Subharmonic Resonance Stability Prediction for Turbomachinery with Active Magnetic Bearings

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ABSTRACT

A finite element computer program for evaluation of dynamic stability of turbomachinery supported by active magnetic bearings has been developed. The new program includes the capability to study the effect of sensor noncollocation and frequency dependent bearing characteristics on the predicted stability of high performance turbomachinery. This paper presents a summary of the verification of the PC-based computer program. The final case study presents the predicted stability variation considering the influence of sensor noncollocation for an eight stage compressor supported by active magnetic bearings.

INTRODUCTION

The evaluation of dynamic stability is now a standard calculation for all new turbomachinery designs and for many field re-rates. The usual calculation procedure follows the methods as outlined in several technical papers published in the early to mid seventies [1,2,3,4]. The majority of industry standard codes use the transfer matrix solution procedure for calculation of undamped critical speeds, forced response sensitivity, and dynamic system stability. The transfer matrix stability programs are capable of only solving for a given finite number of eigenvalues, usually the lowest eigenvalues. Other solution techniques using the finite-element method for rotating shafts have been developed [5,6] which give all the system eigenvalues. Ruhl [1] had first documented this solution technique and compared the transfer matrix and finite element solution for accuracy in regard to the number of nodes selected in the rotor model. The introduction of the personal computer has given the transfer matrix solution procedures an advantage due to the relatively small storage requirement as compared to an equivalent node finite element solution. The use of extended memory in personal computers will make it possible to examine realistic turbomachinery systems by the finite element method.

Another technology that has gained acceptance in recent years is the use of active magnetic bearings for industrial turbomachinery [8,9]. The rotor dynamic evaluation of such machinery has relied upon standard evaluation procedures for the determination of forced response and stability. Recent work reported in the literature [10,11] had been directed at a modified transfer matrix solution for the evaluation of undamped critical speeds and forced response considering the frequency dependent complex stiffness and sensor noncollocation. The iterative procedure required for the transfer matrix solution technique is not necessary when the finite element method is utilized.

The purpose of this paper is to document the results of a finite element stability analysis [7] which has been modified to account for sensor noncollocation and includes the frequency dependent evaluation for the bearings of active magnetic bearing machinery. The results of the modified computer program are first compared to the transfer matrix solution for undamped synchronous critical speeds. The modified computer program is then compared to the results of an existing transfer matrix stability computer program for the case of sensor collocation. The final results show the influence of sensor noncollocation on the predicted damped critical speeds and log decrement of a multi-stage compressor including the frequency dependent bearing stiffness and damping. The evaluation of sub-harmonic whirl includes the evaluation of the frequency dependent support properties at a selected sub-harmonic mode. The procedure has been automated to automatically converge on a given number of sub-harmonic frequency dependent eigenvalues. The results will show the importance of considering both the sensor noncollocation and the frequency dependent characteristics for turbomachinery with active magnetic bearings.

THEORY OF FINITE ELEMENTS FOR ROTOR DYNAMICS

The finite element method has become a versatile tool in modern engineering design and analysis. With the advancement in high-speed machinery and increase in the power/weight ratio, the determination of the rotor dynamic critical speeds, mode-shapes, and the influence of bearing stiffness and damping become more important for the running of the rotors. In this regard, the finite element method can be utilized to determine the required characteristics.

The present work involves the development of a computer program for the stability analysis of rotor-bearing systems. This is based on the finite modeling of the rotor and its bearings. The finite element model uses a distributed parameter formulation for the mass and stiffness matrices thus accounting for the actual distribution of the mass. The model includes the effects of rotatory inertia, gyroscopic moments and axial loads. This program can handle both distributed mass and lumped mass with inertia effects. The details of the basic finite element model are given in reference [7].

With the advancement in the magnetic bearings, this program is capable of handling noncollocation problems. The sensor noncollocation is conveniently handled by inserting the stiffness of the bearing at the appropriate location. Subharmonic oscillations and the dynamic stability of the rotor bearing system can be studied using this program. The program automatically selects and uses the stiffness and damping properties corresponding to the subharmonic frequencies, which are speed dependent properties. The influence of the bearing cross-coupling or the aerodynamic cross-coupling on the stability of the rotor can be studied with this program. The output is provided in the form of eigenvalues and eigenvectors with the damping ratio in the form of log decrement and the damped natural frequency. The eigenvectors give the damped mode shape for nonsynchronous whirl.

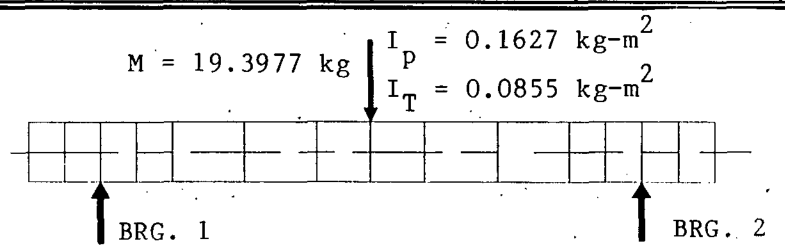
INITIAL VERIFICATION OF FINITE ELEMENT PROGRAM

The finite element computer program is being developed to eventually allow complex, multi-level analysis of machinery supported by either active magnetic bearings or standard fluid-film or anti-friction bearing. The current program runs on a personal computer and is capable of analysis with distributed parameter and/or added lumped inertia properties. One major goal for developing the modified finite element program was to verify the existing transfer matrix critical speed computer program that can calculate the influence of sensor noncollocation by an iterative calculation procedure. The current transfer matrix program has limitations on the range of stiffness and sensor position variation which limits its use for some types of machinery.

The uniform beam model given in Table I was selected as an initial trial rotor bearing system for undamped analysis. The transfer matrix program, CRTMB2 [10], calculates the forward circular synchronous criticals considering either collocated or noncollocated sensors. The results for the first three forward modes are given in Table II with the corresponding results obtained from the finite element program. The results are very encouraging with typically less than 1% deviation for the various conditions examined. The movement of the undamped critical speed, as influenced by sensor position, is in total agreement with previous trends reported in the literature [10].

TABLE I: DATA FOR THE UNIFORM DIAMETER MODEL

Section	Length m	Diameter m	Section	Length m	Diameter m
1	.0508	.0508	8	.0782	.0508
2	.0508	.0508	9	.1016	.0508
3	.0508	.0508	10	.1016	.0508
4	.0508	.0508	11	.0408	.0508
5	.1016	.0508	12	.0508	.0508
6	.1016	.0508	13	.0508	.0508
7	.0782	.0508	14	.0508	.0508



The evaluation of complex non-synchronous eigenvalues is required for stability evaluation of turbomachinery. An existing computer program, ROBEST [4], has been used to verify the gyroscopic and damped critical speed evaluation of the new program. This comparison considers only sensor collocation since ROBEST cannot account for proper sensor position.

The uniform beam results for a lightly damped condition is given in Table III for rotor speeds of 1000.0 and 20000.0 RPM. The percent deviation is typically less than 1.5% which gives added confidence in the computer program results. With this initial rotor system verification showing excellent correlation to existing proven computer programs, the next concern was the calculation of stability and damped natural frequencies for a realistic rotor configuration. The results of the further verification and presentation of new findings are given in the following section.

TABLE II SYNCHRONOUS UNDAMPED CRITICAL SPEEDS COMPARISON WITH CRTMB2

CASES	CRITICAL SPEEDS (RPM)	RESULTS OF TRANSFER MATRIX	RESULTS OF FEM	% ERROR w.r.t. TRANSFER MATRIX
COLLOCATED	First	2657.3	2630.0	-1.027
	Second	6455.5	6420.0	-0.550
	Third	12330.5	12320.0	-0.085
SENSOR LOCATED 4" OUTBOARD	First	2510.2	2564.0	2.143
	Second	7269.7	7230.0	-0.546
	Third	13076.7	13130.0	0.408
SENSOR LOCATED 2" OUTBOARD	First	2580.4	2615.0	1.341
	Second	6873.7	6820.0	-0.781
	Third	12707.9	12715.0	0.056
SENSOR LOCATED 2" INBOARD	First	2742.5	2715.0	-1.003
	Second	6013.9	6000.0	-0.231
	Third	11950.3	12000.0	0.416
SENSOR LOCATED 4" INBOARD	First	2833.3	2842.0	0.307
	Second	5536.5	5485.0	-0.930
	Third	11572.8	11640.0	0.581

EVALUATION OF A MULTI-STAGE COMPRESSOR

The model of an eight-stage compressor used in previous technical reports [10,11] was selected as a good proof-case for the new computer program. The compressor was originally modeled with 60 stations for the transfer matrix analysis of undamped and damped critical speed evaluation. The resulting first four mode shapes and frequencies for a bearing stiffness of $K = 1.3 \times 10^5$ lb/in are shown in Fig. 1 together with the general outline of the rotor with the bearing centerlines indicated. The finite element program is not capable of analysis of the 60-station rotor model due to the 640 K limit of memory on a standard personal computer. This limitation will be removed in a future version of the program compilation. It has been standard practice to reduce the number of elements even for transfer matrix programs. ROBEST has a major-mass option which was incorporated specifically for allowing reduced run times on older mainframes and personal computers. The newer 486 class personal computers are capable of very fast execution and 60 stations is not a problem since eight eigenvalues can be easily computed in less than 30 seconds total run time with writes to disk. If the system is reduced to 19 stations, the execution time is

TABLE III-- NON-SYNCHRONOUS CRITICAL SPEEDS COMPARISON WITH ROBEST

RUNNING SPEED (RPM)	CRITICAL SPEEDS (RPM)		% ERROR w.r.t TRANSFER MATRIX METHOD
	TRANSFER MATRIX	F.E.M.	
1000.0	3432.4	3437.3	0.143
	3433.4	3438.3	0.142
	20151.0	20116.1	-0.173
	20952.0	20917.4	-0.165
	42872.0	43497.2	1.458
	42888.0	53514.5	1.461
	53548.0	54318.9	1.440
	54492.0	55240.8	1.374
20000.0	3423.5	3428.3	0.140
	3442.3	3447.3	0.147
	13534.0	13530.9	-0.023
	27989.0	28053.7	0.231
	42721.0	43332.5	1.431
	43040.0	43679.8	1.487
	47629.0	48471.0	1.768
	67789.0	68158.5	0.545
Point mass added at center of the rotor			19.3977 kg
Polar moment of inertia of the mass			0.1627 kg-m^2
Diametrical moment of inertia of mass			0.0855 kg-m^2
Stiffness of the bearings			1.755 x 10^9 N/m
Damping of the bearings			17550.0 N-s/m

reduced to less than 15 seconds. (Finite element analysis for all eigenvalues requires nearly 4.5 min. on the same machine.) The accuracy of the reduced major-mass rotor model is illustrated by the results given in Table IV for the compressor operating at N = 5000 RPM with bearing stiffness and damping values of K = 1.3 x 10^5 lb/in and C = 125 lb-s/in. The log decrement values are to within 1% for the first three modes and less than 3% for the fourth mode. The mode of concern for instability of high-speed turbomachinery is typically the first or lowest mode. The accuracy shown here is totally acceptable for turbomachinery design and analysis. The resulting 18-section model for use in the finite element program is given in Table V. The rotor is modeled with lumped inertia values which is equivalent to what the transfer matrix stability program, ROBEST, considers. Table VI documents the frequency dependent bearing properties considered in this report, which were generated from standard control theory by the magnetic bearing vendor.

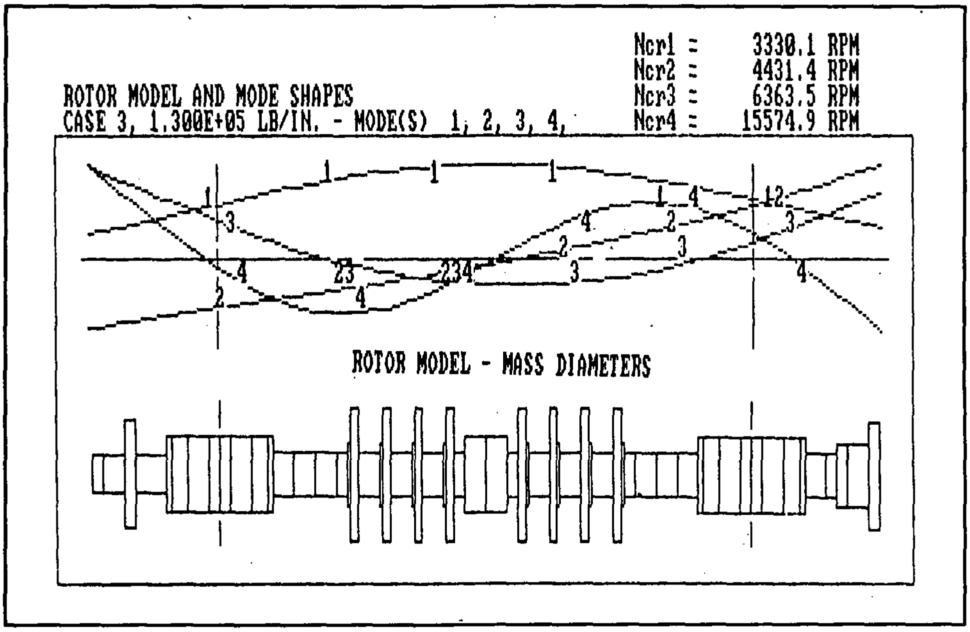


Figure 1 Mode Shapes and Undamped Natural Frequencies for 60-Station Compressor Model
(K = 1.3 x 10^5 lb/in)

TABLE IV COMPARISON OF 60-STATION AND 19-STATION MODEL

N = 5000.0 RPM K´= 1.3x10^5 lb/in C = 125 lb-s/in								
		60 Station			19 Station			% dif
Mode	Type	N_D rev/min	A sec^{-1}	δ dim	N_D rev/min	A sec^{-1}	δ dim	Δ δ/δ
1	F*	3347.0	-43.9	.787	3341.1	-43.44	.780	-.89
1	B	3328.4	-41.73	.752	3321.9	-41.21	.744	-1.06
2	B	4253.0	-94.51	1.333	4251.6	-94.44	1.333	0.0
2	F	4365.0	-97.62	1.342	4363.7	-97.50	1.341	-.07
3	B	5858.8	-54.6	.559	5830.3	-54.74	.563	.70
3	F	6136.6	-53.75	.525	6104.4	-53.79	.529	.76
4	B	13687.	-16.43	.072	13824.	-17.06	.074	2.7
4	F	14481.	-16.67	.069	14611.	-17.42	.071	2.9

*NOTE: F = forward whirl, B = backward whirl

The results from ROBEST considering collocated sensors and both synchronous and non-synchronous first mode whirl are shown in Fig. 2 as log decrement versus aerodynamic excitation at midspan. The mode plotted is the first forward whirl mode which is the one typically of concern. The results indicate approximately 18,000 lb/in cross-coupling threshold for synchronous whirl bearing characteristics and 30,000 lb/in for bearing characteristics to the whirl frequency rather than the running speed synchronous frequency. The results for synchronous whirl and various sensor positions are shown in Fig. 3. The collocated results are in excellent agreement with the transfer matrix results given in Fig. 2. Fig. 3 shows the results for sensor noncollocation for 3 in. inboard and outboard. The condition of sensors inboard by a small axial distance is superior to outboard (i.e., outboard case has the larger negative log decrement). What has not been shown before for rotating

TABLE V: 19-STATION ROTOR MODEL

SECTION	LENGTH m	DIAMETER m	MASS kg	DIAMETER INERTIA kg-m^2	POLAR INERTIA kg-m^2
1	.2286	.0927	.1181E+02	-.928E-01	.5862E-01
2	.0762	.1014	.2304E+02	-.781E-01	.9866E-01
3	.0762	.1014	.1485E+02	.1284E-01	.5869E-01
4	.1016	.1014	.1566E+02	.1073E-01	.5973E-01
5	.1524	.1041	.1043E+02	-.157E-01	.2730E-01
6	.0762	.1071	.1816E+02	.3676E-01	.1448E+00
7	.0762	.1071	.1505E+02	.5560E-01	.1335E+00
8	.0762	.1071	.1505E+02	.5560E-01	.1335E+00
9	.0762	.1271	.1592E+02	.5028E-01	.1289E+00
10	.0952	.1271	.1576E+02	.1777E-01	.6849E-01
11	.0571	.1031	.1583E+02	.4783E-01	.1302E+00
12	.0762	.1071	.1312E+02	.4980E-01	.1166E+00
13	.0762	.1071	.1505E+02	.5560E-01	.1335E+00
14	.1524	.1041	.1816E+02	.3676E-01	.1448E+00
15	.1016	.1014	.1043E+02	-.157E-01	.2730E-01
16	.0762	.1014	.1566E+02	.1073E-01	.5973E-01
17	.0762	.1014	.1485E+02	.1284E-01	.5869E-01
18	.2286	.1137	.2278E+02	-.776E-01	.8127E-01
19			.2042E+02	-.776E-01	.1067E+00

TABLE VI: FREQUENCY DEPENDENT PARAMETERS

SPEED RPM	STIFFNESS N/m	DAMPING N-s/m
1000.0	1.949E+07	3.089E+03
2555.6	2.142E+07	1.229E+04
4111.1	2.229E+07	2.019E+04
5666.7	2.405E+07	2.423E+04
7222.2	2.774E+07	2.967E+04
8777.8	3.195E+07	3.388E+04
10333.3	3.652E+07	3.581E+04
11888.9	4.126E+07	3.458E+04
13444.4	4.564E+07	2.879E+04
15000.0	4.951E+07	1.773E+04

machinery, is the strong instability for sensors inboard by larger amounts (7 in. inboard for this example case).

This is less than double the current design position for this centrifugal compressor. Both a first and second mode would be unstable, even at zero cross-coupling! The first mode is a backward mode and becomes stable as the forward driving cross-coupling is increased. The forward mode becomes more unstable for larger cross-coupling values. Some indication of potential problems can be observed in the synchronous critical speed mode shapes given in Fig. 4. For case (d), the second mode has a mode between the bearing and sensor at the right end location. This case has the sensor 6 in. inboard. Case (e), for sensor 7 in. inboard, indicates the first and second modes have switched mode shape. The lowest mode is now a conical mode and the second is a bending, cylindrical

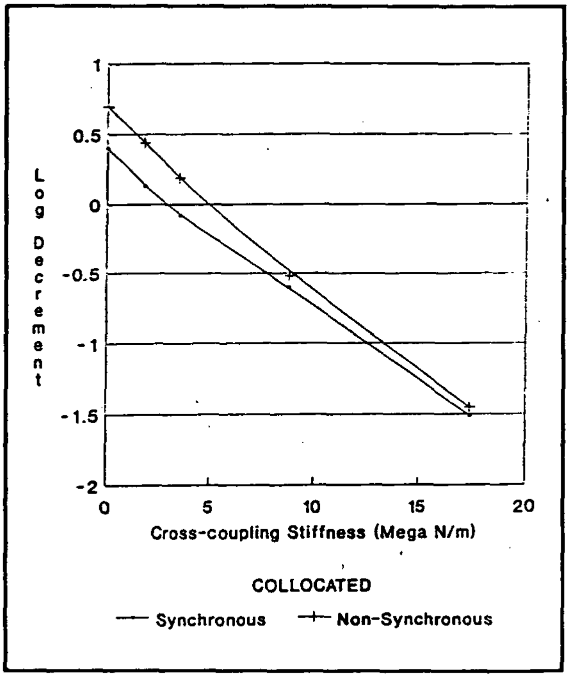


Figure 2 Stability Map for an 8-Stage AMB Supported Compressor (Synchronous Speed Bearing Characteristics)
N = 14000 RPM, transfer matrix solution

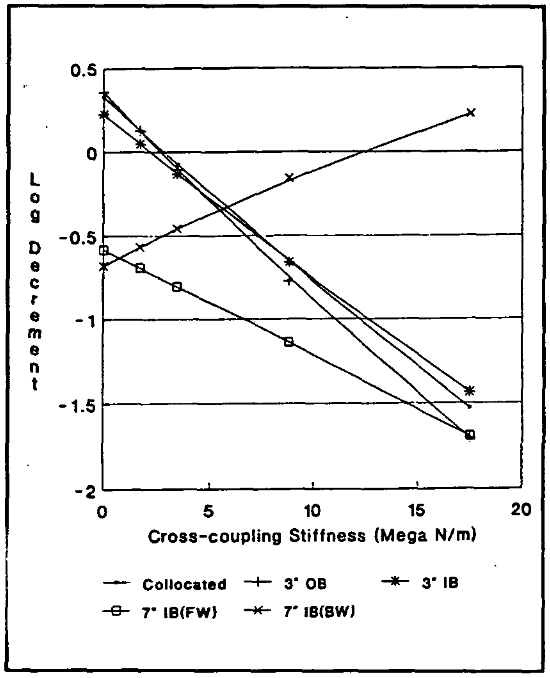


Figure 3: Stability Map for 8-Stage AMB Supported Compressor for Various Sensor Positions (Synchronous Speed Bearing Characteristics; N = 14000 RPM; Finite Element Solution)

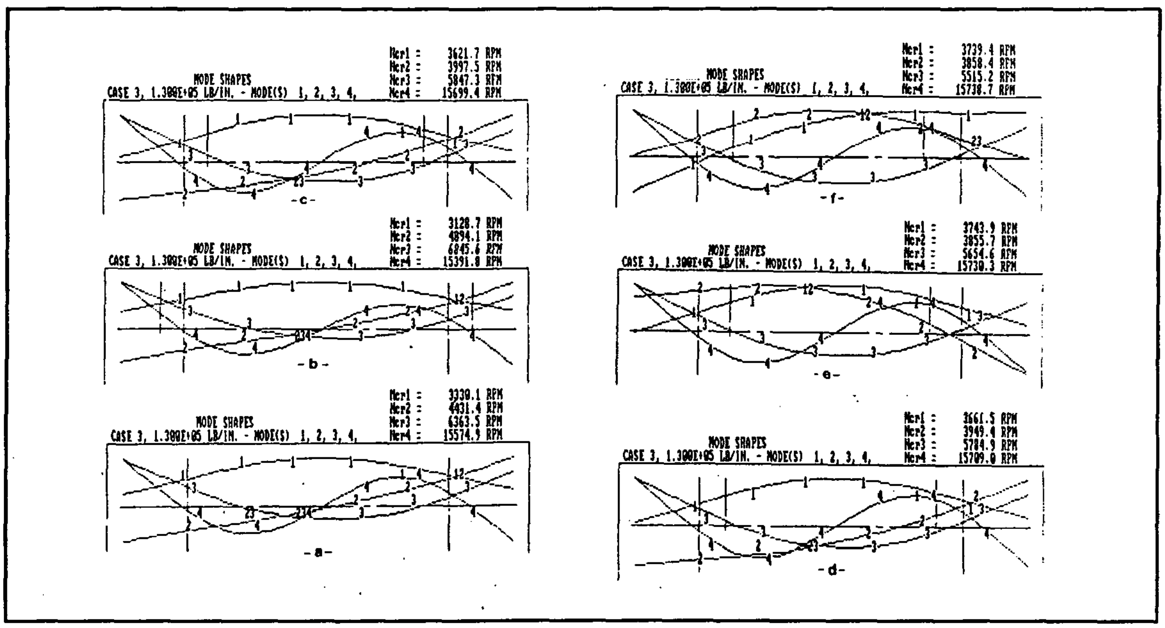


Figure 4: Synchronous Mode Shape for an 8-Stage Compressor with Various Sensor Positions (Sensor position given as: a - collocated; b - 3 in. outboard; c - 3 in. inboard; d - 4 in. inboard; e - 6 in. inboard; f - 7 in. inboard)

mode. It is this same sensor position that gives the strong instability in the synchronous damped (and accurate) stability analysis results shown in Fig. 3.

The results for the nonsynchronous whirl stability analysis of the compressor are shown in Fig. 5. The system is slightly more stable as was predicted by the collocated analysis results from ROBEST. The influence of sensor position is once again shown to be of great importance by these results. The reduced stiffness at the lower whirl frequency has increased the stability such that the 7 in. inboard sensor location mode change does not drive the system unstable at zero cross-coupling. The accurate prediction of rotor stability for active magnetic bearing machinery must consider the effects of sensor position. The program will be extended to allow synchronous response calculation for general elliptic response, with consideration of sensor position.

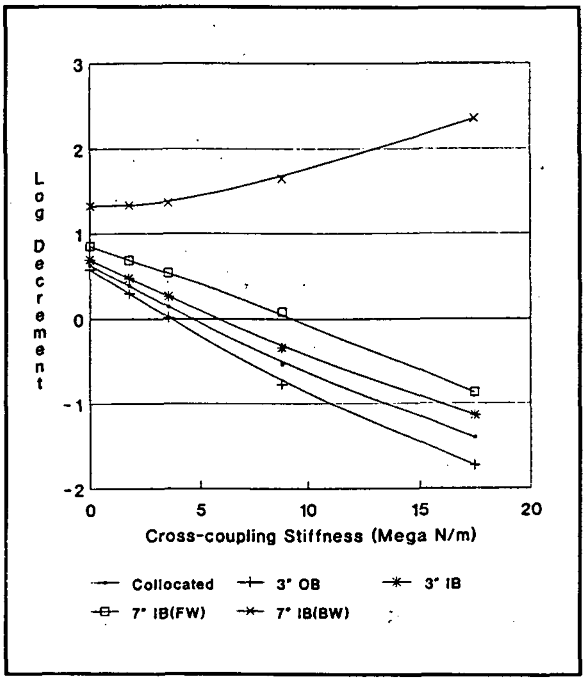


Figure 5: Stability Map for the 8-Stage AMB Supported Compressor for Various Sensor Positions (Nonsynchronous Speed Bearing Characteristics; N = 14000 RPM; Finite Element Method)

CONCLUSIONS

1. The results reported from this research are in general agreement with the results of the iterative transfer matrix solution for consideration of sensor noncollocation in active magnetic bearing machinery.
2. The frequency dependent bearing characteristics should be used for proper stability analysis prediction. Lower stiffness with moderate to high damping was shown to prove to increase the system instability threshold for resistance to aerodynamic excitation (or labyrinth seal excitation).
3. Sensor position must be considered for proper stability prediction. The finite

element method gives a straight-forward analysis capability without the iterative calculation procedure required by the transfer-matrix solution technique.

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