2nd International Symposium on Magnetic Beaing
July 12-14, 1990, Tokyo, Japan

DIGITAL CONTROL SYSTEM FOR MAGNETIC BEARINGS WITH AUTOMATIC BALANCING

T.Higuchi* , T.Mizuno** and M.Tsukamoto***
*Institute of Industrial Science, University of Tokyo,Minato-ku,Tokyo 106,Japan
**Faculty of Engineering, Saitama University,Shimo-Okubo Urawa 338,Japan
***Asahi Chemical Industry Co.,Ltd.,Fuji-shi,Shizuoka 416,Japan

Abstract

The principles and features of a control system for magnetic bearings with an unbalanced rotor are discussed. Unbalance of the rotor causes whirling motion of the rotor and vibratory force acting on bearings. Two special ways to control magnetic bearings with an unbalanced rotor have been presented. One is compensation for unbalance and the other is automatic balancing control. When the spinning axis corresponds with the principal axis of inertia by the automatic balancing control, no reaction force acts on the base of magnetic bearings. In this paper, control system for automatic balancing using observer for totally active magnetic bearing is discussed and implemented by a digital control system. The relationship between the compensation for unbalance and the automatic balancing control is clarified. The effectiveness of the proposed control system is demonstrated by numerical simulation and experiments.

1. Introduction

In a rotating machine unbalance of the rotor causes whirling motion of the rotor and vibratory force transmitting to the base through bearings. These problems can be solved by using a totally active magnetic bearing system, which is able to control its suspension force dynamically. There are two methods to solve the problems caused by unbalance. The first is called compensation for unbalance. With this method, the rotor precisely rotates around the geometrical axis. The other is called an automatic balancing control system in which the spinning axis corresponds with the principal axis of inertia. No reaction force acts on the base of magnetic bearings with this method.

The authors developed a compensator for unbalance using an observer for totally active magnetic bearings[1]. Habermann et al. developed compensation for unbalance and automatic balancing control by utilizing coordinate conversion between fixed coordinate and rotating coordinate and by filtering tuned with the spinning frequency[2][3][4]. Reinig et al. studied compensation for unbalance and automatic balancing control using an observer, but they discussed on only a simplified model of 2 degrees-of-freedom magnetic bearing[5]. In this paper, a compensator for automatic balancing using an observer for totally active magnetic bearing is developed.

The control systems of compensation for unbalance and automatic balancing are implemented by digital controller. The nonlinear characteristics of electromagnet is compensated for linearity. The relationship between compensation for unbalance and automatic balancing control is discussed from the standpoint of the observability. The effectiveness of the proposed control system is demonstrated by numerical simulation and experiments.

2. Modeling

2-1. Structure of magnetic bearings[6]

Fig.1 shows a totally active DC-type magnetic bearing system. It consists of an axial magnetic bearing, two radial magnetic bearings, a motor-stator and a rotor. The axial bearing has a pair of electromagnets and each radial magnetic bearing has two pairs of electromagnets. To describe dynamics of a magnetic bearing system, we define coordinate axes and forces acting on the rotor as shown in Fig.2; in the equilibrium the center of mass of the rotor G is at the coordinate origin O and the axis of the rotor corresponds with z-axis. In this paper, we assume that the rotor is a rigid body and that the rotor spins at a constant angular velocity.

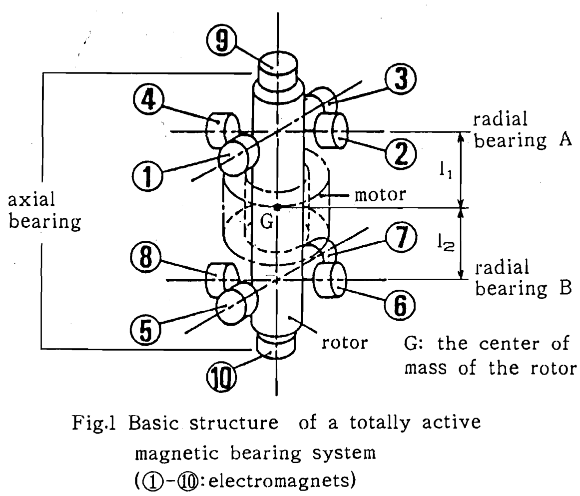


Fig.1 Basic structure of a totally active magnetic bearing system (①-⑩: electromagnets)

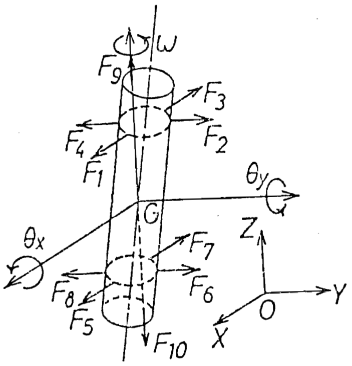


Fig.2 Coordinates and forces acting on the rotor

2-2. Linearization of controlled system

In designing a control system, it is usually assumed that the system to be controlled has linear characteristics. However, electromagnets used for magnetic bearings have nonlinear characteristics as follows: (1)

$F_n = Q_n \cdot I_n^2 / D_n^2, \quad n=1,\ldots,10,$

F_n: the attractive force of electromagnet n,
Q_n: the coefficient of electromagnet n,
I_n: the coil current,
D_n: the gap between the magnet pole faces and the reaction surface.

In conventional method characteristics of electromagnets are linearized by adding bias current into the coil current. The force of a pair of electromagnets can be given by

$$\begin{aligned} F_p - F_m &= Q_p \cdot I_p^2/D_p^2 - Q_m \cdot I_m^2/D_m^2 \\ &= Q\cdot(I_o+i)^2/(D_o-d)^2 - Q\cdot(I_o-i)^2/(D_o+d)^2 \\ &= 4Q\cdot I_o/D_o^2 \cdot i + 4Q\cdot I_o^2/D_o^3 \cdot d \end{aligned} \quad (2)$$

where,
$Q=Q_p=Q_m$, $I_p=I_o+i$, $I_m=I_o-i$, $D_p=D_o-d$, $D_m=D_o+d$,
D_o,d: stationary and incremental component of D_n,
I_o: bias current, i:feedback current,
(p,m)=(1,3),(2,4),(5,7),(6,8),(9,10)
:the numbers of a pair of electromagnets.

To linearize sufficiently, the bias current must be adequately larger than the feedback current. We introduce a direct linearization method[7] which linearize the characteristics of electromagnet by the following nonlinear compensation:

$F \geqq 0 \; ; \; I_p = D_p\sqrt{F/Q}\; , \; I_m = 0$ (3a)
$F < 0 \; ; \; I_p = 0 \; , \; I_m = D_m\sqrt{-F/Q}$ (3b)

F: the force acting on the rotor by a pair of electromagnets given by the feedback

This method renders the bias current unnecessary. Therefore, heat generation and consumption of electric power of electromagnet are minimized. The eddy current in the rotor caused by the high speed spinning becomes sufficiently small and so the heat generation of the rotor and rotation loss are also minimized.

2-3. State equations

The state equations of the magnetic bearings with the direct linearization method are developed as the following procedures. The dynamics of the rotor is described as inertial system with the gyroscopic effect. The whole system can be divided to subsystems related to translation and rotation.

The subsystem of translation along x-axis and y-axis is described as

$$\frac{d}{dt}x_t = A_t^c x_t + B_t^c u_t \quad (4)$$

$x_t = [x_g, v_x, y_g, v_y]^T \; , \; u_t = [F_x, F_y]^T$

$$A_t^c = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B_t^c = \begin{bmatrix} 0 & 0 \\ b_t & 0 \\ 0 & 0 \\ 0 & b_t \end{bmatrix}.$$

The subsystem of rotation around x-axis and y-axis is described as

$$\frac{d}{dt}x_r = A_r^c x_r + B_r^c u_r \quad (5)$$

$x_r = [\theta_x, \omega_x, \theta_y, \omega_y]^T \; , \; u_r = [M_x \, , \, M_y]^T$

$$A_r^c = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -a_k \\ 0 & 0 & 0 & 1 \\ 0 & a_k & 0 & 0 \end{bmatrix}, \quad B_r^c = \begin{bmatrix} 0 & 0 \\ b_r & 0 \\ 0 & 0 \\ 0 & b_r \end{bmatrix}.$$

The subsystem of translation along z-axis is described as

$$\frac{d}{dt}x_z = A_z^c x_z + B_z^c u_z \quad (6)$$

$x_z = [z_g, v_z]^T \; , \; u_z = F_z$

$$A_z^c = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B_z^c = \begin{bmatrix} 0 \\ b_z \end{bmatrix},$$

where,
x_g, y_g, z_g: displacement of center of mass G,
v_x, v_y, v_z: velocity of center of mass G,
θ_x, θ_y: tilting angle of the rotor axis,
ω_x, ω_y: derivative of θ_x and θ_y,
ω_z: angular velocity of spinning,
F_x, F_y, F_z: force of electromagnet acting on the rotor,
M_x, M_y: moment of electromagnet acting on the rotor,
$a_k = I_a/I_r \cdot \omega_z$,
$b_t = 1/m$, $b_r = 1/I_r$, $b_z = 1/m$,
m: mass of the rotor,
I_a, I_r: polar and transverse mass moments of inertia of the rotor.

The forces F_x, F_y, F_z and the moments M_x, M_y are given by

$F_x = (F_1-F_3)+(F_5-F_7), \; M_x = -(F_2-F_4)l_1 + (F_6-F_8)l_2,$ (7a)
$F_y = (F_2-F_4)+(F_6-F_8), \; M_y = (F_1-F_3)l_1 - (F_5-F_7)l_2,$ (7b)
$F_z = F_9 - F_{10},$ (7c)

where,

l_1, l_2: distance between the center of mass G and the magnets of radial bearing A and B as shown in Fig.1.

3. Control system with automatic balancing

3-1. Model with unbalance

When the rotor is unbalanced, that is, the geometrical axis of the rotor is different from the principal axis of inertia, the rotor spins with whirling motion. Unbalance of the rotor can be divided to static unbalance and dynamic unbalance. The static unbalance affects only the subsystem related to translation along x and y axes. The dynamic unbalance affects only the subsystem related to rotation. If the parameter a_k equals to zero in the subsystem related rotation, it has the same form in the state equations as the subsystem related to translation along x and y axes. There is no influence of unbalance on the subsystem related to translation along z-axis. It is allowed to discuss only the subsystem related to rotation without loosing generality.

To construct a digital control system, we discretize the system $(A_r{}^c, B_r{}^c)$ using a zero-order holder and let (A,B) denote the obtained digital system. State variable x_r and input u_r are represented by x and u, respectively, in shortening. The digital system is written as follows:

$$x(k+1)=Ax(k)+Bu(k)\ . \tag{8}$$

Because of unbalance, disturbance force seems to act on the rotor. It is represented by the following free system:

$$\frac{d}{dt}w_u=A_d{}^c w_u\ ,\quad w_u=[M_{dx}, M_{dy}]^T, \tag{9}$$

where,

M_{dx}, M_{dy}: moment caused by unbalance,

$$A_d{}^c=\begin{bmatrix}0 & -\omega_z\\ \omega_z & 0\end{bmatrix}.$$

To construct a digital control system, we discretize the free system $A_d{}^c$ using a zero-order holder and let A_d denote the obtained digital free system. Expansion system including disturbance can be written as follows:

$$x_{eu}(k+1)=A_u x_{eu}(k)+B_u u_u(k)\ , \tag{10a}$$

$$y_u(k)=C_u x_{eu}(k)\ , \tag{10b}$$

$$x_{eu}(k)=\begin{bmatrix}x_u(k)\\ w_u(k)\end{bmatrix},\quad A_u=\begin{bmatrix}A & B\\ 0 & A_d\end{bmatrix},$$

$$B_u=\begin{bmatrix}B\\ 0\end{bmatrix},\quad C_u=[C\ 0]\ ,\quad C=\begin{bmatrix}1&0&0&0\\0&0&1&0\end{bmatrix},$$

where,

x_u: state variable based on the geometrical axis of the rotor,

y_u: output generated from sensor signals.

This augmented system can be represented by another form. Instead of looking upon unbalance as disturbance force, we can think of unbalance as observatory disturbance. That is, the sensors should indicate the tilting angle of the principal axis of inertia. But because of unbalance, sensor outputs include geometrical errors; the difference between the geometrical axis of the rotor and the principal axis of inertia. The geometrical errors can be also represented by the following free system: Note that it is the same form as eq.(9).

$$\frac{d}{dt}w_a=A_d{}^c w_a\ ,\quad w_a=[\theta_{dx}, \theta_{dy}]^T, \tag{11a}$$

where,

θ_{dx}, θ_{dy}: difference between the geometrical axis of the rotor and the principal axis of inertia.

An expansion system including geometrical errors can be written as follows:

$$x_{ea}(k+1)=A_a x_{ea}(k)+B_a u_a(k)\ , \tag{12a}$$

$$y_a(k)=C_a x_{ea}(k)\ , \tag{12b}$$

$$x_{ea}(k)=\begin{bmatrix}x_a(k)\\ w_a(k)\end{bmatrix},\quad A_a=\begin{bmatrix}A & 0\\ 0 & A_d\end{bmatrix},$$

$$B_a=\begin{bmatrix}B\\ 0\end{bmatrix},\quad C_a=[C\ I\ ]\ ,\quad C=\begin{bmatrix}1&0&0&0\\0&0&1&0\end{bmatrix},$$

where,

x_a: state variable based on the principal axis of inertia,

y_a: output generated from sensor signals.

3-2. Structure of controller

The authors developed compensation for unbalance using an observer[1]. In this method, a system to be controlled is represented by eq.(10). The disturbance force is estimated by a minimal order observer and cancelled by bearing force according to the estimation. In this paper we propose an automatic balancing system using an observer for totally active magnetic bearings. In this method, the system to be controlled is represented by eq.(12) and the state variables represent the motion of the principal axis of inertia. By constructing feedback system utilizing these state variables, the rotor spins around the principal axis of inertia. Since the state variables cannot be measured directly, they are estimated by an observer. In this paper, we use a full-order observer. Controllers of compensation for unbalance and automatic balancing are given by follows; ^ shows estimated value.

compensation for unbalance

$$\hat{x}_{eu}(k+1)=(A_u-K_uC_u)\hat{x}_{eu}(k)+B_u u_u(k)+K_u y_u(k)\ , \tag{13a}$$

$$u_u(k)=F_u\hat{x}_{eu}(k)\ , \tag{13b}$$

$$\hat{x}_{eu}(k)=\begin{bmatrix}\hat{x}_u(k)\\ \hat{w}_u(k)\end{bmatrix}, \quad K_u=\begin{bmatrix}K_{ux}\\ K_{uw}\end{bmatrix},$$

$$F_u=[F\ \ -I],$$

automatic balancing control

$$\hat{x}_{ea}(k+1)=(A_a-K_aC_a)\hat{x}_{ea}(k)+B_au_a(k)+K_ay_a(k), \qquad (14a)$$

$$u_a(k)=F_a\hat{x}_{ea}(k), \qquad (14b)$$

$$\hat{x}_{ea}(k)=\begin{bmatrix}\hat{x}_a(k)\\ \hat{w}_a(k)\end{bmatrix}, \quad K_a=\begin{bmatrix}K_{ax}\\ K_{aw}\end{bmatrix},$$

$$F_a=[F\ \ 0],$$

where,

K_u, K_a: gain matrices of observer

F: feedback matrix of regulator

In these forms, we can hardly understand the structure of the whole system. We rewrite the whole system using estimating errors:

$$\xi_u=\hat{x}_u-x_u, \quad \eta_u=\hat{w}_u-w_u$$
$$\xi_a=\hat{x}_a-x_a, \quad \eta_a=\hat{w}_a-w_a. \qquad (15)$$

Then, we get the following equations:

compensation for unbalance

$$\begin{bmatrix}x_u(k+1)\\ \xi_u(k+1)\\ \eta_u(k+1)\\ w_u(k+1)\end{bmatrix}=\begin{bmatrix}A+BF & BF & -B & 0\\ 0 & A-K_{ux}C & B & 0\\ 0 & -K_{uw}C & A_d & 0\\ 0 & 0 & 0 & A_d\end{bmatrix}\begin{bmatrix}x_u(k)\\ \xi_u(k)\\ \eta_u(k)\\ w_u(k)\end{bmatrix}$$

$$y_u(k)=[\ C\ \ 0\ \ 0\ \ 0\ ]\begin{bmatrix}x_u(k)\\ \xi_u(k)\\ \eta_u(k)\\ w_u(k)\end{bmatrix}$$

$$u_u(k)=[\ F\ \ F\ \ -I\ \ -I\ ]\begin{bmatrix}x_u(k)\\ \xi_u(k)\\ \eta_u(k)\\ w_u(k)\end{bmatrix} \qquad (16)$$

automatic balancing control

$$\begin{bmatrix}x_a(k+1)\\ \xi_a(k+1)\\ \eta_a(k+1)\\ w_a(k+1)\end{bmatrix}=\begin{bmatrix}A+BF & BF & 0 & 0\\ 0 & A-K_{ax}C & -K_{ax} & 0\\ 0 & -K_{aw}C & A_d-K_{aw} & 0\\ 0 & 0 & 0 & A_d\end{bmatrix}\begin{bmatrix}x_a(k)\\ \xi_a(k)\\ \eta_a(k)\\ w_a(k)\end{bmatrix}$$

$$y_a(k)=[\ C\ \ 0\ \ 0\ \ I\ ]\begin{bmatrix}x_a(k)\\ \xi_a(k)\\ \eta_a(k)\\ w_a(k)\end{bmatrix}$$

$$u_a(k)=[\ F\ \ F\ \ 0\ \ 0\ ]\begin{bmatrix}x_a(k)\\ \xi_a(k)\\ \eta_a(k)\\ w_a(k)\end{bmatrix} \qquad (17)$$

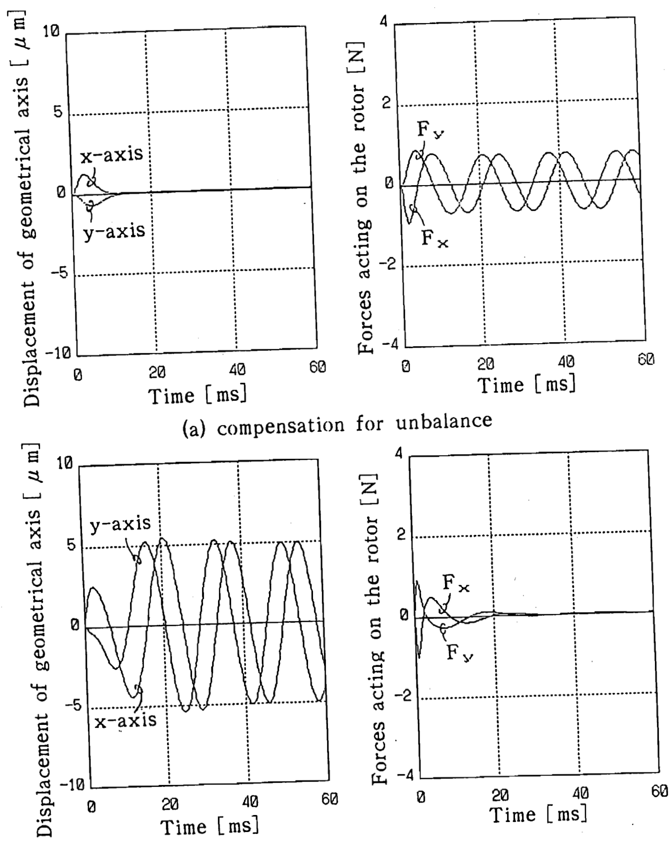
Let us consider these systems in the standpoint of the observability. Generally, an unobservable system can be written as eq.(18):

$$\begin{bmatrix}x_1(k+1)\\ x_2(k+1)\end{bmatrix}=\begin{bmatrix}A_{11} & 0\\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}x_1(k)\\ x_2(k)\end{bmatrix}, \qquad (18a)$$

$$y(k)=[C_1\ \ 0\ ]\begin{bmatrix}x_1(k)\\ x_2(k)\end{bmatrix}. \qquad (18b)$$

In the case of eq.(18), state variable x_2 cannot be observed from output y and A_{22} is the unobservable subsystem. Comparing eq.(16) and eq.(18), A_d is an unobservable subsystem from output y_u in the system of compensation for unbalance. Therefore, the unbalance gives no influence to output y_u. On the other hand, A_d is an unobservable subsystem from input u_a in the automatic balancing system described by eq.(17). With the automatic balancing control, the state variables are based on the principal axis of inertia and no influence can be transferred to input u_a. Therefore, the rotor rotates around the principal axis of inertia. The compensation for unbalance and the automatic balancing control are opposing each other. The controllers of those methods also have opposing structures; the system of the compensation for unbalance is unobservable from output and the automatic balancing system is unobservable from input.

In these systems the poles of regulator and



(b) automatic balancing

rotation speed : 3600[rpm]

Fig.3 Calculated results

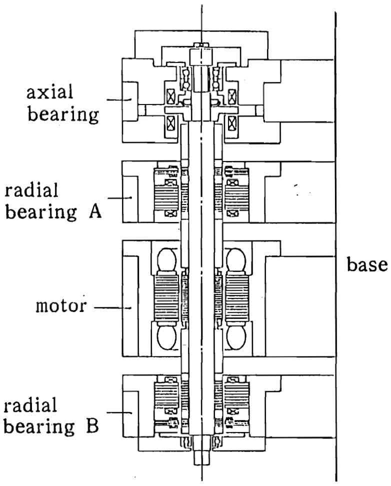


Fig.4 Structure of the experimental setup

Table 1 Parameters of the experimental setup

m	0.998	[kg]
Ir	4.26×10^{-3}	[kg·m^3]
Ia	1.01×10^{-4}	[kg·m^3]
l_1	42	[mm]
l_2	88	[mm]
$Q_1 \sim Q_8$	2.5	[N·mm^2/A^2]
$DO_1 \sim DO_8$	0.3	[mm]

those of observer can be assigned independently. The poles of regulator are assigned by utilizing the linear quadratic optical control theory. The poles of observer are assigned by applying the same theory for its dual system.

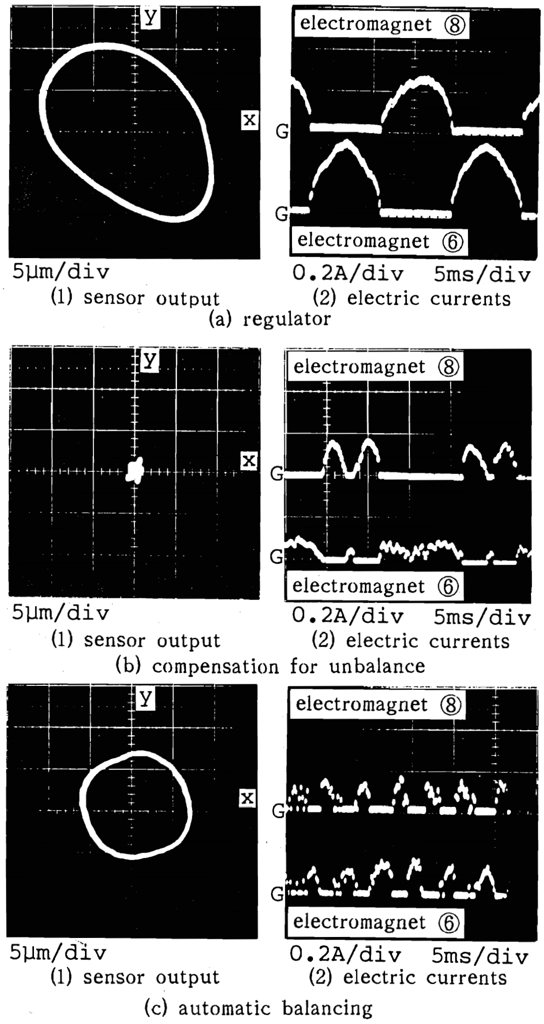
4. Simulation and Experiments

The effectiveness of the proposed control system is verified by numerical simulation and experiments. To compare the two control methods, we also examine compensation for unbalance.

Fig.3 shows a calculation result. Displacement of the geometrical axis converges to zero by compensation for unbalance and the forces acting on the rotor converges to zero by automatic balancing control as expected.

Fig.4 shows the section view of the experimental setup. Table 1 explains its parameters. The control system is implemented by digital controller using a digital signal processor[8]. It carries out the above control law and linearization within 0.2 msec.

Fig.5 shows some experimental results. In Fig.5(a) there is a whirling motion in response of a regulator system which is designed as if there were no unbalance. Fig.5(b) shows a result of the compensation for unbalance. The whirling motion are controlled fairly well. In the response of regulator as shown in Fig.5(a), there is an oscil-



rotation speed : 3600[rpm]

Fig.5 Experimental results

lating magnet-current synchronizing with spinning of the rotor. Fig.5(c) shows a result of automatic balancing. The magnet-current which has the frequency component of the spinning is eliminated in spite of the whirling motion. Therefore, the rotor must be rotated around the principal axis of inertia. Essentially, we have to observe attractive force of magnet. In the case of Fig.5, however, we can substitute magnet-current into attractive force of magnet, because the whirling motion is adequately smaller than the gaps between poles of magnets and the rotor.

5. Conclusion

The control system for automatic balancing using an observer for totally active magnetic bearings was developed.

The relationship between the compensation for unbalance and the automatic balancing control was discussed from the standpoint of the observability.

The control system for automatic balancing was implemented by a digital controller. The effectiveness of proposed control system was verified by numerical simulation and experiments.

References

[1]Mizuno,T. and Higuchi,T.: Compensation for Unbalance in Magnetic Bearing System(in Japanese). Trans. SICE, Vol.20, pp.1095-1101(1984).

[2]Habermann,H. and Brunet,M.: The Active Magnetic Bearing Enables Optimum Control of Machine Vibrations, ASME International Gas Turbine Conf., 85-GT-221(1985)

[3]Habermann,H. et al.: Device for Damping the Critical Frequencies of a Rotor Suspended by a Radial Electromagnetic Bearing, United States Patent 4,128,795(1978)

[4]Habermann,H. et al.: Device for Compensating Synchronous Disturbances in Magnetic Suspension of a Rotor, United States Patent 4,121,143(1978)

[5]Reinig,K. and Desrochers,A.: Disturbance Accommodating Controllers for Rotating Mechanical Shaft, ASME Journal of Dynamic Systems, Measurement, and Control, 108, March, pp.24-31(1986)

[6]Mizuno,T. and Higuchi,T.: Design of the Control System of Totally Active Magnetic Bearing ---Structures of the Optimal Regulator---, Proc. of 1st Int. Symp. on Design and Synthesis, Tokyo(1984)

[7]Tsuda,M.,Nakamura,Y.,and Higuchi,T.: Design and Control of Magnetic Servo Levitation, Proc. of 20th Int. Conf. on Industrial Robots, pp.693-700(1989)

[8]Tsuda,M.,Nakamura,Y.,and Higuchi,T.: High-Speed Digital Controller for Magnetic Servo Levitation of Robot Mechanism,Proc. of 1st Int. Symp. on Experimental Robotics, pp.229-243(1989)