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DESIGN of MAGNETIC BEARING CONTROLLERS BASED ON DISTURBANCE ESTIMATION

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Abstract

In order to minimize AC power dissipation in the electromagnet or vibratory force transmitted to the base through the bearing, two control methods are presented which use disturbance estimation generated by the observer for unbalance. One is designed to eliminate periodic component from control input asymptotically. By using this method, even an unbalance rotor can be suspended stably with fixed exciting current. The other is designed to eliminate oscillatory component from bearing force. This method enables the rotor to rotate about its principal axis. Another control method is presented which feeds back rate-plus-displacements of the center of mass and the principal axis of rotor. Such signals are obtained by compensating synchronous observatory disturbances by using the output of the observer. It is shown that the three control methods are equivalent if the bias magnetic flux is null.

1. Introduction

An unbalance on a rotor causes whirling motions of the rotor and alternating forces of the bearings in rotating machines. The authors have shown a magnetic bearing control system in which an unbalanced rotor can be suspended without whirling[1]. The concept of designing the control system was that the effects of unbalance were estimated by an observer and canceled by the electromagnetic forces of the bearing. With the control method applied, the position of the rotor can be maintained at the desired position with high precision. However, it is suitable for some machines to minimize the variation of exciting currents or vibratory force transmitted to the base through the bearing. In this paper, control methods are presented which can eliminate stationary alternating components from exciting currents or electromagnetic forces and keep them constant by using the output of the observer. Another control method is presented which feeds back the estimated signals of the displacement of the principal axis; such signals are obtained by compensating synchronous observatory disturbances by use of the output of the observer. It will be shown that the proposed three control methods are equivalent when the bias magnetic flux is null. Effectiveness of the proposed control methods are verified by simulation.

2. Modeling

A model, which is used for investigation of a typical totally active magnetic bearing system dynamics, is shown in **Fig.1.** Since the rotor is treated as a rigid body in this paper, it has six degrees of freedom of motion. In order to keep the rotor rotating about an fixed axis, the magnetic bearing has to control five degrees of freedom of motion. The eight electromagnets, which are numbered as ①,..., ⑧ in **Fig.1,** are used to control two translational motions and two rotational motions in the radial directions. The two electromagnets which are numbered as ⑨, ⑩ are used to control one translational motion in the axial direction.

To derive the equations of motion, a coordinate frame **O**-xyz fixed in space is defined as shown in **Fig.2;** the origin **O** corresponds to the center of the rotor **S** in the desired position and z-axis corresponds to its rotating axis.

The attractive force of the n^{th} magnet is represented as F_n. The directions of $F_1,\ldots,F_8$ are also shown in **Fig.2.** For small motions about the stationary, F_n can be approximated by a linear relation:

$$F_n=F_0-Gd_n+Hi_n \quad n=1,\ldots,8 \tag{1}$$

where

F_0: stationary force

G,H: coefficients of the linearized model of the magnet

i_n: incremental current flowing through the winding

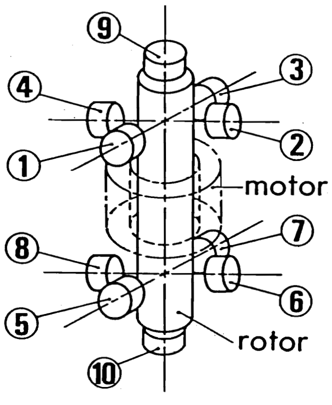

Fig.1 Model of a totally active magnetic bearing (①- ⑩ : electromagnets)

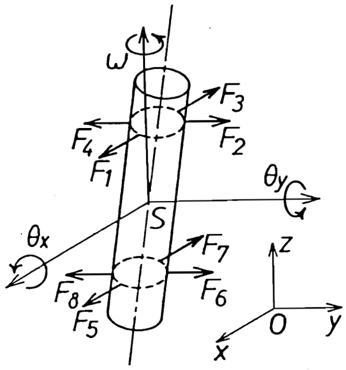

Fig.2 Coordinate axes and forces acting on the rotor

d_n:incremental gap between the rotor and the magnet

Each d_n is determined by the translational and rotational displacements of rotor.

When the rotor is driven to rotate at a constant speed ω, the equations of motion in the radial directions are given by[1]

$$\left.\begin{aligned} & m\ddot{x}_s-4Gx_s=H(i_1-i_3+i_5-i_7)+m\varepsilon\omega^2\cos(\omega t+\alpha) \\ & m\ddot{y}_s-4Gy_s=H(i_2-i_4+i_6-i_8)+m\varepsilon\omega^2\sin(\omega t+\alpha) \\ & I_r\ddot{\theta}_x+I_a\omega\dot{\theta}_y-4Gl^2\theta_x \\ & \quad =H(-i_2+i_4+i_6-i_8)l+(I_r-I_a)\tau\omega^2\cos(\omega t+\beta) \\ & I_r\ddot{\theta}_y-I_a\omega\dot{\theta}_x-4Gl^2\theta_y \\ & \quad =H(i_1-i_3-i_5+i_7)l+(I_r-I_a)\tau\omega^2\sin(\omega t+\beta) \end{aligned}\right\}(2)$$

where

- m:mass of the rotor
- I_a, I_r:polar and transverse mass moments of inertia of the rotor
- l:distance between the center of the rotor and the magnets
- α, β:parameters on angular location of static and dynamic unbalance
- ε:eccentricity of the rotor (amount of static unbalance)
- τ:angle between the rotational axis and the principal axis (amount of dynamic unbalance)
- x_s, y_s:displacements of the rotor center S in x and y directions
- θ_x, θ_y:angular displacements of rotor axis about x and y axes

It is clear from eq.(2) that the total system can be divided into two subsystems; one is related to translation and the other is related to rotation. In each subsystem the dynamics is expressed in the form[1,3]:

$$\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t)+\mathbf{D}\mathbf{w}(t) \quad (3)$$

$$\dot{\mathbf{w}}(t)=\mathbf{E}\mathbf{w}(t) \quad (4)$$

where

Table 1 Variables and Coefficients in each subsystem

symbol	subsystem related to translation	subsystem related to rotation
x_1	x_s	θ_x
x_2	y_s	θ_y
u_1	$i_1-i_3+i_5-i_7$	$-i_2+i_4+i_6-i_8$
u_2	$i_2-i_4+i_6-i_8$	$i_1-i_3-i_5+i_7$
w_1	$\varepsilon\omega^2\cos(\omega t+\alpha)$	$(1-c)\tau\omega^2\cos(\omega t+\beta)$
w_2	$\varepsilon\omega^2\sin(\omega t+\alpha)$	$(1-c)\tau\omega^2\sin(\omega t+\beta)$
a	$4G/m$	$4Gl^2/I_r$
b	H/m	Hl/I_r
c	0	I_a/I_r

$$\mathbf{x}=[x_1\ \dot{x}_1\ x_2\ \dot{x}_2]^T,\quad \mathbf{u}=[u_1\ u_2]^T,\quad \mathbf{w}=[w_1\ w_2]^T$$

$$A=\begin{bmatrix}0 & 1 & 0 & 0\\ a & 0 & 0 & c\omega\\ 0 & 0 & 0 & 1\\ 0 & -c\omega & a & 0\end{bmatrix}\quad B=\begin{bmatrix}0 & 0\\ b & 0\\ 0 & 0\\ 0 & b\end{bmatrix}\quad D=\begin{bmatrix}0 & 0\\ 1 & 0\\ 0 & 0\\ 0 & 1\end{bmatrix}$$

$$E=\begin{bmatrix}0 & -\omega\\ \omega & 0\end{bmatrix}$$

The variables and coefficients are defined as shown in **Table 1.**

It is remarked that the effects of unbalance are considered to be exogenous disturbances to the system

$$\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t) \quad (5)$$

and the disturbance model has the state form given by eq.(4).

3. Disturbance Estimation

Since it is difficult to measure the effects of unbalance $\mathbf{w}(t)$ in real time, we will construct an observer which asymptotically generates the disturbance state $\mathbf{w}(t)$ from measurable signals $\mathbf{x}(t)$ and $\mathbf{u}(t)$; later we will use the estimation for regulation control or disturbance compensation control.

According to the theory of observers, a minimal-order observer, which estimates $\mathbf{w}(t)$, is given by

$$\dot{\mathbf{z}}(t)=(\mathbf{E}-\mathbf{V}\mathbf{D})\mathbf{z}(t)+(-\mathbf{V}\mathbf{A}+\mathbf{E}\mathbf{V}-\mathbf{V}\mathbf{D}\mathbf{V})\mathbf{x}(t)-\mathbf{V}\mathbf{B}\mathbf{u}(t) \quad (6)$$

$$\hat{\mathbf{w}}(t)=\mathbf{z}(t)+\mathbf{V}\mathbf{x}(t) \quad (7)$$

where

$$\mathbf{z}=[z_1,\ z_2]^T,\quad \hat{\mathbf{w}}=[\hat{w}_1,\ \hat{w}_2]^T$$

$$\mathbf{V}=\begin{bmatrix} 0 & \sigma & 0 & \nu \\ 0 & -\nu & 0 & \sigma \end{bmatrix}$$

For convergence the parameter σ must satisfy

$$\sigma > 0 \tag{8}$$

4. Control Laws

In this chapter we will construct a control system in which the exciting currents or the attractive forces of the magnets converge to their set-point values although an unbalanced rotor is suspended stably. Another control method is also presented which feeds back the estimated signals on the motions of the center of mass and the principal axis instead of the center of configuration and the geometrical center axis.

Define complex variables as

$$\left.\begin{aligned} x&=x_1+jx_2 \\ u&=u_1+ju_2 \\ w&=w_1+jw_2 \\ z&=z_1+jz_2 \\ \hat{w}&=\hat{w}_1+j\hat{w}_2 \end{aligned}\right\} \tag{9}$$

and denote each Laplace-transformed variable by the capital. Since this paper focuses on stationary states, we will ignore initial conditions $\mathbf{x}(0)$ and $\mathbf{z}(0)$. Then by using these variables the dynamics of the control system is written as

$$X(s)=\frac{1}{t_o(s)}(bU(s)+W(s)) \tag{10}$$

$$W(s)=\frac{w(0)}{s-j\omega} \tag{11}$$

$$\hat{W}(s)=\frac{\sigma-j\nu}{s+\sigma-j(\omega+\nu)}W(s) \tag{12}$$

where the polynomial $t_o(s)$ is defined as

$$t_o(s)=s^2-jc\omega s-a \tag{13}$$

The initial condition w(0) can be represented as

$$w(0)=X_e\omega^2 \tag{14}$$

where

$$X_e=\varepsilon e^{j\alpha} \tag{15}$$

in the subsystem related to translation, and

$$X_e=(1-c)\tau e^{j\beta} \tag{16}$$

in the subsystem related to rotation. Moreover, we define f(t) as

$$f(t)=ax(t)+bu(t) \tag{17}$$

and x_b as

$$x_b(t)=x(t)-X_e e^{j\omega t} \tag{18}$$

or

$$X_b(s)=X(s)-\frac{W(s)}{t_n(j\omega)} \tag{19}$$

where

$$t_n(s)=s^2-jc\omega s \tag{20}$$

The variable f(t) represents force or moment acting on the rotor and $x_b(t)$ represents displacement of the center of mass or angular displacement of the principal axis.

For convenience we define [X] for a Laplace-transformed variable X(s) as the coefficient of $(s-j\omega)^{-1}$ when we expand X(s) into partial fraction form. If X(s) has $j\omega$ as a simple pole, [X] is given by

$$[X]=\lim_{s\to j\omega}(s-j\omega)X(s) \tag{21}$$

This quantity represents the amplitude and phase of sinusoidally varying component of x(t).

4.1 Current Regulation Control

If control input u(t) becomes null in each of the subsystems related to translation and rotation, the exciting currents of magnets will be automatically kept constant. Thus, in the following we treat the control input u(t) as a variable to be regulated.

The control strategy is to decompose the control u(t) into two parts,

$$u(t)=u_x(t)+u_w(t) \tag{22}$$

where $u_x(t)$ and $u_w(t)$ are chosen to stabilize a closed-loop system (5) and to eliminate sinusoidally varying component from the control input itself, respectively.

We give u_x in an linear feedback form[2]:

$$U_x(s)=-k(s)X(s) \tag{23}$$

where

$$k(s)=p_v s+(p_d-jp_c) \tag{24}$$

The feedback coefficients p_d, p_v and p_c are selected to satisfy the stability conditions:

$$(s1)(bp_d-a)bp_v+bp_c c\omega>0 \tag{25}$$

$$(s2)(bp_d-a)(bp_v)^2-(bp_c)^2+bp_v bp_c c\omega>0 \tag{26}$$

Next, we determine $u_w(t)$ as follows. First, we will find $U_w(s)$ such that

$$U(s)=0 \tag{27}$$

From eqs.(22) and (24), U(s) is written as

$$U(s)=-k(s)X(s)+Uw(s) \tag{28}$$

Substitution of eq.(10) into eq.(28) yields

$$U(s)=\frac{1}{t_c(s)}(-k(s)W(s)+t_o(s)U_w(s)) \tag{29}$$

Hence, the solution of eq.(27) is

$$U_w(s)=\frac{k(s)}{t_o(s)}W(s) \tag{30}$$

Second, replace W(s) by the estimate $\hat{W}(s)$ in the control law (30) as

$$U_w(s)=\frac{k(s)}{t_o(s)}\hat{W}(s) \tag{31}$$

From eqs.(12), (29) and (31),

$$\begin{aligned} U(s)&=-\frac{k(s)(s-j\omega)}{t_c(s)(s+\sigma-j(\omega+\nu))}W(s) \\ &=-\frac{k(s)w(0)}{t_c(s)(s+\sigma-j(\omega+\nu))} \end{aligned} \tag{32}$$

and from eq.(10) and (32),

$$X(s)=\frac{(s-j\omega)W(s)}{t_c(s)(s+\sigma-j(\omega+\nu))}+\frac{(\sigma-j\nu)W(s)}{t_o(s)(s+\sigma-j(\omega+\nu))} \tag{33}$$

Using eq.(32) to compute [U] yields

$$[U]=0 \tag{34}$$

Thus, the control input u(t) converges to zero. However, the second term in the right-hand side in eq.(33) shows that the closed-loop system falls into instability.

Third, in order to recover stability, we replace $t_o(s)$ by $t_o(j\omega)$ in the control law (31) as

$$U_w(s)=\frac{k(s)}{t_o(j\omega)}\hat{W}(s) \tag{35}$$

Then, substitution of eq.(35) into eq.(29) leads

$$U(s)=-k(s)\cdot\frac{(\sigma-j\nu)t_o(s)-(s+\sigma-j(\omega+\nu))t_o(j\omega)}{t_c(s)(s+\sigma-j(\omega+\nu))t_o(j\omega)}W(s) \tag{36}$$

Hence,

$$[U]=0 \tag{37}$$

From eqs.(10),(12),(29) and (35),

$$X(s)=[1+\frac{bk(s)(\sigma-j\nu)}{t_o(j\omega)(s+\sigma-j(\omega+\nu))}]\frac{W(s)}{t_c(s)} \tag{38}$$

This equation reveals that the rotor remains stably suspended and yields

$$[X]=-\frac{w(0)}{(1-c)\omega^2+a} \tag{39}$$

As a result, we can realize a stable suspension with asymptotical elimination of the sinusoidally varying component from the exciting currents by determining the control input as

$$U(s)=-k(s)(X(s)-\frac{\hat{W}(s)}{t_o(j\omega)}) \tag{40}$$

Then, the stationary components of f(t) and $x_b(t)$ are obtained as follows:

$$[F]=\frac{-aw(0)}{(1-c)\omega^2+a} \tag{41}$$

$$[X_b]=(\frac{1}{(1-c)\omega^2}-\frac{1}{(1-c)\omega^2+a})w(0) \tag{42}$$

In a magnetic bearing instrument using permanent magnet to provide the bias magnetic flux, DC power dissipation in the coil can be reduced to zero for steady load by the virtually zero power system[5]. In contrast with this, the proposed current regulation control enables zero AC power dissipation under periodically changing load due to unbalance.

4.2 Force Regulation Control If f(t) converges to zero in each subsystem, the electromagnetic forces will come to constant values. Thus, in the following we treat f(t) as a variable to be regulated.

From eqs.(9) and (17),

$$X(s)=\frac{1}{t_n(s)}(F(s)+W(s)) \tag{43}$$

From eqs.(17), (28) and (43),

$$F(s)=\frac{1}{t_n(s)}[(a-bk(s))W(s)+bt_n(s)U_w(s)] \tag{44}$$

This implies that F(s) becomes null if $U_w(s)$ is given by

$$U_w(s)=(k(s)-\frac{a}{b})\frac{W(s)}{t_n(s)} \tag{45}$$

We replace $W(s)$ by $\hat{W}(s)$ to realize the control law (45) and $t_n(s)$ by $t_n(j\omega)$ not to lose internal stability:

$$U_w(s)=(k(s)-\frac{a}{b})\frac{\hat{W}(s)}{t_n(j\omega)} \tag{46}$$

Then $F(s)$ is

$$F(s) =\frac{(bk(s)-a)}{t_c(s)}\left[\frac{(\sigma-j\nu)t_n(s)}{(s+\sigma-j(\omega+\nu))t_n(j\omega)}-1\right]W(s) \tag{47}$$

Hence,

$$[F]=0 \tag{48}$$

When the control input is given by eq.(46), the displacement $X(s)$ and control input $U(s)$ are obtained as follows:

$$X(s) =\left[\frac{(bk(s)-a)(\sigma-j\nu)}{t_n(j\omega)(s+\sigma-j(\omega+\nu))}+1\right]\frac{W(s)}{t_c(s)} \tag{49}$$

$$U(s) =\left[(k(s)-\frac{a}{b})\frac{(\sigma-j\nu)t_o(s)}{s+\sigma-j(\omega+\nu)t_n(j\omega)}-k(s)\right]\frac{W(s)}{t_c(s)} \tag{50}$$

Hence,

$$[X]=\frac{w(0)}{t_n(j\omega)}=X_e \tag{51}$$

$$[U]=-\frac{aw(0)}{bt_n(j\omega)} \tag{52}$$

From eq.(51),

$$[X_b]=0 \tag{53}$$

This implies that the rotor rotates about its the principal axis and the state of "automatic balancing"[4] is realized when force regulation control is applied.

4.3 Observatory disturbance compensation

We can design control system based upon a model that describes the motions of the gravity center and the principal axis instead of the geometrical center and axis. The motions, however, cannot be measured directly by the position sensors built in the magnetic bearing instrument. In this case the signals generated by the position sensors are considered to contain a sinusoidal disturbance due to configuration error. This observatory disturbance can be asymptotically eliminated by using the output of observer as

$$\hat{X}_b(s)=X(s)-\frac{\hat{W}(s)}{t_n(j\omega)} \tag{54}$$

We determine the control input to stabilize the motions of gravity center and principal axis by feedback compensation such as

$$U(s)=-k(s)\hat{X}_b(s) \tag{55}$$

Then, displacement $X(s)$, control input $U(s)$ and control force $F(s)$ are obtained as follows:

$$X(s)=\frac{1}{t_c(s)}\left[\frac{bk(s)(\sigma-j\nu)}{t_n(j\omega)(s+\sigma-j(\omega+\nu))}+1\right]W(s) \tag{56}$$

$$U(s)=\frac{k(s)}{t_c(s)}\left[\frac{t_o(s)(\sigma-j\nu)}{t_n(j\omega)(s+\sigma-j(\omega+\nu))}-1\right]W(s) \tag{57}$$

$$F(s) =\left[a-bk(s)\left(1-\frac{t_n(s)(\sigma-j\nu)}{t_n(j\omega)(s+\sigma-j(\omega+\nu))}\right)\right]\frac{W(s)}{t_c(s)} \tag{58}$$

Hence,

$$[X]=(1+\frac{a}{t_c(j\omega)})\frac{w(0)}{t_n(j\omega)} \tag{59}$$

$$[U]=-k(j\omega)\frac{aw(0)}{t_c(j\omega)t_n(j\omega)} \tag{60}$$

$$[F]=\frac{aw(0)}{t_c(j\omega)} \tag{61}$$

From (19) and (59),

$$[X_b]=\frac{aw(0)}{t_c(j\omega)t_n(j\omega)} \tag{62}$$

In this control system the rotor does not rotate around its principal axis and both the exciting current and bearing force vary sinusoidally unless the parameter a is equal to zero.

4.4 Comparison of the control methods Using real-valued variables the control input is given

Table 2 Data for numarical simulation

controlled object		designed controller	
a	1.00	p_d	2.73
b	1.00	p_v	2.34
c	0.02	p_c	0
ω	1.00	σ	0.25
X_e	0.98+j0	ν	0
		z(0)	**0**

by

$$\mathbf{u}(t)=\mathbf{P}_1\mathbf{x}(t)+\mathbf{P}_2\hat{\mathbf{w}}(t) \tag{63}$$

where $\mathbf{P}_1$ is

$$\mathbf{P}_1=-\begin{bmatrix} p_d & p_v & -p_c & 0 \\ p_c & 0 & p_d & p_v \end{bmatrix} \tag{64}$$

and $\mathbf{P}_2$ is

(1)current regulation control

$$\mathbf{P}_2=\frac{1}{(1-c)\omega^2+a}\begin{bmatrix} p_d & -\omega p_v+p_c \\ \omega p_v-p_c & p_d \end{bmatrix} \tag{65}$$

(2)force regulation control

$$\mathbf{P}_2=\frac{1}{(1-c)\omega^2}\begin{bmatrix} p_d-\frac{a}{b} & -\omega p_v+p_c \\ \omega p_v-p_c & p_d-\frac{a}{b} \end{bmatrix} \tag{66}$$

(3)observatory disturbance compensation

$$\mathbf{P}_2=\frac{1}{(1-c)\omega^2}\begin{bmatrix} p_d & -\omega p_v+p_c \\ \omega p_v-p_c & p_d \end{bmatrix} \tag{67}$$

It is clear from eqs.(65),(66) and (67) that the above mentioned three control methods are identical if the parameter a equals zero. This parameter represents the degree of instability caused by the bias magnetic field that provides a linear control force-current characteristics. We can realize a magnetic bearing with zero-bias field by compensating nonlinearity of the characteristics. Then, an unbalanced rotor can be suspended stably with fixed exciting currents and bearing forces.

5. Numerical Simulation and Analysis on Frequency Characteristics

To confirm the effectiveness of the proposed control methods, several numerical simulations are carried out. The values of the parameters used in the simulations are listed in **Table 2**; they represent normalized values that are obtained by describing the system equation by dimensionless variables[1,2,3].

Figures 3 and **4** show responses when the current regulation control and the force regulation control are applied; each regulation control start at t=0 and the stationary response of the system using the simple feedback control

$$\mathbf{u}(t)=\mathbf{P}_1\mathbf{x}(t) \tag{68}$$

is shown for t<0. Figure 3 shows that with the convergence of estimation sinusoidally varying component of control input disappears in the current regulation control system. The same is said of bearing force in the force regulation system (Fig.4).

Next, we show several numerical studies on the performance of the system using the simple feedback control or using alternatively the current regulation control and the force regulation control in add to (68). We design the controllers for ω=1.0 (**case a**) and 2.0 (**case b**). Other design parameters pd, pv, pc, σ and ν are set to be same as used in the simulation.

The amplitudes of displacement, control input and bearing force at each rotational speed are shown in **Fig.5** (case a) and **Fig.6** (case b);

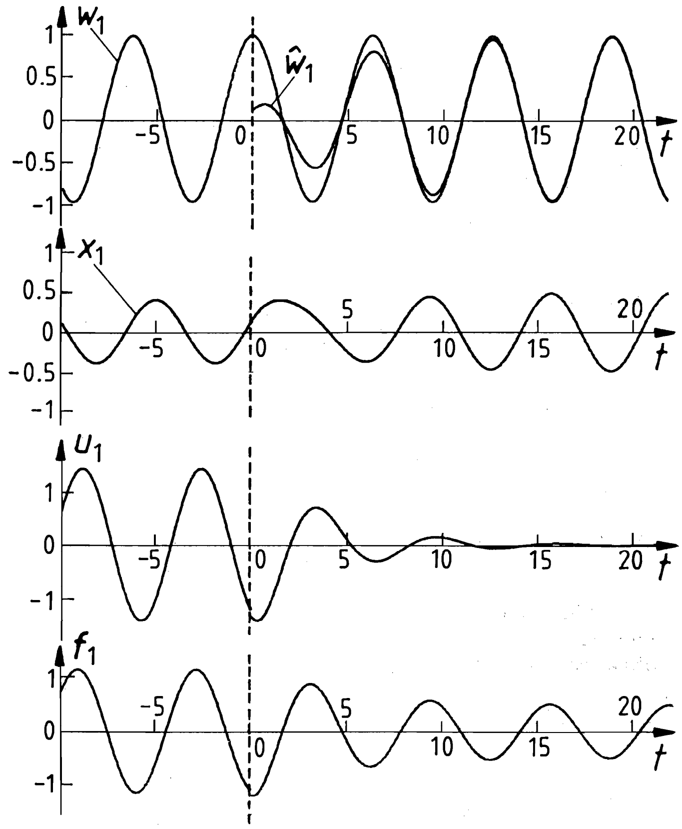

Fig.3 Response when the current regulation control is applied at t=0

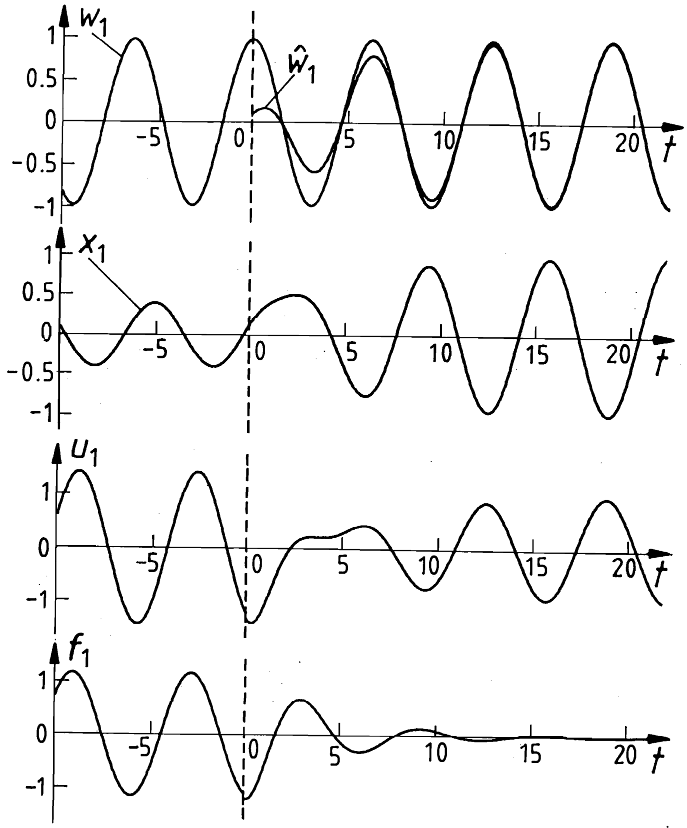

Fig.4 Response when the force regulation control is applied at t=0

it is to be remarked that in both cases the parameters of controller are fixed although the rotational speed changes. We can see again that at the designed frequency control input and bearing force become null in the current and force regulation control systems, respectively. It is seen that the performance of the system using the current regulation control becomes very similar to that of the system using the force regulation control when they are designed for a high rotational speed.

6. Conclusions

The possibility of suspending a rotor without any synchronous variation in the exciting current or bearing force in a magnetic bearing has been shown. Whereas the virtually zero power system enables zero DC power dissipation in the coil under steady load, the proposed current regulation control enables zero AC power dissipation under periodically changing load due to unbalance. When the force regulation control is applied, the rotor is rotating about its inertial axis. When the bias magnetic field can be reduced to zero, the exciting current and force of electromagnet are to be regulated at the same time. In this case the observatory compensation control yields the same result.

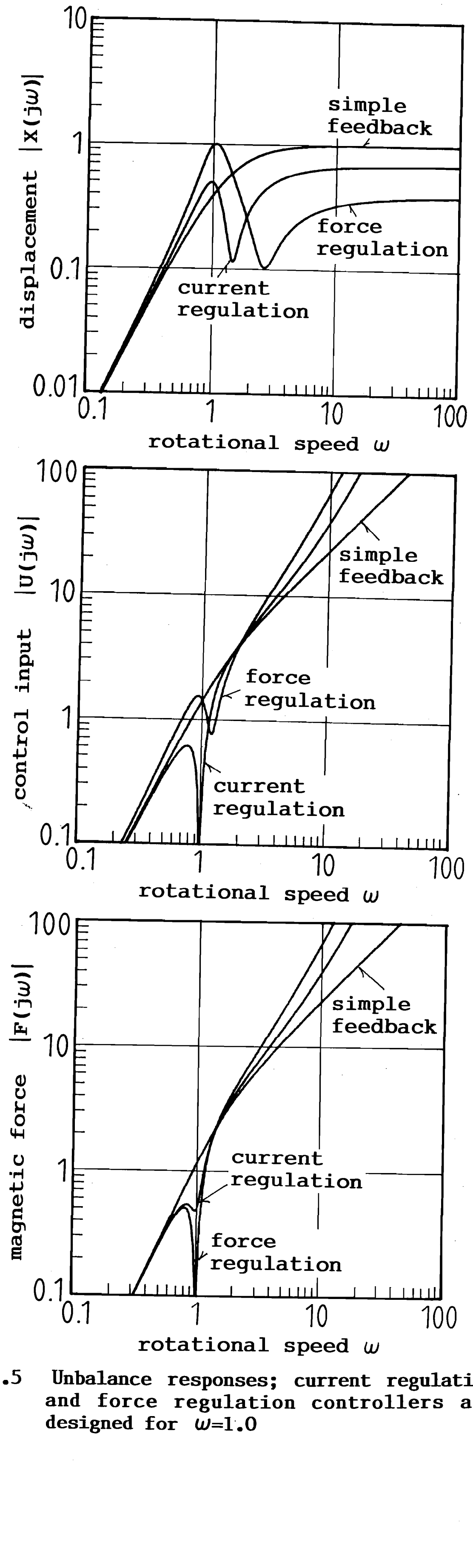

Fig.5 Unbalance responses; current regulation and force regulation controllers are designed for ω=1.0

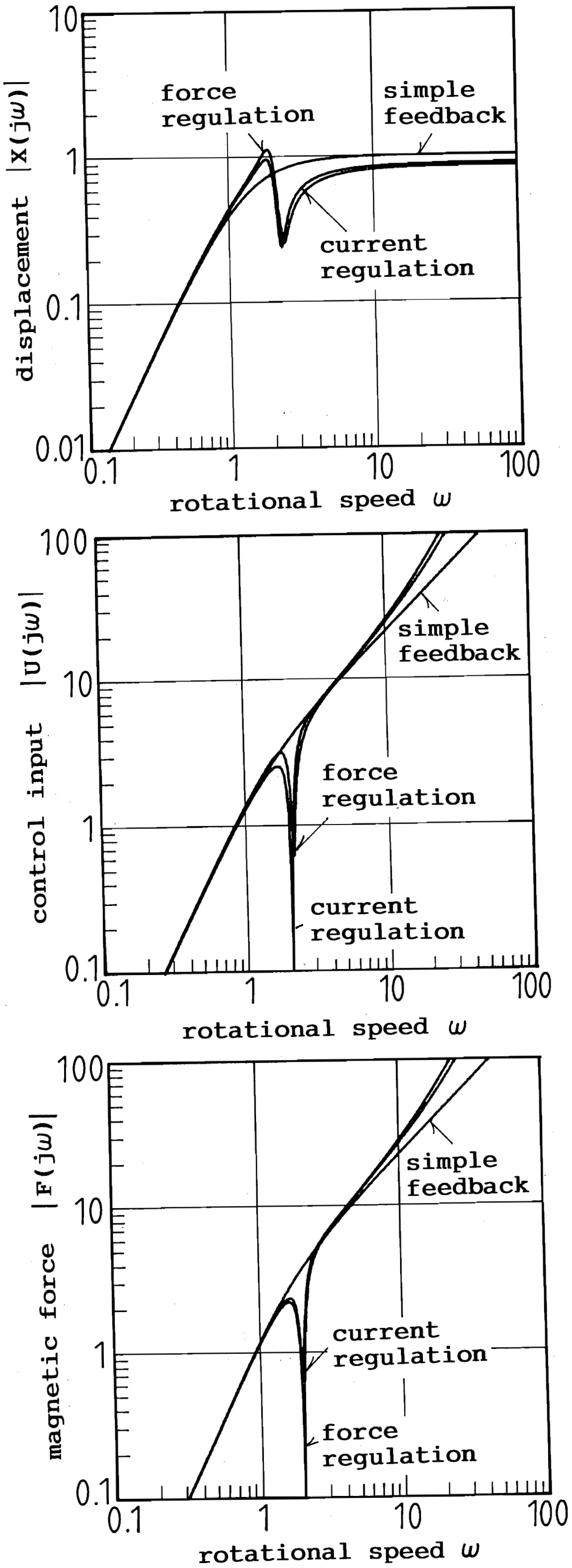

Fig.6 Unbalance responses; current regulation and force regulation controllers are designed for ω=2.0

Since we have shown the control methods based on the same state equation and observer that was used in our previous works, it will be possible to apply the developed methods of analysis on parameter variations and modification of the structure of controller[3].

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