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LINEAR COMPENSATION FOR MAGNETIC BEARINGS

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Abstract

By employing magnetic attraction and feedback control of the rotor's levitating position, active magnetic bearings can suspend rotors without mechanical contact. However, magnetic attraction has a significant non-linear relationship vis-a-vis the air gap between the rotor and the electromagnet, and vis-a-vis the current which flows in the electromagnet. Also, the dynamic characteristics of magnetic attraction are influenced by the air gap. Therefore, in the presents of an external force, both the levitating position and current vary too greatly to obtain stable status. To solve this problem, the author proposes some compensation methods to compensate for the electromagnet's non-linear characteristics within the linear system.

1. Introduction

For recent mechanical systems, smaller size, lighter weight, higher speed, and higher accuracy have been required. Magnetic bearings are suitable to meet those requirements. A magnetic bearing suspends a rotor without mechanical contact by employing magnetic force. Since no other media are required, magnetic bearings can be used under special conditions such as high or low temperature, high pressure, or vacuum. They are used for turbo molecular pumps or machine tool spindles since they remain clean and free from wear, and maintenance is not necessary[1,..,6]. Also it is possible to use the active magnetic bearings at speeds which exceed the rotor bending mode's critical speed, since these bearings have variable bearing stiffness[7,8]. While the active magnetic bearings have the above-mentioned features, their magnetic attraction has significant non-linear characteristics. Generally, a control system was supposed to be a linear system, assuming that the air gaps and electromagnets' currents would vary slightly. Since most designs were based on this theory, it is often the case that a rotor would not stop vibrating once it exceeded the linear range.

Some ideas to compensate for non-linearity of the electromagnet have been considered[9,10,11]. Untill now, compensation has been required in order to compensate for non-linearity over a wide range of air gaps, without using a sensor except in the case where a displacement sensor was used for magnetic bearings.

In this paper, first of all, I will propose three methods to compensate for the stationary non-linearity of electromagnet. These three methods are called "bias method" (B method), "square root method" (R method), and "combination method" (C method), respectively[12]. These methods compensate for the proportional relationship between a command signal given by a magnetic bearing controller and the force acting on the rotor even if the air gap between the rotor and electromagnet is changed. Also, they have the function of holding constant the loop gain in the control system.

Next, a method to compensate for the dynamic non-linearity of electromagnet will be described. In this method, dynamic characteristics between a command signal given by a magnetic bearing controller and the force acting on the rotor are compensated for, i.e., they are kept constant, even if the air gap between the rotor and electromagnet is changed. Also, a function to hold constant the initial stability of the control system is available using this method.

Typical experiments of two stationary linearizing methods will be explained below.

2. Ordinary Methods

Fig.1 shows a model diagram of a push-pull type radial magnetic bearing. In this figure, a pair of electromagnets are provided in a vertical axis.

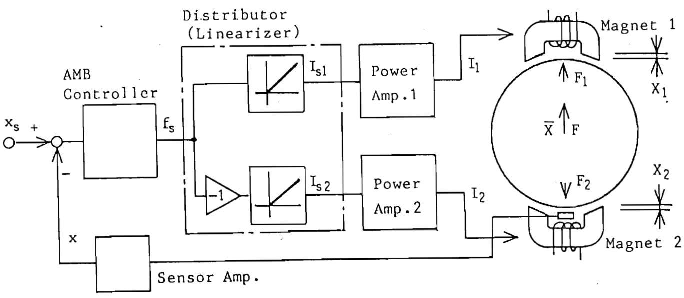


Fig.1 Magnetic Bearing Control Model

A control system is also shown in the same axis. However, in actuality, the identical magnets and control systems are provided in the horizontal axis as well. Magnetic attraction is generated by current supplied from a power amplifier to levitate a rotor in the center. The following equations are given for electromagnet 1(Subindex is omitted).

$$Rm=\frac{1}{A}\left(\frac{l}{\mu}+\frac{2X}{\mu_o}\right) \quad (1)$$

$$NI=Rm\Phi \quad (2)$$

$$F=\frac{\Phi^2}{\mu_o A} \quad (3)$$

F:attractive force
Φ:magnetic flux
N:number of coil turns
I:elecrtromagnet coil current
A:elecrtromagnet core area
Rm:magnetic reluctance
X:air gap
l:magnet core path length
μ_o:air permeability
μ:magnet core permeability

According to the above three equations, the relation between attractive force F, air gap X and current I will be as shown in Equation (4).

$$F=K\left(\frac{I}{X+X_{oo}}\right)^2 \quad (4)$$

$$X_{oo}=l\mu_o/2\mu, \quad K=\mu_o AN^2/4$$

X_{oo} is equivalent to the air gap, and it is determined by the magnet core path length(l). As such, X_{oo} is a constant determined by l. Since magnet core permeability is much greater than that of air, the Equation (4) can be approximated as follows:

$$F=K\left(\frac{I}{X}\right)^2 \quad (5)$$

In this equation, it is shown that the magnet has typical non-linear characteristics since its attractive force is in inverse proportion to the squared air gap. If both the gap and current change by dX and dI around a value X_o and I_o, and both are squared so as to be closely approximated, then this system can be considered as a linear system as shown in Equation (6).

$$F=K\frac{I_o^2}{X_o^2}\left(1+\frac{2}{I_o}dI+\frac{2}{X_o}dX\right) \quad (6)$$

Since power amplifier current has an equation $I_1=K_i I_{s1}$ (K_i : power amplifier conversion coefficient, I_{s1} : current command), there is no proportional relation except the linear relation between current command and attractive force. However, the relationship expressed by Equation (6) is not valid if either the air gap or current is changed greatly.

There have been some simple methods to compensate for non-linear characteristics of an electromagnet. Fig.2 shows one of these methods, in which bias is simply added to the distributor output stage shown in Fig.1. This method is called the ordinary bias method (OB method). In this configuration, there are these relations.

$I_1=K_i I_{s1}$, $I_{s1}=f_o+f_s$
$I_2=K_i I_{s2}$, $I_{s2}=f_o-f_s$
$F_1=K\left(\frac{I_1}{X_1}\right)^2$, $F_2=K\left(\frac{I_2}{X_2}\right)^2$
$X_1=X_o-\overline{X}$, $X_2=X_o+\overline{X}$
$F=F_1-F_2$
f_o:bias
fs:controller command signal
X_o:air gap in the center position
$\overline{X}$:deviation from X_o

Equation (7) is obtained from the above equations.

$$F=KK_i^2\left[\frac{(f_o+f_s)^2}{X_1^2}-\frac{(f_o-f_s)^2}{X_2^2}\right]$$
$$=4KK_i^2\frac{f_o^2}{X_o^2}\left(\frac{f_s}{f_o}+\frac{\overline{X}}{X_o}\right) \quad (7)$$

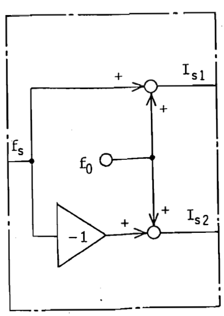


Fig.2 OB Method

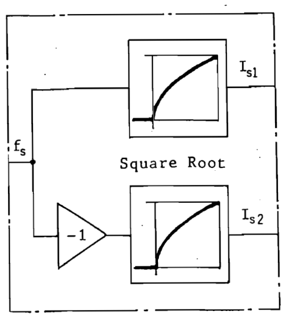


Fig.3 OR Method

When the value of $\overline{X}$ is close to 0, controller command signal fs and resultant force F acting on the rotor have a proportional relation by which a linear system is obtained. However, in this method, the linear relation cannot hold if a rotor deviates from the center position. In this OB method, bias f_o can be added by a distributor as shown in Fig.2. Another way to produce the same effect using the same OB method is to wind another coil around an electromagnet to apply constant current. Both methods have the same characteristics.

Fig.3 shows the second ordinary method, where a square root circuit is employed in the output stage of the distributor shown in Fig.1. This method is called the ordinary root method (OR method). In this method, Equation (8) is obtained as follows:

$$I_{s1}=\sqrt{f_s}\,(0=<f_s),\quad I_{s2}=\sqrt{-f_s}\,(f_s<0)$$
$$F=KK_i^2\frac{f_s}{X_1^2}\quad(0=<f_s)$$
$$F=KK_i^2\frac{f_s}{X_2^2}\quad(f_s<0)\qquad(8)$$

According to these equations, it can be seen that controller command fs and force F acting on the rotor have a proportional relation only when the rotor is levitating in the center. However, when the rotor deviates from the center, linearity is lost and stability will be reduced even in OB method.

As described above, when the two ordinary linearizing methods are used, controller command fs and force F acting on the rotor have a proportional relation only when the rotor is located in the center. It is possible to design and obtain (by using linear control system theory) a stable control system. However, when the rotor deviates a great deal from the center, linearity is lost and stable control is not available. Therefore, while a rotor is moving normally around the center, the stable control status could be retained. However, when large vibration is generated by external force or at startup, normal operation is very diffcult to achieve.

3.Proposed Methods

3.1 Bias Method (B Method)

Fig.4 shows the first method proposed in this paper. This method improves upon the OB method. A multiplier is provided in the output stage of the distributor shown in Fig.2. The most remarkable feature of this method is that a command signal added to the bias f_o is multiplied by another, separate signal which corresponds to the air gap between the electromagnet and the rotor. This air gap signal χ_1 or χ_2 is obtained by adding/subtracting a displacement sensor signal χ (which detects rotor deviation $\overline{X}$) to/from signal χ_o corresponding to air gap X_o in the center position.

The two power amplifier commands are :

$$I_{s1}=\chi_1(f_o+f_s),\quad I_{s2}=\chi_2(f_o-f_s)$$

When the displacement sensor's sensing factor is k_P, two air gap signals are

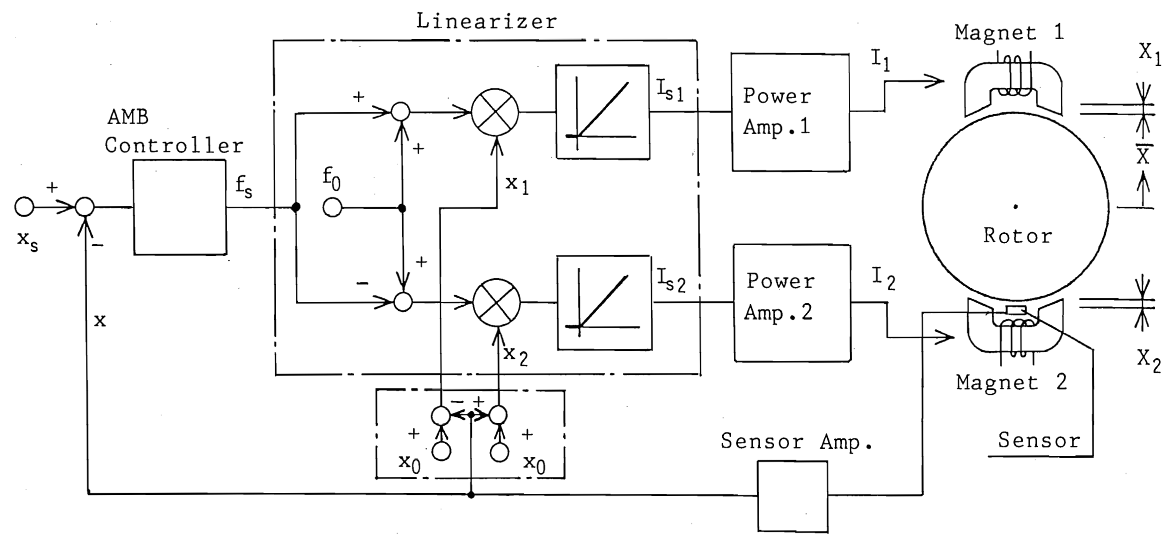


Fig. 4 AMB Block Diagram (B Method)

expressed : $\chi_1 = k_P X_1, \chi_2 = k_P X_2$
According to these relations, Equation (9) which shows the relation between controller command fs and force F is obtained,expressed as:

$$F = F_1 - F_2$$
$$= K[\{\frac{K_i \chi_1 (f_o + f_s)}{X_1}\}^2 - \{\frac{K_i \chi_2 (f_o - f_s)}{X_2}\}^2]$$
$$= 4KK_i^2 k_P^2 \{(f_o + f_s)^2 - (f_o - f_s)^2\}$$
$$= kf_s \quad (9)$$
$$(k = 4KK_i^2 k_P^2 f_o)$$

This equation does not have a variable rotor deviation $\overline{X}$, that is to say, this equation is effective with or without considering a rotor levitating position. Thus, stable controlability is assured wherever a rotor is located only if the control system can be stabilized in the center position. This is because loop gain in the control system is not changed even if the rotor levitating position is changed.

As described above, in this method, by using two electromagnets which are applied with current, the attractive force difference can be in proportion to the controller command value. Controllr command fs becomes 0 when external force does not affect a rotor. However, current is always applied to two electromagnets since bias is added in the linearizer. Therefore, in addition to Joule heat generated in the electrmagnets' coils, heat loss is generated by eddy current in a rotor when the rotor rotates at high speed.

Since two electromagnets are required, this method is more applicable to rotating machine bearings than to magnetic levitation systems.

3.2 Square Root Method (R Method)

Fig.5 shows the second method proposed in this paper, a diagram of a linearizer. This method improves upon the OR method. A multiplier is provided in the output stage of the distributor shown in Fig.3. Then a command signal from the square root circuit is multiplied by a signal which corresponds to the air gap. The square root circuit just mentioned is provided in place of a bias addition circuit as in the B method. The two power amplifier commands are:

$$I_{s1} = \chi_1 \sqrt{f_s} \quad (0 =< f_s)$$
$$I_{s2} = \chi_2 \sqrt{-f_s} \quad (f_s < 0)$$

From these equations and displacement sensor's sensing factor k_P, Equation (10) which shows the relation between conroller command fs and force F, can be obtained as follows:

$$F = K(\frac{K_i \chi_1 \sqrt{f_s}}{X_1})^2$$
$(0 =< f_s$
only electromagnet 1 works.)
$$= K(\frac{K_i \chi_2 \sqrt{-f_s}}{X_2})^2$$
$(f_s < 0$
only electromagnet 2 works.)
$$= kf_s \quad (10)$$
$$(k = KK_i^2 k_P^2)$$

Since this equation does not have a variable rotor deviation $\overline{X}$, it can be established regardless of the rotor levitating position. Therefore, once an initial setting is stabilized, stable control status can be assured.

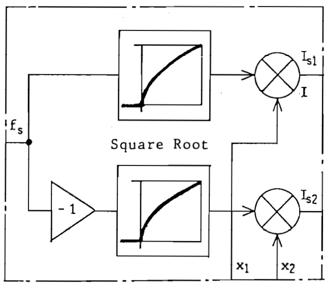


Fig.5 Linearizer in R Method

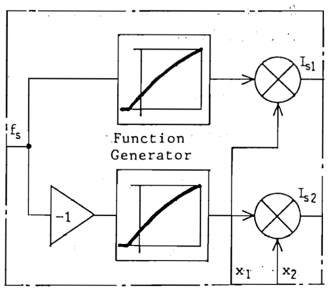


Fig.6 Linearizer in C Method

Because of the fact that two electromagnets are applied with current, B method cannot be employed for a bearing with only one electromagnet, while R method can be applied to bearings with either one or two electromagnets. When an external disturbance load is zero, the controller command and power amplifier command become zero and no current flows. Therefore, Joule heat becomes zero and heat loss will not be generated by eddy current in the rotor when the rotor rotates at high speed.

3.3 Combination Methd (C Method)

In B method, since current is always applied to two electromagnets, Joule heat cannot be avoided. Also heat loss is generated by eddy current in a rotor when the rotor rotates at high speed. In R method, due to a square root circuit, input-output ratio of the circuit becomes larger when the command is close to zero. A slight variation of the controller command value will change greatly the electromagnet coil's current. Thus, response of the force affecting a rotor becomes less when the controller command is close to zero because of electric circuit and electromagnet inductance. In other words, one of the B method features is that there is no problem with reduced response. The combination method (C method), combining B and R methods and with intermediate features, is slightly more effective. Fig.6 shows a linearizer for this method.

In C method, two current commands I_{s1} and I_{s2} are in accordance with the following equations, corresponding to controller command f_s:

$$
\begin{aligned}
&(1)\ -1 \leq f_s < -a: && I_{s1}=0 \\
& && I_{s2}=\chi_2\sqrt{-f_s} \\
&(2)\ -a \leq f_s < a: && I_{s1}=\frac{\chi_1\sqrt{a}}{2}\left(1+\frac{f_s}{a}\right) \\
& && I_{s2}=\frac{\chi_2\sqrt{a}}{2}\left(1-\frac{f_s}{a}\right) \\
&(3)\ a \leq f_s \leq 1: && I_{s1}=\chi_1\sqrt{f_s} \\
& && I_{s2}=0 \\
&(0<a<1)
\end{aligned}
\qquad (11)
$$

Fig.7 shows the relation between controller command f_s, and signal f_{s1} or f_{s2} which has not yet been multiplied by air gap signal χ_1 or χ_2. In the above-mentioned three ranges, (1) and (3) are exactly the same as the range of R method. While (2) is the same as that of B method. However, inclination is changed according to the size of value a. From these equations and displacement sensor's sensing factor kp, Equation (12) showing the relation between controller command fs and force F can be obtained as follows:

$$
F=F_1-F_2=kf_s \qquad (k=KK_i{}^2k_P{}^2) \qquad (12)
$$

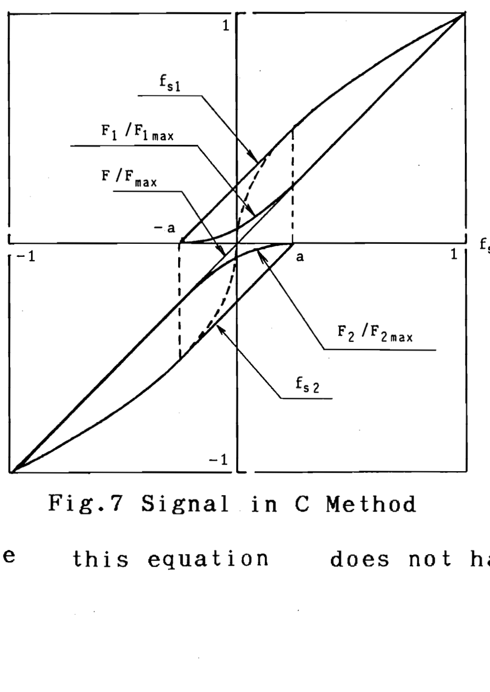


Fig.7 Signal in C Method

Since this equation does not have variable $\overline{X}$ as do the other two methods, it can be establised regardless of the rotor levitating position. Fig.7 also shows action forces F1, F2, and resultant force F.

In the above three methods (B, R, C methods), special sensors are not required. Apparently, by only employing displacement sensors used for magnetic bearings, we can, ipso facto, linearize magnet non-linearity. In any case, the most remarkable feature is that a multiplier is used to multiply command signal by each electromagnet's gap signal. However, to obtain a linearizer with higher accuracy, Equation (4) $X_{oo}=1\mu_o/2\mu$ must not be omitted. This equation can be established only by adding χ_{oo} to gap signal χ_1 or χ_2.

4 Experimental Results

An experiment to compare two of the proposed methods with the ordinary methods was performed. For this experiment, a magnetic bearing motor of 5-axis control type was used. Fig.8 shows a configuration of its radial bearing. A PID controller was used to control magnetic bearings. Each method mentioned above was applied to the same single axis. B method was applied to all the other 4 axes.

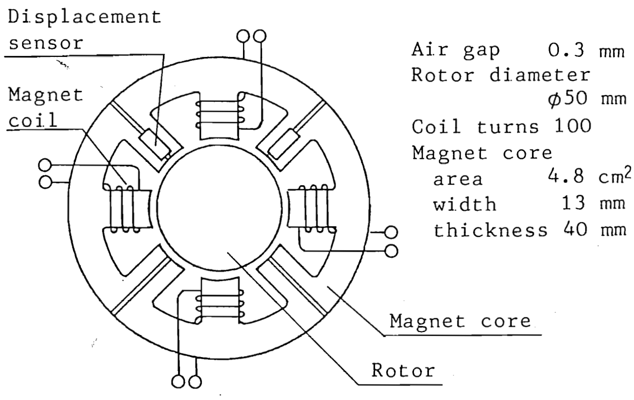
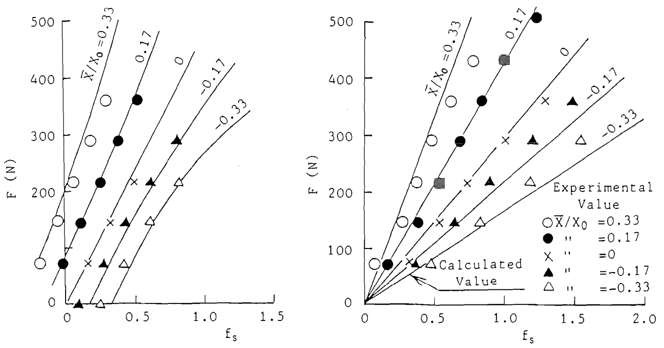


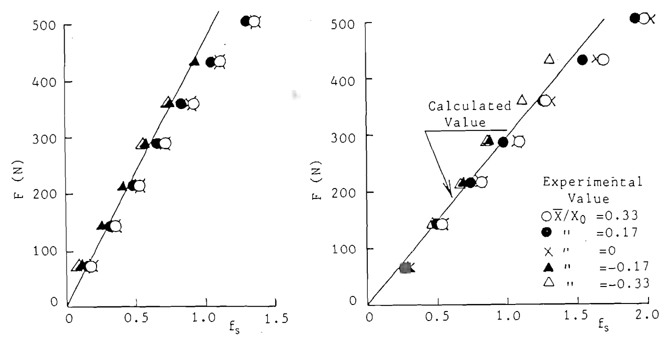
Fig.8 Radial Bearing Configuration

First, stationary characteristics were compared. Fig.9 shows a diagram where the relation between controller command fs and force F was measured using the ordinary methods.

In the OB method, when rotor deviation is zero ($\overline{X}=0$), action force F has a proportional relation with command fs. However, when the deviation is not 0, F does not pass the zero point and become non-linear. In the OR method, regardless of rotor deviation, F and fs have a proportional relation. However, eccentricity changes the ratio. Fig.10 shows the experiment results of two proposed methods (B method, R method), in the measurable range. In both cases, the relation between command fs and action force F is proportional. According to these results, great effect could be obtained by improving the OB method. On the other hand, improvement could not be expected for the OR method. The gradient in the B method which was approx. 1.6 times larger than that in the R method is given by setting bias f_o to 0.4. By setting f_o to 0.25, all gains can coincide.



(a)OB Method (b)OR Method
Fig.9 fs-F Characteristics in Ordinary Method



(a)B Method (b)R Method
Fig.10 fs-F Characteristics in New Method

Fig.11 shows the results of measured dynamic characteristics in new(B) and ordinary(OB) bias methods. Both new and ordinary square root methods have the same result as ①. In the OB method, gain becomes high at 40Hz and 1500Hz. The peak at 1500Hz is due to resonance with vibration peculiar to rotor bending. To lower this peak, loop gain is reduced to 80%. There is larger vibration around 35Hz as shown in ③, and stiffness is lowered.

In Fig.12, 10% recovery time after half the period is compared. In OR method, there is not a remarkable difference between when a large impulse is given and when the levitating position is eccentrized. Generally, difference by amplitude of impulse exciting force is small. However, the recovery time is shorter in the bias or square root methods proposed in this paper, which show us how well they are improved.

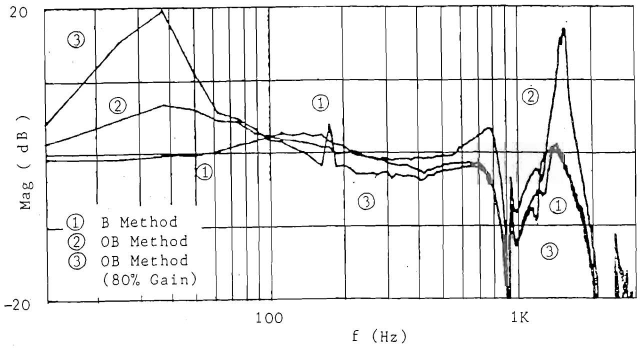


Fig.11 Bode Diagram of B and OB Method

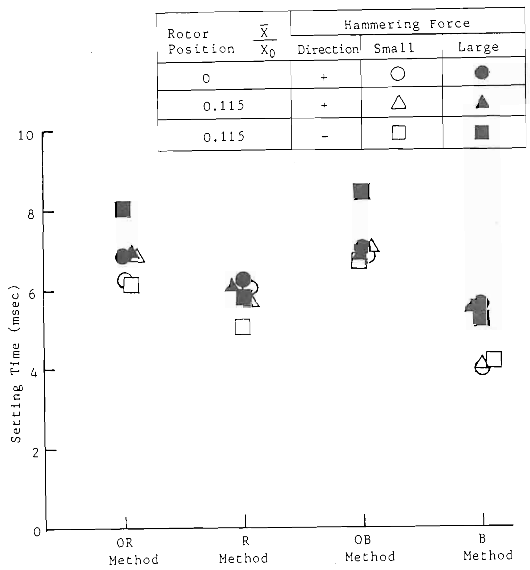


Fig.12 Setting time for each Methods (semi period 10%)

5. Applications

A few examples of actual applications of the proposed methods will be described. Fig.13 shows a magnetic levitation unit , developed by YASKAWA, to transport a semi-conducter wafer in a vacuum tank. To levitate a carrier, there are two ways:(1)only electromagnets for lifting are used or (2)electromagnets for lifting and small-size electromagnets for pulling down are used. 2nd way makes use of an overhang system. Both ways employ the R method. Before introduction of the R method, it had been impossible to levitate a carrier. By applying the R method, unstable control due to electromagnet non-linear characteristics could be solved and stable levitating became possible.

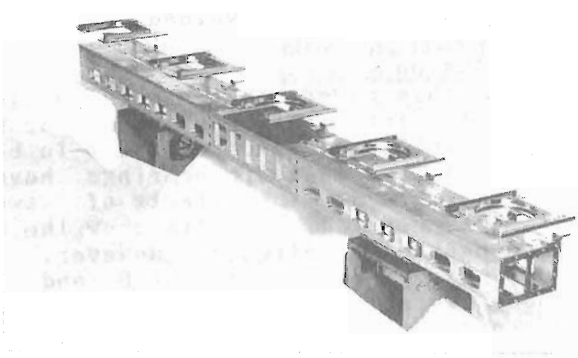


Fig.13 Magnetic Levitation Unit

A typical external force is applied when a bearing is activated. The following describes a state when a motor with 4- or 5-axis control magnetic bearings is started up. In the B or R method, stable levitating exists when a rotor is placed in the center position without any special circuits provided. Levitating without making any acoustic noise is possible especially in the R method. However, in the ordinary methods, any such levitating is very difficult to realize.

Another typical large external force is applied when a rotor rotates at high speed because of unbalance mass. This vibration is especially large as the rotor exceeds or passes through the critical speed of the rotor bending mode. In this case, special compensation circuits are required to function as dampers of the rotor bending mode. However, by using either the B or R method together with these special circuits, stability can be improved. When B method is used for an experiment where the critical speed of the rotor bending mode is approx. 90,000rpm (1,500Hz), the bearing section is sometimes skewed greatly (approx. 50μm) owing to unbalance mass. But bearing stability is always available[8]. However, in the OR methods, it is sometimes impossible to suspend the rotor when a rotor starts to make a large deviation.

6. Dynamic linear compensation

Stationary non-linear characteristics of an electromagnet could be compensated for by multiplying command signal by gap signals as mentioned above. It is also considered that electromagnet dynamic characteristics are compensated for since they are changed by the air gap and affected by control system conditions. Dynamic characteristics are changed since coil inductance varies according to the air gap. Therefore, by providing a compensation circuit to cancel such change, the characteristics can be compensated for.

In relation to electromagnets, the 2 following equations, in addition to Equations (1) through (3) are establised[9].

$$E=L\frac{dI}{dt}+RI \qquad (13)$$

$$E=N\frac{d\Phi}{dt}+RI \qquad (14)$$

E:electromagnet coil terminal voltage
L:electromagnet coil inductance
R:electromagnet coil resistance

From the above equations and Equation (2), Equation (15) is obtained.

$$L=\frac{2K}{X+X_{OO}} \qquad (15)$$

For better current rise and for smaller loss, power MOS-FET is used as a power amplifier in the PWM method to perform current feedback control. When coil-applied voltage is increased, the second term in Equation (13) can be omitted and the current rising rate becomes: $dI/dt=E/L$.
Assuming that current is applied in sine wave $I=I_O\sin 2\pi ft$ and that maximum response frequency fmax is provided at a point where maximum rising rate $|dI/dt|=2\pi fI_O$ coincides with rising rate E/L, then the equation $f_{max}=E/2\pi I_O L$ is obtained. By using this equation, when current time constant T is defined as $T=1/2\pi f_{max}$ Equation (16) can be obtained:

$$T=\frac{2\pi I_O L}{E}=k_L L=\frac{k_X}{X+X_{OO}} \qquad (16)$$

$$(k_L=2\pi I_O/E,\ k_X=4\pi KI_O/E)$$

From the above equation, time constant T is varied by the air gap and is more or less in inverse proportion to the air gap.

Assume that X_{OO} is one tenth of central gap X_O, rotor deviation is one half of X_O and the cut-off frequency of the rotor levitating in its central position is $f_c=1/2\pi T(X_O)$. In this case, phase and gain are varied according to rotor deviation in the range shown in Table 1. Phase and gain of magnetic attractive force will be larger since it is in proportion to the square of the current. Therefore, there is no problem when control system relative stability is large enough. Otherwise, instability may be created by gap variation.

f	$0.5f_0$	f_0	$1.5f_0$
Phase (deg)	-17~ -40	-32~ -59	-43~ -68
Mag/Mag(f=0)	0.95~0.77	0.85~0.51	0.73~0.37

Table.1

Fig.14 shows a block diagram of a compensator to compensate for variation of dynamic characteristics. In this circuit, the time constant of differentiator is changed according to air gap signal χ. The circuit is inserted in the stage before a power amplifier. One-side input of the differentiator is multiplied by coefficient k_3 to change the time constant variable. Coefficient k_3 is obtained by a function generator with an air gap signal as input signal and it(k_3) functions as in Equation (17):

$$k_3=\frac{k_1}{\chi+\chi_{oo}}-k_2 \quad (17)$$

$(k_1=k_x k_P/RC,\ k_2=r/R)$

r,R : resister resistance

C : capacitor capacity

The transfer function of configurated compensator is as follows:

$$\frac{I_{s1}^*}{I_{s1}}=\frac{1+(r+k_3R)CS}{1+rCS}=\frac{1+TS}{1+tS} \quad (18)$$

$(t=rC,\ T=(r+k_3R)C\ )$

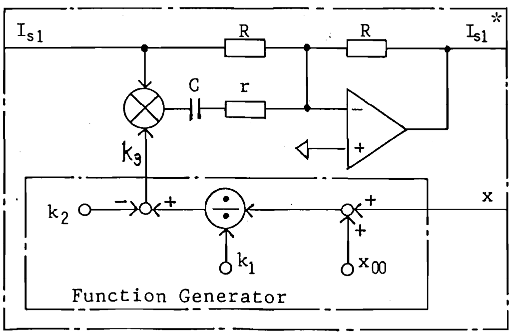


Fig.14 Dynamic Linearizer

By using displacement sensor's sensing factor k_P, we arrive at the equation $T=k_x/(X+X_{oo})$. Therefore, the transfer function between this compensator input signal I_{s1} and electromagnet current I_1 is as follows:

$$\frac{I_1}{I_{s1}}=\frac{K_i}{1+tS} \quad (19)$$

As a result, time constant t does not depend on rotor deviation $\overline{X}$ in the above equation. That is, dynamic characteristics are held constant. Therefore, by using this method, change of dynamic characteristics and unstable control system can be avoided.

7. Conclusion

In this paper, three methods to compensate for stationary non-linear characteristics of electromagnets to be used for active magnetic bearings have been described and the effects of two have been compared. Results show the B method had better effects. However, in actual experiments, both the B and R methods were found to be effective. Concerning compensation for varying dynamic characteristics caused by an air gap, only the method in Fig.14 is proposed. The results of all of the above methods will be checked by performing other tests.

The author anticipates the methods proposed in this paper will be employed widely and will continue to be developed for more practical uses.

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