2nd International Symposium on Magnetic Beaing
July 12-14, 1990, Tokyo, Japan

STABILITY ANALYSIS FOR ROTORS SUPPORTED BY ACTIVE MAGNETIC BEARINGS

H. M. CHEN, D. WILSON, P. LEWIS, J. HURLEY
Mechanical Technology Incorporated
Latham, New York

Abstract

This paper presents an analytical method for tying active magnetic bearing (AMB) controller parameters to rotordynamic stability analysis. The controller, represented by its component transfer functions, is written into a subroutine appended to a finite-element rotordynamic program. The controller parameters can be modified until several lower system modes are stabilized. This method facilitates the controller design and shortens the tuning effort in installing rotor-AMB systems.

Introduction

Turbomachinery users and manufacturers are gradually accepting AMBs for the advantages they offer, i.e., elimination of the lubrication system and ability to control stiffness and damping. Stiffness and damping can be manipulated to desired values without any mechanical modifications by changing resistors and capacitors of an analog controller, or by changing equivalent parameters of a digital controller. However, the "desired values" are not yet well defined for these new rotor-AMB systems.

In conventional rotor systems with oil-film bearings, the stability of natural modes above operating speed is seldom a problem. However, it can be a controller design struggle to stabilize up to two or three of these modes in a rotor-AMB system. If the controllers are not properly tuned, it is not unusual for a rotor on AMBs to vibrate out of control during start-up. As reported in a recent conference*, a rotor was damaged during the installation and controller-tuning process. Tuning is a time-consuming process that involves adjusting and/or modifying the controller until the rotor-AMB system is stable and well damped at the lower modes.

Guiding the controller design with system stability analysis reduces the risk and effort of tuning. However, there is a lack of suitable computing tools for rotor-AMB systems. The stability programs developed during the last two or three decades for conventional bearings are awkward for use with rotor-AMB systems for the following reasons:

- AMB stiffness and damping are functions of rotor whirl frequency, not rotor speed.
- AMB reaction forces are proportional to displacements measured at rotor locations offset axially from the bearing. [1]**

To account for these unusual AMB features, Chen presented an extended state variable approach in which AMB dynamic behavior was represented by a set of first-order differential equations appended to the rotor equations of motion. [2] A different method is presented herein to facilitate the controller design.

Stability Analysis Formulation

Bearing force is defined as

$$F_b = -K_i\, G(S)\, Y_p + K_m\, Y_b \qquad (1)$$

where

F_b	=	AMB force exerted on rotor
K_i	=	Current stiffness
K_m	=	Magnetic stiffness
Y_b	=	AMB center displacement
Y_p	=	Displacement at AMB sensor
$G(S)$	=	Controller transfer function
S	=	Laplace variable.

The stiffnesses K_i and K_m are functions of bias (steady state) currents in the magnetizing coils and are key parameters in the linearized control of AMBs.

The transfer function G(S) represents a series of component transfer functions that are typically multiplied together as shown in Equation 2.

$$G(S) = G_1(S)\, G_2(S)\, G_3(S)\, G_4(S) \qquad (2)$$

where G_1, G_2, G_3, and G_4 are, for example, the transfer functions of the sensor, PID controller, phase compensator, and power amplifier/coils, respectively.

Additional filtering schemes, such as band-pass or band-reject (notch) filters, can be added to Equation 2 as transfer function multipliers.

A radial AMB has two identically controlled axes, each represented by Equation 1. The two axes are perpendicular to each other and are usually located at 45° angles from vertical to share the rotor weight. Therefore, the dynamic properties of the axes, i.e., stiffness and damping, can be made identical. Since AMBs are generally an order of magnitude less stiff than the conventional bearings and their pedestals or casings, for most turbomachinery applications it is adequate to assume circular rotor whirl orbits in the stability analysis. The resulting equations of motion of a rotor can be represented by Equation 3.

$$[M][\ddot{X}] + [C][\dot{X}] + [K][X] = \{F\} \qquad (3)$$

where

[M]	=	Rotor mass matrix; an assembly of beam element 4 x 4 consistent mass matrices, and concentrated masses and transverse moments of inertia. [3]
$\{X\}'$	=	$(Y_1, \theta_1, Y_2, \theta_2 \ldots\ldots)$; state vector
[C]	=	diag. $(0, -jI_{P1}\,\Omega, 0, -jI_{P2}\Omega \ldots\ldots)$; diagonal matrix containing only gyroscopic terms
j	=	$\sqrt{-1}$

*REVOLVE '89 - A Symposium On Dry Seals and Magnetic Bearings, September 26, 1989. Calgary, Canada.

**Numbers in brackets indicate references cited herein.

Y_i = Linear displacement of i^{th} station

θ_i = Angular displacement of i^{th} station

I_{pi} = Polar moment of inertia of i^{th} station

Ω = Rotating speed (rad/sec)

$[K]$ = Rotor stiffness matrix; an assembly of beam element 4×4 stiffness matrices, including the shear effect [3]

$\{F\}'$ = $(\text{---}0\text{---}F_{b1}\text{---}0\text{---}F_{b2}\text{---}0\text{---})$; forcing vector containing only bearing reactions for stability analysis

F_{bm} = Reaction of m^{th} AMB.

For a rotor model containing n stations, the above matrices and vectors would have an order of 2n.

Stability Analysis Method

Substituting the AMB forces of Equation 1 into the forcing vector {F} of Equation 3, and rearranging {F} to the left-hand side of Equation 3, a homogeneous system matrix equation is established for eigenvalue/eigenvector solution (damped system modes). Since each F_b term is a function of rotor whirl frequency, an iterative numerical scheme is required to obtain the solution. The steps in this scheme are:

1. Generate a conventional undamped natural frequency map at a fixed rotor speed. The AMBs are represented by incremental stiffnesses. At each stiffness, calculate the undamped natural frequencies. The natural frequencies for a given mode, calculated at different stiffnesses, are connected in a curve.
2. Superimpose the AMB stiffnesses as functions of whirl frequency on the map and locate the approximate system modal frequencies.
3. For each mode, begin by using these approximate frequencies as the rotor whirl frequency and calculate the corresponding stiffness and damping of the AMBs. Use these dynamic properties to calculate the damped system modes. If the assumed whirl frequency is equal to (or close to) the calculated damped frequency, a solution for one mode is obtained. If not, use the newly calculated frequency (or one close to it) as the new assumed whirl frequency and repeat this step.

In step 3, the AMB stiffness and damping coefficients at an assumed rotor whirl frequency are calculated through a program subroutine that implements a closed-form solution of Equation 2.

Numerical Example

Figure 1 shows a fan rotor supported by two radial AMBs and a thrust AMB. The rotor has a large fan wheel at one end. The radial AMBs, which are identical, each have 8 poles, 1-in. length, and a 2.5-in. journal outer diameter. Pertinent rotor and bearing data are listed on Figure 1. Dynamically, the AMBs are tuned to have approximately the stiffness and damping shown in Figure 2.

A stability analysis program was coded in Fortran with a static condensation option for saving computing time. [4] Twelve of the rotor model's 28 stations were taken as "active" stations, as indicated by the circled numbers in Figure 1.

The undamped natural frequencies at a rotating speed of 1800 rpm were calculated at bearing stiffnesses of 1×10^4, 4×10^4, and 1×10^5 lb/in. The natural frequency map for these calculations is presented in Figure 3. The superimposed AMB stiffness curve shows the first four forward modes as 55, 85, 250, and 480 Hz, respectively.

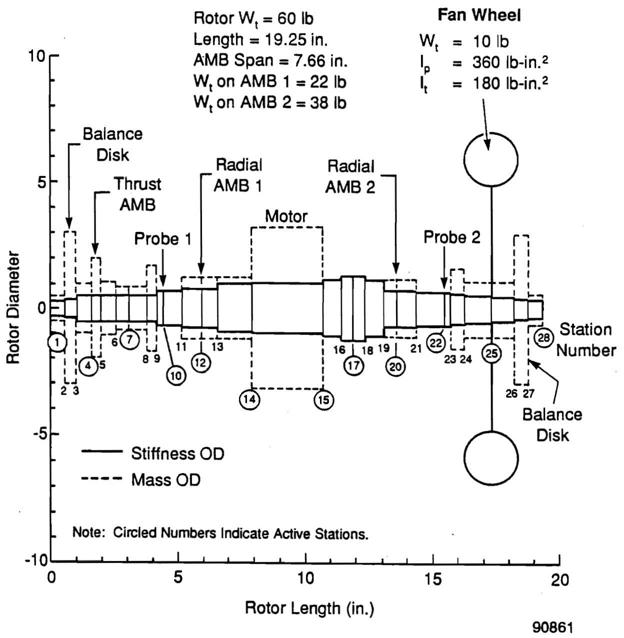

Fig. 1 Fan Rotor Model

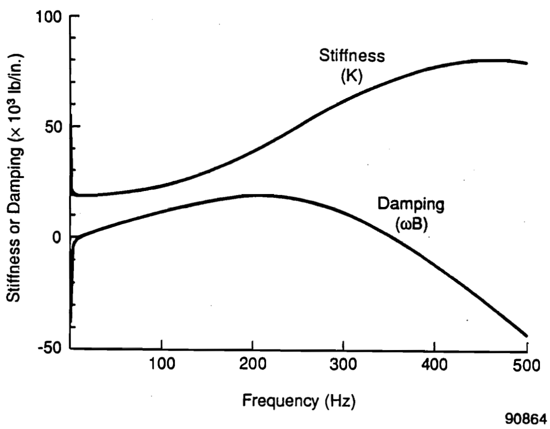

Fig. 2 AMB Stiffness and Damping

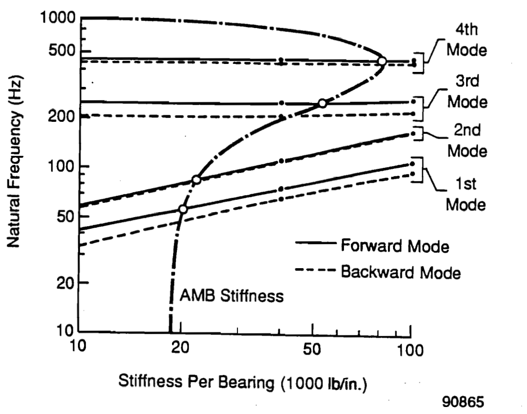

Fig. 3 Natural Frequency Map at 1800 rpm

The undamped mode shapes calculated at 4×10^4 lb/in./bearing are plotted in Figure 4. The mode shapes show that the displacements of an AMB and its associated probe have the same sign (i.e., positive value or negative value). Therefore, the probe locations are correct for the AMB control. The third mode has a node at one AMB, which could make controlling this mode difficult. These mode shapes provide a reference for evaluating the damped results.

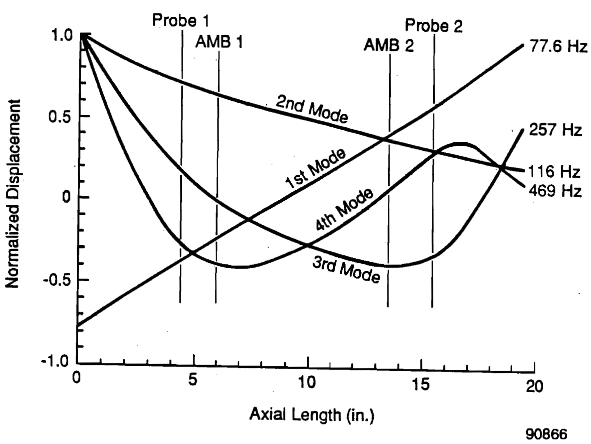

Fig. 4 Undamped Mode Shapes at 1800 rpm (4×10^4 lb/in./bearing)

To perform the damped analysis, a magnetic bearing subroutine that is applicable to both AMBs, MAGBRG, was written according to the control scheme in Figure 5. The subroutine's first two data statements contain all the essential control parameters and can be changed at the keyboard. The subroutine is compiled separately and is called by the main program. The subroutine Fortran statements are simple and straightforward.

Damped natural frequencies are plotted against assumed whirl frequencies with a 45° straight line superimposed. The log decrements of the calculated whirl modes are marked on the plot. Numerical iteration efforts to calculate the exact damped frequencies and the associated log decrements are not necessary as long as there are calculated points nearby. However, for cases of significant gyroscopic effect, this plot should be constructed twice — once at zero speed and once at full speed.

The results of searching the damped natural modes are presented in Figure 6. Only the forward modes are included as an example. The first two modes are well damped. This should be evident by examining their mode shapes in Figure 4 and the dynamic properties of Figure 2. The third mode (first bending) is very lightly damped because a node exists at one AMB. The fourth mode (second bending) is unstable. Although the rotating speed is low, the fourth mode is at a frequency of high stiffness and is likely to be excited.

To improve the damping of the two bending modes, a notch filter is placed to the left of each mode. As shown in Figure 7, these bending modes are riding on the right-hand slopes of the notches where large phase leads exist. Even with these large phase leads, the third mode log decrement is only slightly improved. The fourth mode, however, became stable.

Conclusions

Stability analysis is crucial to the design process for rotors supported by AMBs, since instability can be detrimental to these systems. Separating the dynamics of the AMB from the dynamics of the rotor during the analysis imposes uncertainty in the stability results and can lead to lengthy tuning time during system installation. This paper presents an analytical method for tying the AMB controller design directly to the rotor mechanical stability. The controller parameters can be readily modified until satisfactory log decrements are achieved for several lower modes. This method, when properly computerized, will take away the guesswork and in the future may make the installation tuning process unnecessary.

References

[1] KIRK, R. G., et al.: "Influence of Active Magnetic Bearing Sensor Location on the Calculated Critical Speeds of Turbomachinery," ASME, 12th Biennial Conference on Mechanical Vibration and Noise, Montreal, Canada, September 17-21, 1989.

[2] CHEN, H. M.: "Magnetic Bearing and Flexible Rotor Dynamics," STLE Annual Meeting at Cleveland, Ohio, May 9-12, 1988.

[3] ZORZI, E. S., and NELSON, H. D.: "Finite Element Simulation of Rotor-Bearing Systems with Internal Damping," ASME Paper No. 76-GT-89.

[4] COOK, R. D.: "Concepts and Applications of Finite Element Analysis," John Wiley and Sons, Inc., 1974, pp 242-48.

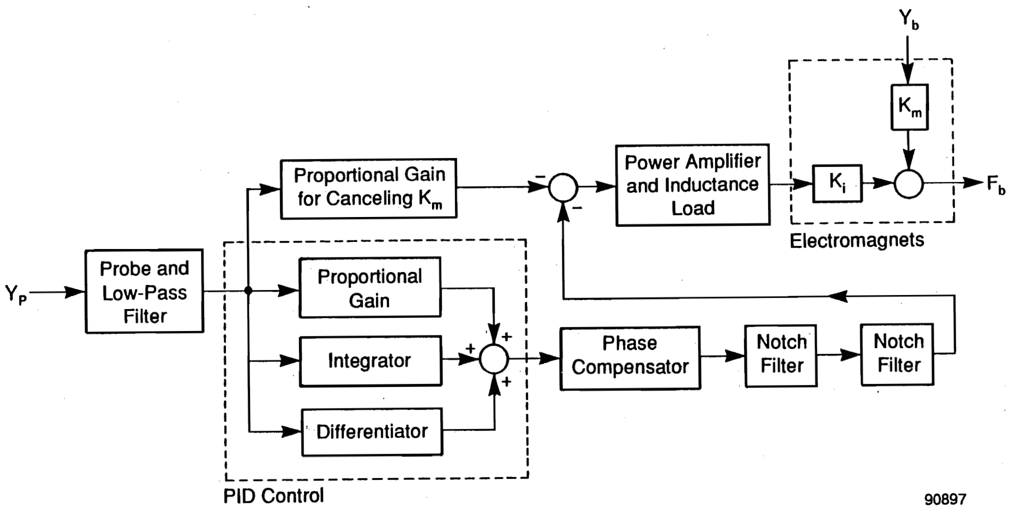

Fig. 5 Control Scheme Implemented in Subroutine MAGBRG

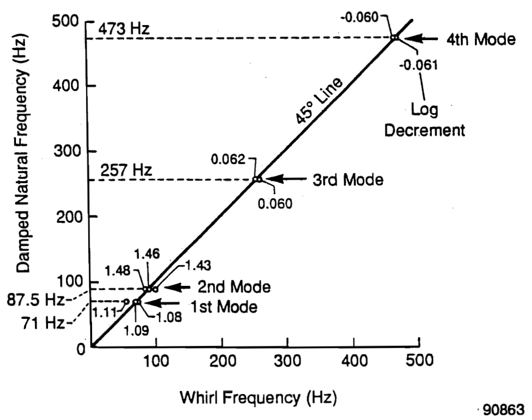

Fig. 6 Stability Analysis Results for Forward Modes at 1800 rpm

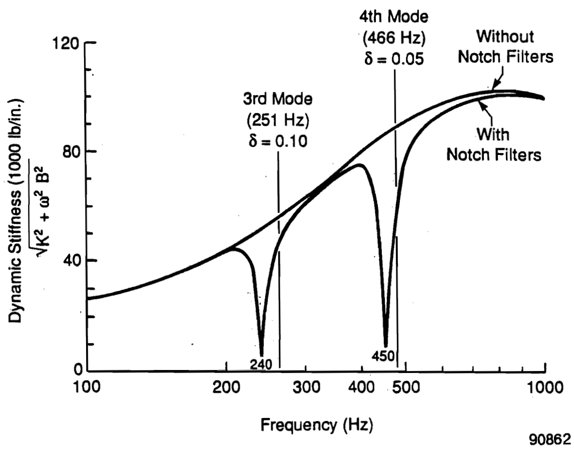

Fig. 7 Bending Mode Control Using Notch Filters