulations.

NUMERICAL EXAMPLE

To illustrate the use of the above stability analysis method for a rotor-magnetic-bearing system, an industrial pump [8, 10] is used here as an example. The pump, which was designed to run at 3600, is shown as a rotordynamic model in Figure 2. This system has two identical radial AMBs. Figure 3 is an AMB block diagram for one control axis. Note that a notch filter with the center frequency at 60 Hz and a Q-factor of 10 is implemented in series with other control components. Figure 4 presents the dynamic properties of the radial AMB.

Chen and Ku [8] have shown the notched system to be unstable. Figure 5 is an undamped critical speed map on which the AMB stiffness curves are overlaid. Four extra interactions occur near the operating speed due to the notch filter, two slightly below and two slightly above the operating speed. Each interaction implies two vibration modes, one forward and one backward. The two intersections below the operating speed represent four modes and are unstable [8]. To find the unstable modes accurately, an eigenvalue search analysis below 60 Hz was performed using the Secant method. The results are presented in Table I. As expected, the four unstable modes were very close in frequencies and all had a positive growth factor.

For each natural frequency, the computer program required several iterations to obtain convergent results. As in other multistep root-search algorithms, the iteration times depend on system conditions, convergence criteria, and starting estimates. A good starting estimate can be obtained by plotting an undamped critical map, such as that shown in Figure 5. For an AMB system implemented with a notch filter, a small convergence criterion and a relaxation factor less than one are recommended. Therefore, more iterations are necessary to obtain convergent results. For a normal system with the relaxation factor equal to one, three to five iterations are adequate to obtain an accurate natural frequency. Figure 6 shows an iteration history for the first eight natural frequencies of the example system without the notch filter. Table II presents the numerical values of those damped natural frequencies.

CONCLUSIONS

A two-level, finite-element-based rotordynamic computer program employing the Secant method for stability analysis of AMBs has been developed. Given an initial estimate of the first natural frequency, the program automatically searches for all the damped natural frequencies. The numerical example presented herein, which has several very close natural frequencies, has demonstrated the effectiveness of this method.

REFERENCES

1. Chen, H. M., 1988. "Active Magnetic Bearing Design Methodology - A Conventional Rotordynamics Approach." 15th Leeds-Lyon Symposium on Tribology, Leeds, U.K., September 6-9.

2. Chen, H. M., 1988. "Magnetic Bearings and Flexible Rotor Dynamics." STLE Annual Meeting, Cleveland, Ohio, May 9-12, paper No. 88-AM-2D-1.

3. Kirk, R. G., et al., 1989. "Influence of Active Magnetic Bearing Sensor Location on the Calculated Critical Speeds of Turbomachinery." Proceedings of the 12th Biennial ASME Conference on Mechanical Vibration and Noise; Rotating Machinery Dynamics, Montreal, Canada, September 17-21, pp. 309-316.

4. Schmied, J., 1990. "Experience with Magnetic Bearings Supporting a Pipeline Compressor." Proceedings of the 2nd International Magnetic Bearing Symposium, Japan, July 12-16, pp. 47-55.

5. Chen, H. M., et al., 1990. "Stability Analysis for Rotors Supported by Active Magnetic Bearings." Proceedings of the 2nd International Magnetic Bearing Symposium, Japan, July 12-16, pp. 325-328.

6. Chen, H. M. and C.-P. R. Ku, 1991. "Forced Responses of Submerged Rotors in Magnetic Bearings." Proceedings of the ROMAG Magnetic Bearings and Dry Seals Conference, Washington, D.C., March 13-15.

7. Chen, H. M. and C.-P. R. Ku, 1991. "Vibration Control of Submerged Rotors Using Magnetic Bearings." Proceedings of the Conference on Recent Advances in Active Control of Sound and Vibration, Blacksburg, Virginia, April 15-17, pp. 391-407.

8. Chen, H. M. and C.-P. R. Ku, 1991. "Virtual Balancing of Rotors Supported by Magnetic Bearings." Proceedings of the 13th Biennial ASME Conference on Mechanical Vibration and Noise; Rotating Machinery and Vehicle Dynamics, Miami, Florida, September 22-25, ASME DE-Vol. 35, pp. 65-68.

9. Zorzi, E. S., and H. D. Nelson, 1977. "Finite-Element Simulation of Rotor-Bearing Systems with Internal Damping." <u>ASME Journal of Engineering for Power</u>, (January):71-76.

10. Blair, B. J., et al., 1990. "A Canned Pump with Magnetic Bearings for Industrial Use - Laboratory Testing." Proceedings of the 25th IECEC, Reno, Nevada, August 12-17, pp. 475-481.

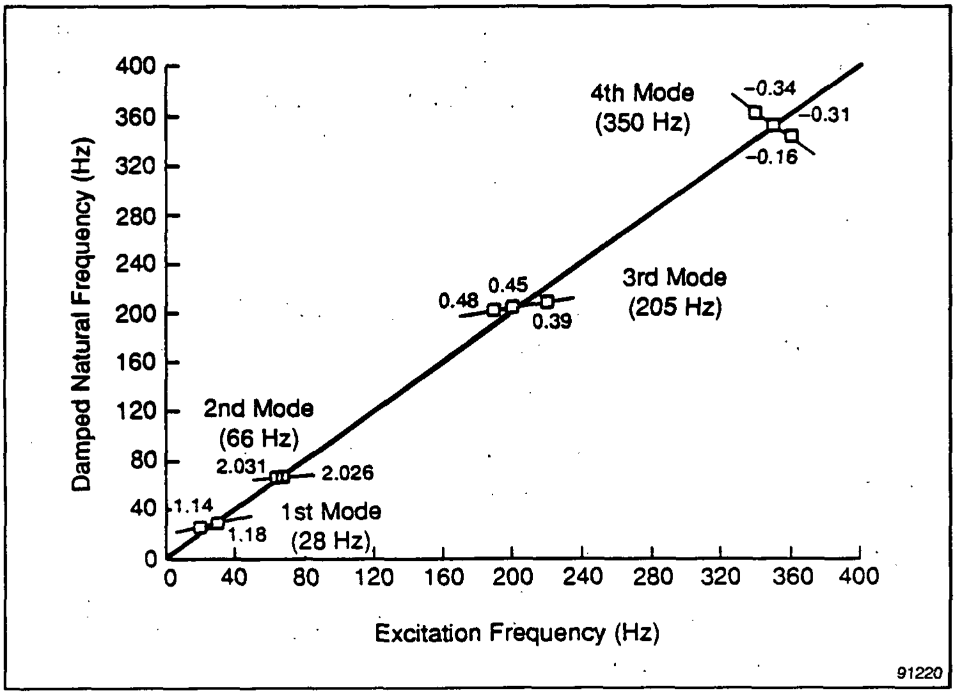


Figure 1. Sample Results of a Stability Analysis

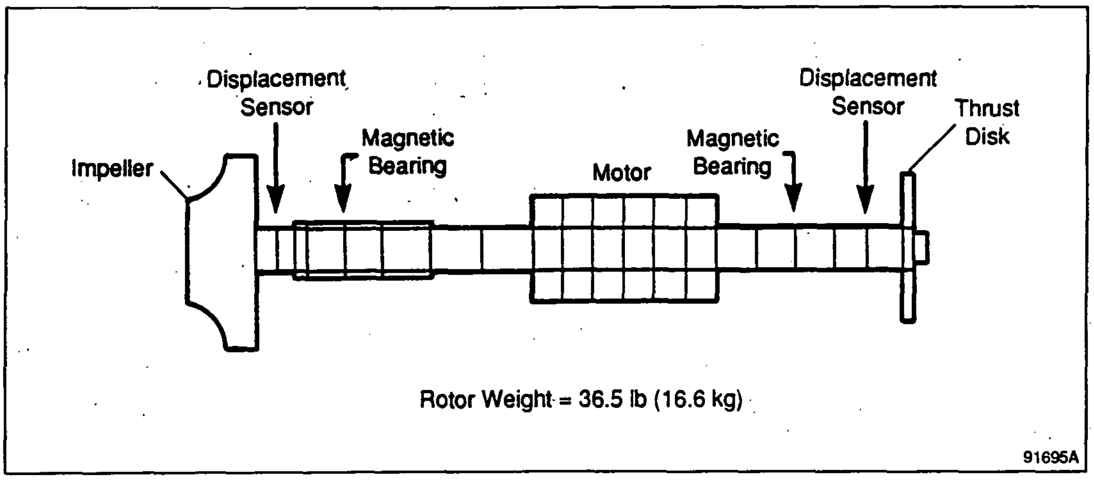
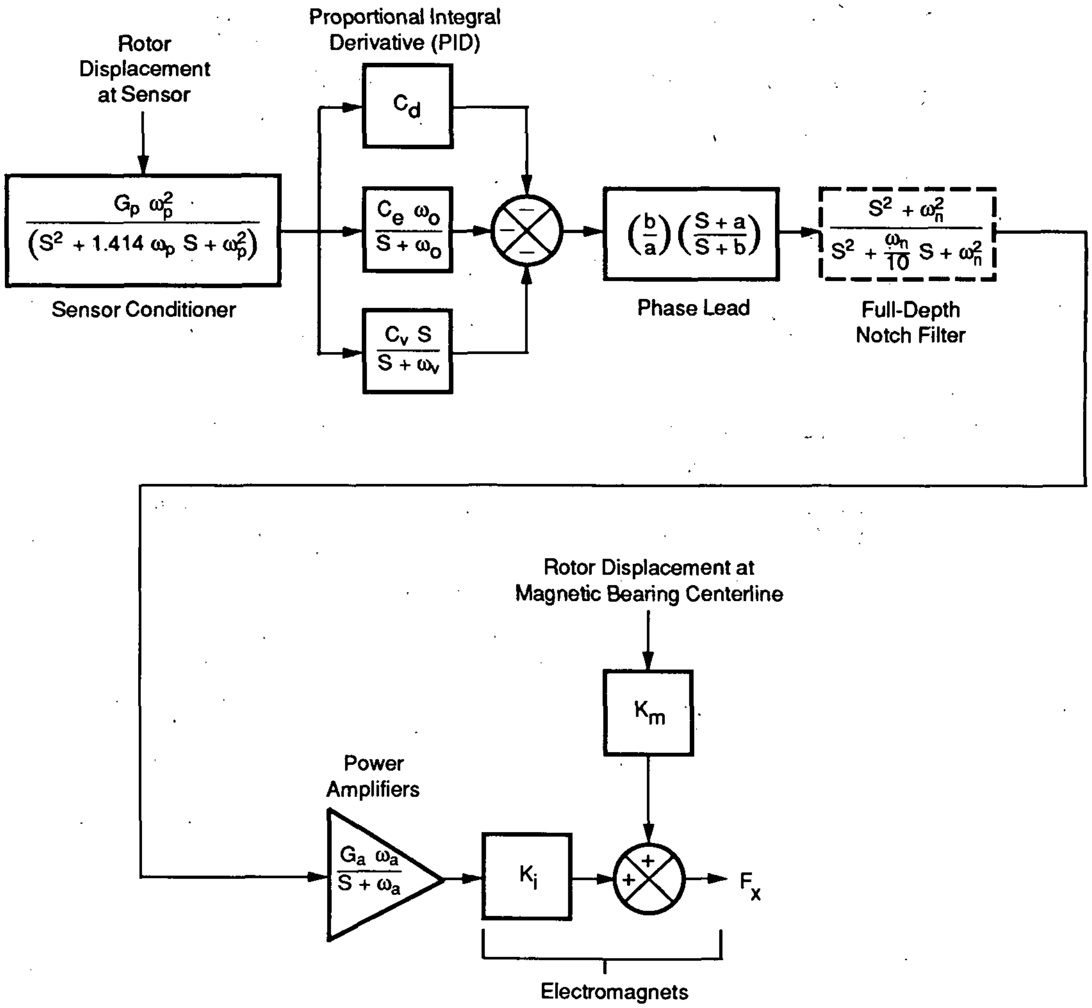


Figure 2. Pump Rotordynamic Model



G_p, ω_p	=	Sensor gain and cut-off frequency, respectively
S	=	Laplace transfer function
C_d, C_e, C_v	=	Proportional, integral, and derivative gains
ω_o	=	Integrator cut-off frequency
ω_v	=	Differentiator cut-off frequency
a, b	=	Lead network frequencies
ω_n	=	Notch filter center frequency
G_a, ω_a	=	Power amplifier gain and cut-off frequency
K_i	=	Current stiffness
K_m	=	Magnetic stiffness
F_x	=	Force on rotor at magnetic bearing centerline

92148(N)

Figure 3. Magnetic Bearing Block Diagram for a Control Axis in the x-Direction

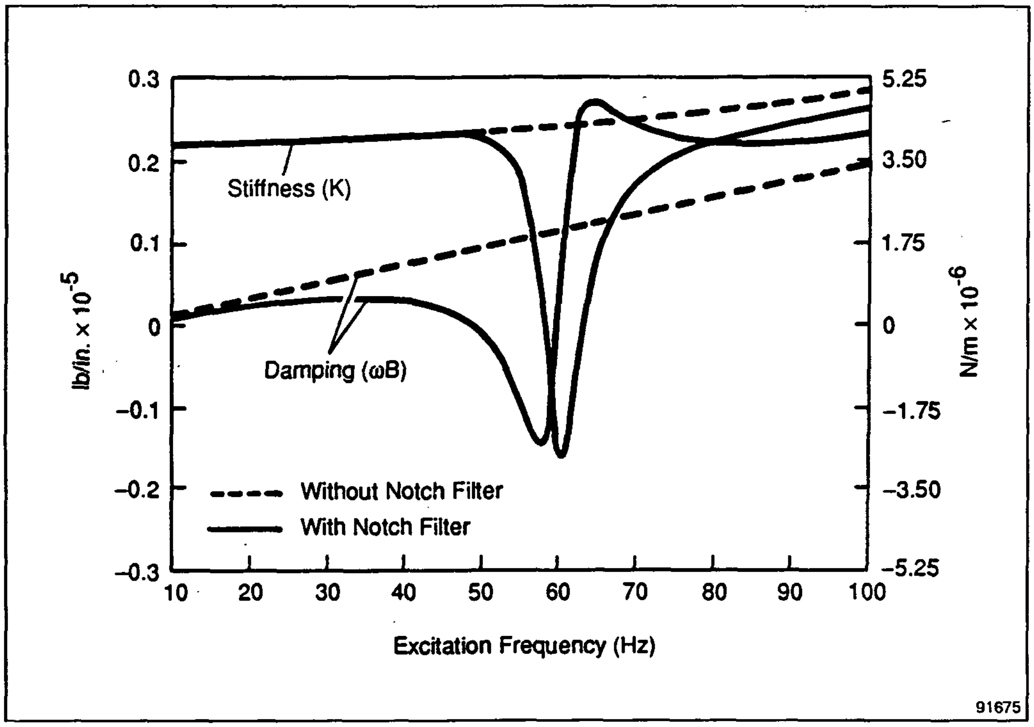


Figure 4. Magnetic Bearing Stiffness and Damping versus Frequency

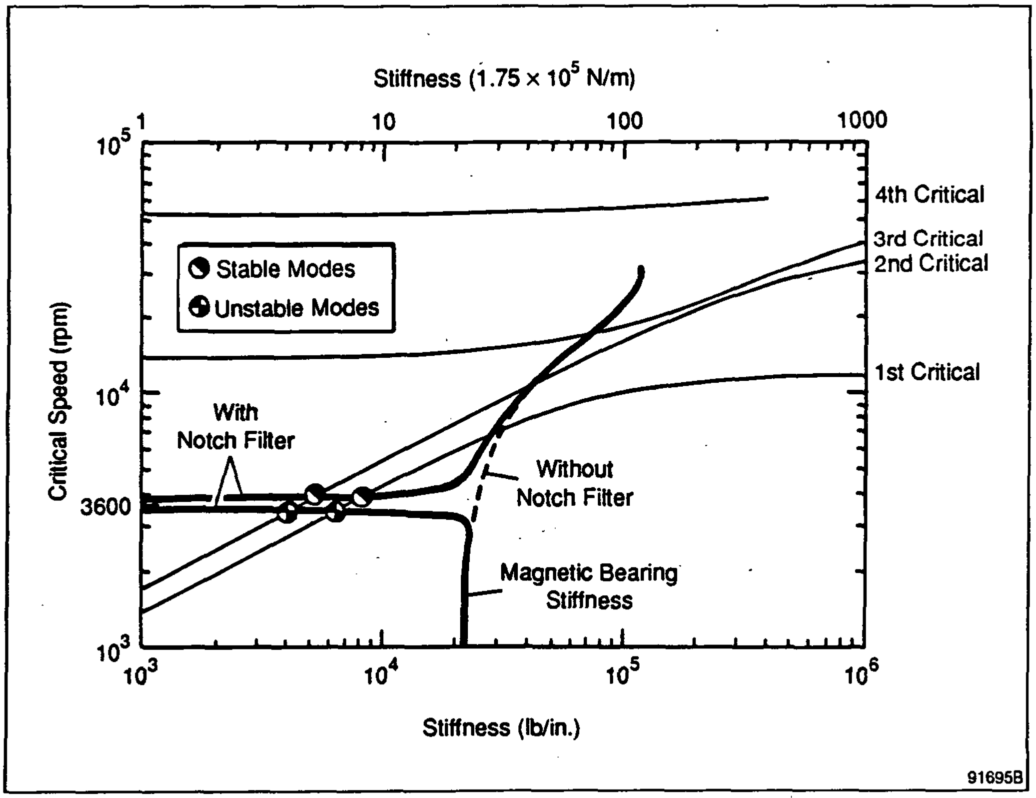


Figure 5. Undamped Critical Speed Map

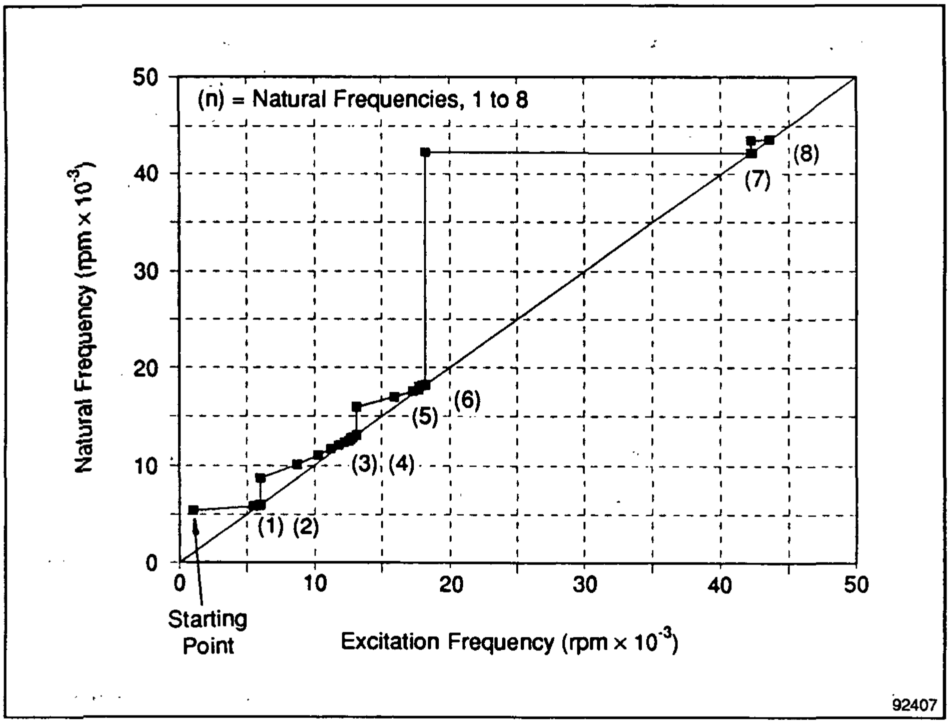


Figure 6. Iteration History of the Pump System without Notch Filter

TABLE I. NATURAL FREQUENCIES AT 3600-rpm ROTATING SPEED WITH NOTCH FILTER

Mode	Eigenvalues		
	Real Part (1/sec)	Imaginary Part	
		(1/sec)	(Hz)
1 – Backward	637.1	356.01	56.66
2 – Forward	660.4	356.17	56.69
3 – Backward	313.4	360.20	57.328
4 – Forward	309.6	360.21	57.329

P412

TABLE II. NATURAL FREQUENCIES AT 3600-rpm ROTATING SPEED WITHOUT NOTCH FILTER

Mode	Eigenvalues			Iteration Times
	Real Part (1/sec)	Imaginary Part		
		(1/sec)	(Hz)	
1 – Backward	-151.7	626	99.7	5
2 – Forward	-153.9	628	99.9	2
3 – Backward	-446.7	1331	211.8	7
4 – Forward	-441.4	1375	218.8	2
5 – Backward	-219.9	1858	295.6	4
6 – Forward	-211.4	1909	303.8	2
7 – Backward	-70.8	4419	703.3	3
8 – Forward	-72.3	4567	726.8	2

P412