On the Static Stability Problem of Magnetic Levitation from Rigid Body to Flexible Plate

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Abstract
It is an important item that how many magnets and control axis are necessary to be able to suspend the body by the magnetic force. The magnet has the attraction force originally,and the couple force i.e. the attraction moment in spite of it is controlled by the position feedback.
This paper presents the conditions of static stability which are necessary in order to determine the arrangements of magnetic bearings especially for flexible body.

1. Introduction

When the body is levitatied by magnetic force, the possibility of levitation is restricted by Earnshow's Theorem. Earnshow's Theorem shows that it is impossible for a position of stable equilibrium, and a current of the electromagnet has to be controlled by position feedback.

It is the important item that how many magnets and control axis are necessary to be able to suspend the body, especially the long body.
The magnet has the attraction force originally and the couple force i.e. attraction moment in spite of it is controlled by the position feedback. We have already presented that it was necessary to consider the couple force of every axis in order to discuss the stability of the full magnetic bearing type rotor(ref.[1]).

This paper presents the relations between the static stability conditions and the magnetic bearing arrangements under considering the above-mentioned original force of magnets theoreticaly.

Recently, many works have continued for applying the active magnetic bearings to the flexible body levitation (ref.[3], [4]). We are studying the magnetic levitation technologies of the thin steal plate in order to make good use of non-contact merit(ref.[2]).

At first,we make it be clear that is the static stability condition considering the magnetic couple force at the rigid body levitation. Secondly we discuss the static stability condition to determine the pitch of the bearings at the flexible beam levitation. And we investigate the instability problem caused by difference between the magnetic field and the position of feedback sensor.

2. Characteristics of Magnetic Bearing

In order to investigate the stability of magnetic levitation system, we have to make it clear that the static characteristic of the active magnetic bearing unit itseif first of all.

The bearing unit consists of twin electromagnets, their drive circuits, a position sensor, and a position feedback line as shown in Fig.1. As it's static characteristic, the transfer function from a displacement of the body to the electromagnetic forces has to be clear. There are two kind of electromagnetic forces for the displacement as follows.

(1)Passive Part:

$$F_P = K_M \cdot X_P \qquad (1)$$

$$K_M = -4K(I_0^2 + I_C^2)/\delta_0^3$$

(2)Active Part:

$$F_A = K_F \cdot I_A \qquad (2)$$

$$I_A = K_P \cdot X_A$$

$$K_F = 4KI_0/\delta_0^2 \qquad (3)$$

where F :electromagnetic force
X :displacement of the body
I_A:control current
K_P:proportional gain
K :electromagnetic constant
δ_O:aimed gap
I_O:bias current
I_C:load balance current
P :passive , A : active

These functions are led as the concentrated system. The displacement X_A used in the active part is measured by sensor, and then means that of one point of the body. On the other hand, the displacement X_P used in the passive part means the distribution of the body, being concerned with the magnetic field. And both the passive part F_P and the active part F_A of the electromagnetic force act on the magnetic field as the distribution. Really, the displacement X_A which is the position feedback signal, only is as a cencentrated system, the others are as the distribution system. Then, the above -mentioned functions are realized, only when the body is rigid and displaces parallel to the electromagnet.

If the body rotates at the axis perpendicular to the displacement X_A, the position sensor can not measure the rotation movement, and the electromagnetic force F_A does not act to the body at all. But the electromagnetic force F_P acts to the body as the moment, as the displacement X_P is concerned with the rotation movement. Then, this phenomena is written as the following equations.

$$M_P = K_\theta \cdot \theta$$
$$K_\theta = \frac{1}{12} K_M \cdot \ell^2 \qquad (4)$$

where M_P :moment (coupled force)
θ :the rotation angle of the body
ℓ :The equivalent electromagneto width

The equation (4) is realized only when the body is rigid too. If the flexible body such as thin plate is levitated by such some magnetic bearing units, there are the following subjects.
(1) We wish that the distance between the magnetic bearing units is as much as possible.
(2) We wish that the width of the electromagnet is as mush as possible.
These subjects means that the number of the bearing units is reduced. The static stablity problem is studied from a view-point of each subject theoretically.

3. Stability Problem of Rigid Body

The static stability problem is studied for the rigid body of the system shown in Fig.2. In Fig.2, the body is suspent by 4 bearing units. Rotational stiffness of both rolling and yawing direction is ensured by each 2 bearing units. The distance of 2 bearing units is not much longer than the width of electromagnet. Then, the passive rotational stiffness K_θ is important to ensure the static stability. The static stability condition is as follows.

$$K_F \cdot K_P + K_M > 0 \qquad (5)$$

for each bearing unit

$$2 \cdot \frac{L^2}{4} (K_F \cdot K_P + K_M) + 2 K_\theta > 0 \qquad (6)$$

for rolling and yawing direction
where L:the distance of bearing units
As K_θ is negative, it is sufficient to satisfy equation (6). Equation (6) becomes the following equation.

$$K_F \cdot K_P + \left(1 + \frac{\ell^2}{12L^2}\right) K_M > 0 \qquad (7)$$

And,it is a very important selection that the body is levitated by either attraction magnetic force or repulsion magnetic force. As using the attraction magnetic force, the stability check of pitching is necessary. And the couple force is added to the left hand of equation (6). Then, the feedback gain K_P increases at using the attraction force.

When the body is levitated by the electromagnet, the plate contacts with electromagnet. In order to clear the reason, the eigenvalue of the model shown in Fig.2 is analized by the program shown in ref.[6]. These results are shown in Fig.3. Fig.3 shows that the rigidity of plate is small to secure the static stability under keeping 4 bearing units. This problem is solved by analizing the precise model such as Fig.2.

4. Stability Problem of Steal Plate

When the thin and long steal plate is levitatied by the magnetic force, it is very important to determine the pitch of bearing units and the width of electro-magnet. To be clear this subject, the static stability problem is studied. When this static stability problem is solved, the following items are assumed.
(1)The steal plate is analized as the beam model (ref.[5]).
(2)The steal plate is free in tension.
(3)The steal plate is not transfered.
And, the precise model is shown in Fig.4.

4.1 Pitch of Bearings and Stability

First of all, the ratio of steal strip area to electromagnet area is determined by the load capacity of bearing. Relations between the ratio of load capacity to strip weight and the thickness of strip is shown in Fig.5 (Ref[2]). The ratio of strip area to electromagnet area must be smaller than the value of the perpendicular axis in Fig.5.

Next the condition which determines the pitch of bearing units is introduced from the study of static stability. In Fig.4, the concentrated model mentioned in paragraph 2 is used as the

characteristics of bearing units. The eigenvalues are calculated as the values of rotational stiffness K_θ are parameter (ref.[5]). These results are shown in Fig.6. Fig.6 shows the following things.
(1)The 3rd eigenvalue is unstable at K_θ= -3.5 N·m/rad. And 2nd eigenvalue is unstable at K_θ =-5.9N·m/rad.
(2)When these eigenvalues become unstable, the mode of 3rd eigenvalue looks like that of 1st eigenvalue at pin support and that of 2nd eigenvalue looks like that of 2nd eigenvalue at pin support.

Then, the following stability criterion is gotten from the comparsion between the bendings stiffness of the above-mentioned mode and the rotational stiffness of bearing unit.

$$\frac{2EI}{L} + 2K_\theta > 0 \qquad (8)$$

for 3rd eigenvalue.

$$1 \Bigg/ \left[\frac{2}{\tilde{K} \cdot L^2} + \frac{L}{12EI} \right] + 2K_\theta > 0 \qquad (9)$$

for 2nd eigenvalue.

where, $\tilde{K}=K_F \cdot K_P + K_M$
EI:bending stiffness of beam
The results compared between the eigenvalue analysis and the criteria equation are shown in Fig.7. This simple criterion coincides with the results of the eigenvalue analysis. Usually, if equation (8) is satisfied, equation (9) is satisfied. Then, the pitch of bearing has to determined from the criterion of equation (8).

4.2 Width of Electromagnet and Stability

There is a question that how much area of electromagnet it is good for one feedback sensor. In order to confirm this question, the eigenvalues of the part of only a bearing unit shown in Fig.8. In Fig.8, both the passive stiffness K_M and the active magnetic force F_A act on the magnetic field which is different from the point of sensor. The root loci with the parameter of $\overline{K}=K_F \cdot K_A$ is shown in Fig.9. As the 2nd eigenvalue is unstable naturally and the 4th eigenvalue is stable, these are omitted. Only 1st and 3rd eigenvalues are plotted in Fig.9. Fig.9 shows the following results.
(1) As feedback $\overline{K}$ increases, 1st eigenfrequency increases and 3rd eigenfrequency decreases.
(2) At $\overline{K}$=4850N/m, 1st eigenfrequency coincides with 3rd eigenfrequency.
(3) When $\overline{K}$ is greater 4850N/m, 1st and 3rd eigenvalues are coupled, and both eigenvalues are dynamic unstable. Fig.10 shows each mode of beam when 2 eigenvalues are both uncoupled and coupled. Such a feedback gain has the effect which is bending the beam.

Then, the following stable criterion is able to be made from the considence of both 1st and 3rd eigenfrequencies.

$$K_F \cdot K_P + K_M > \alpha \frac{48 \cdot EI}{\ell^3} \qquad (10)$$

where, 2·l:width of electromagnet
As the constant α is estimated from these results, α equals 0.15. The bearing unit is determined by the existance of K_F which is satisfied with both equation (8) and (10).

4.3 Results of Total System

To confirm the above-mentioned results, the eigenvalues of the precise model shown in Fig.4 is analized. Dimentions of strip and bearing are as t=0.3mm, L=570mm and l=70mm. The results of eigenvalue analysis are shown in Table 1. This system doesn't satisfy equation (8). Then, 1st eigenvalue is static unstable. And, as $K_F \cdot K_A$=5000N/m, this system doesn't satisfy equation (10). Then, Table 1. shows that 5th and 6th eigenvalues are dynamic unstable. Next, when the width of electromagnet is made smaller, the eigenvalues are shown in Table. 2. As $K_F \cdot K_P$=5000N/m, all of eigenvalues are stable. This system satisfies both equation (8) and (9).

5. Conclutions

According to the theoretical study above mentioned, satisfing the following conditions are necesary to determine the arrangement of magnetic bearing units.
(1) The ratio of strip weight to load capacity is determined.
(2) The pitch of bearing units satifies equation (8).
(3) And, the selected feedback gain satisfies equation (10).

From now on, we will develop these idea to the following status.
(1) From the beam model to the plate model.
(2) The strip is contrained by tention. We succeeded in the levitation of thin steal strip(2m×3m×0.6mm~6.0mm) shown in ref.[2]. The above-mentioned phenomena has to be confirmed by the test in the near future.

References.
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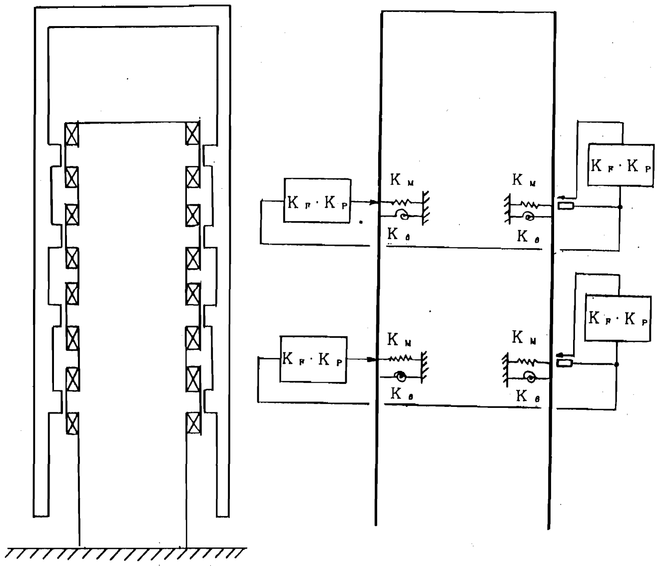


Fig.2 Model of Rigid Body Levitation

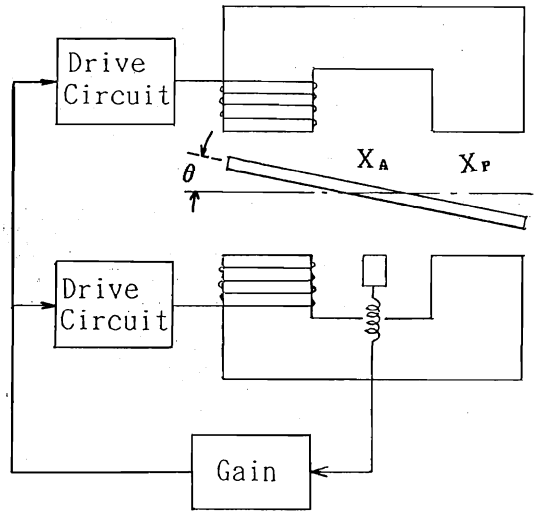


Fig.1 Magnetic Bearing Unit

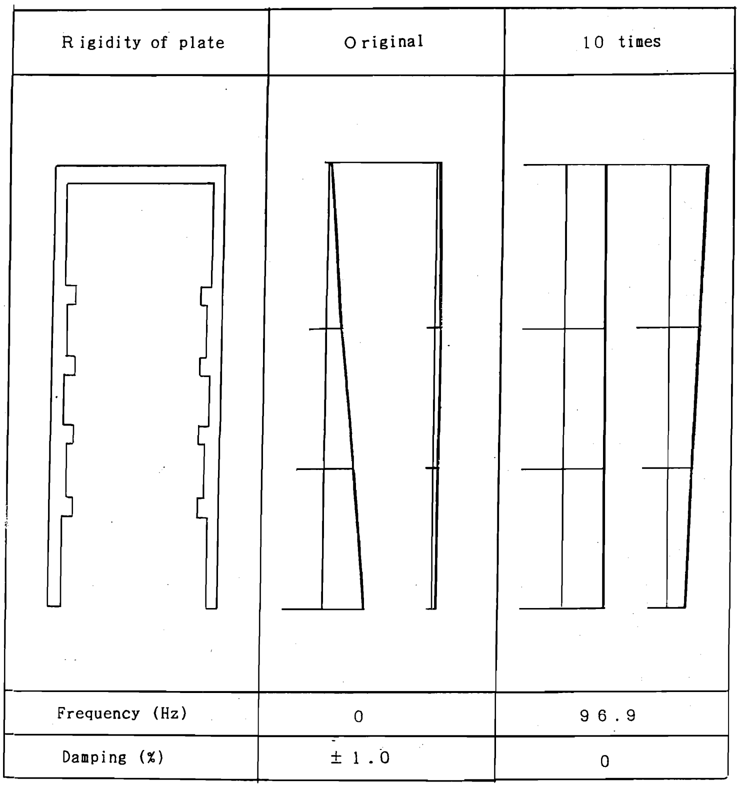


Fig.3 Static Stability Analysis Results

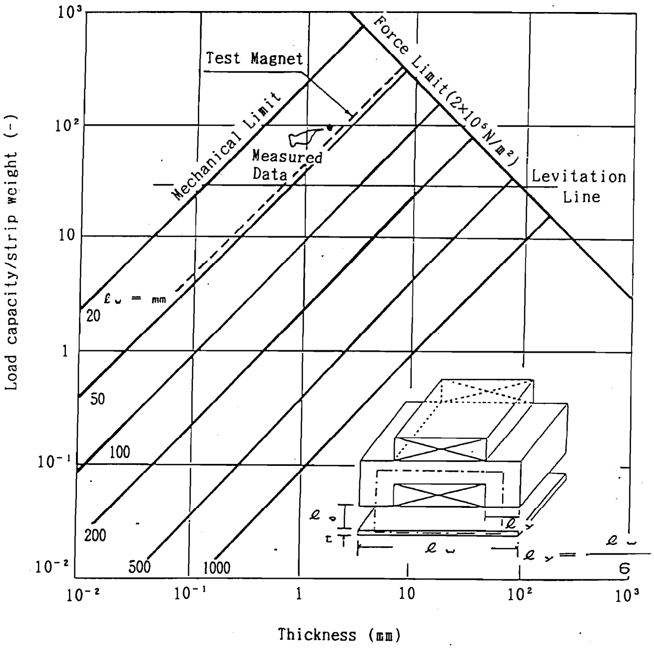


Fig.4 Load Capacity for Steal Plate

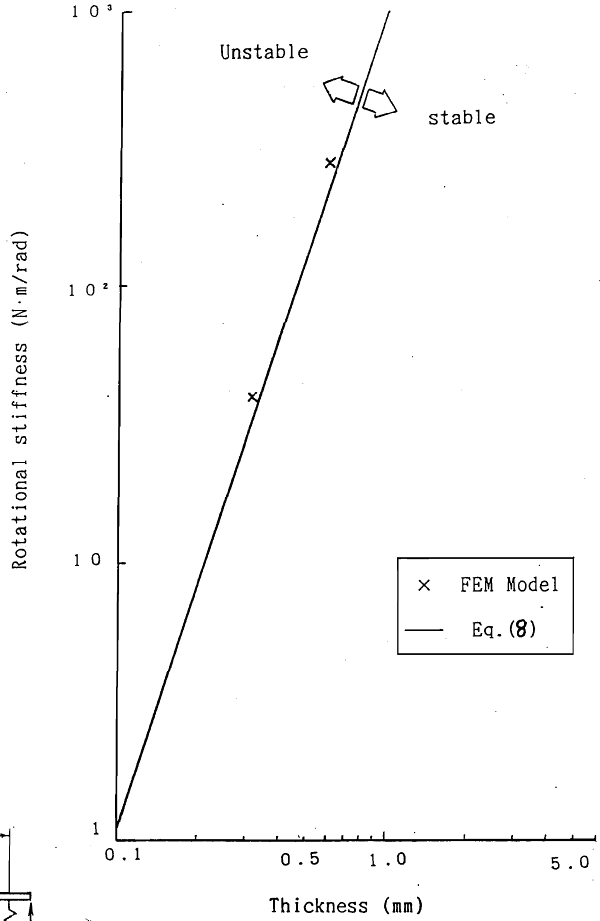


Fig.7 Stability Criterion concerning with Electromagnetic Width

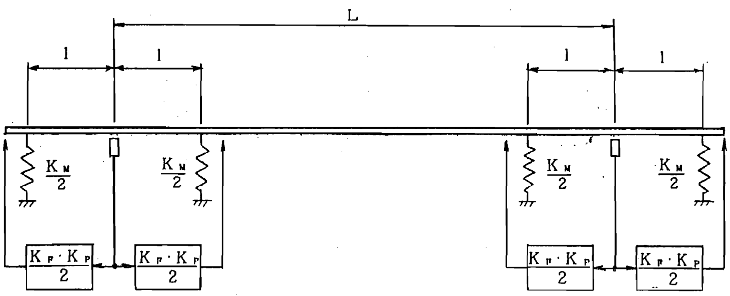


Fig.5 Beam Model of Plate Levitation

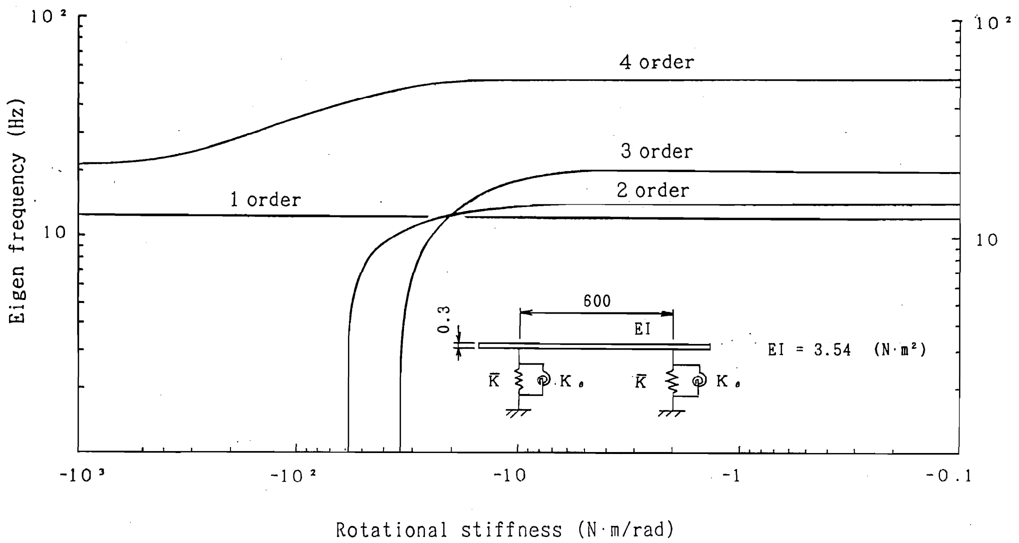


Fig.6 Eigenfrequency & Magneto Stiffness

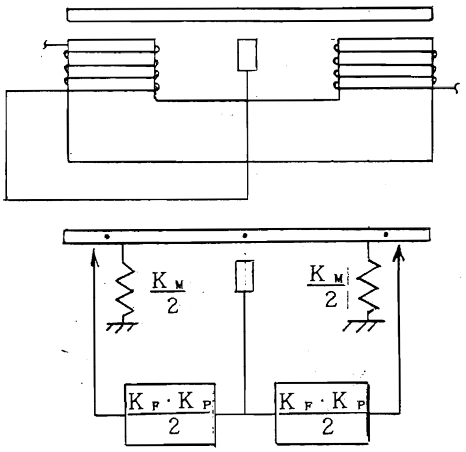


Fig.8 Model of Bearing Unit

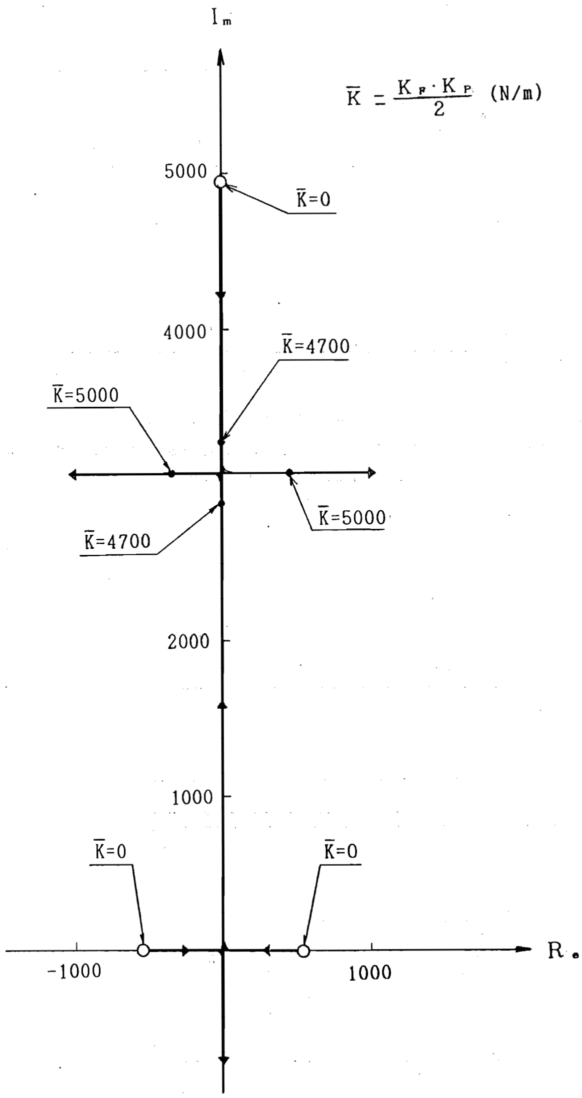
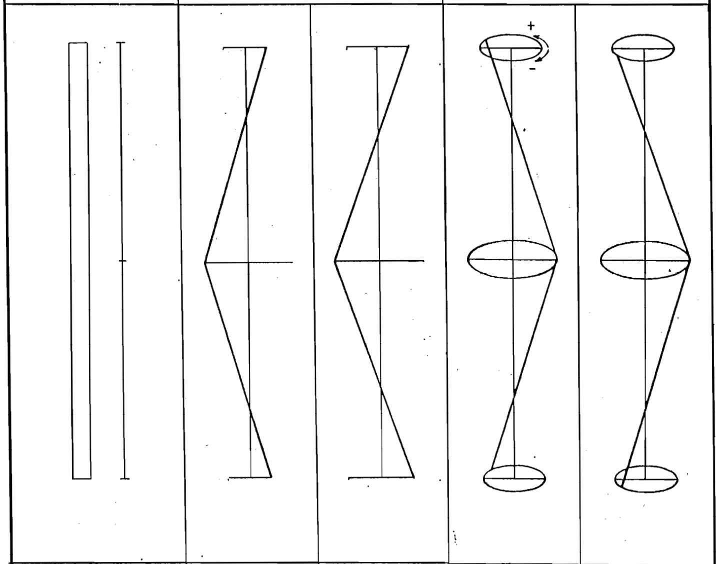


Fig.9 Root Loci

Proportional gain $\bar{K} = \frac{K_F \cdot K_P}{2}$ (N/m)	4700		5000	



FREQUENCY (HZ)	456.5	519.2	489.7	489.7
DAMPING (%)	0	0	−0.145	0.145

Fig.10 Stability Analysis Results

Table 1. Stability Analysis Results (L=570, l=70)

	$K_F \cdot K_P$ = 1000 (N/m)		$K_F \cdot K_P$ = 5000 (N/m)	
	Frequency (Hz)	ζ (-)	Frequency (Hz)	ζ (-)
1-Order	0.	±1.0	0.	±1.0
2-Order	45.6	0.	46.3	0.
3-Order	103.2	0.	150.0	0.
4-Order	107.0	0.	302.6	0.15
5-Order	142.1	0.	302.6	−0.15
6-Order	263.9	0.	351.1	−0.21
7-Order	408.3	0.	351.1	0.21
8-Order	600.6	0.	519.1	0.

Table 2. Stability Analysis Resuits (L=330, l=70, $K_F \cdot K_P$ =5000N/m)

	Frequency (Hz)	ζ (-)
1-Order	76.7	0.
2-Order	315.2	0.32
3-Order	315.2	0.32
4-Order	315.2	−0.32
5-Order	315.2	−0.32
6-Order	619.8	0.
7-Order	1253.2	0.