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Modeling and Control of Magnetic Bearing Systems Achieving a Rotation Around the Axis of Inertia

Fumio Matsumura, Masayuki Fujita, and Kenji Okawa.
Dept. of Electrical and Computer Eng., Kanazawa University
2-40-20 Kodatsuno, Kanazawa 920, Japan

Abstract

This paper deals with the problem of an unbalance vibration in magnetic bearings. A rigorous modeling of a magnetic bearing system to achieve a rotation of the rotor around its axis of inertia is made. Then, an effective control scheme is proposed based on the state-space approach. The objective of the control scheme is to construct a controller which preserve internal stability and reject the disturbances. To solve this problem, output regulator with internal stability are introduced. The effectiveness of the proposed controller is demonstrated by a numerical example.

1. Introduction

Magnetic bearing systems have become practical in many industrial fields and numbers of studies for magnetic bearing systems have been reported. Especially, active control type magnetic bearing systems have been studied because of their several attractive features.

On the other hand, two critical problem exist in a application of the magnetic bearing systems. One is the problem of the interference caused by gyroscopic effect and the other is the problem of the vibration caused by the unbalance on the rotor. We deal with the latter problem.

In this paper, to solve this problem, a rotation of the rotor around the inertial axis is considered. First, a rigorous modeling of a magnetic bearing system in which the rotation of the rotor is on its axis of inertia is developed. Next, an effective control scheme is proposed based on the state-space approach. Finally, the effectiveness of the proposed controller is demonstrated by a numerical example.

2. Modeling

In this section, we derive the state equation of a magnetic bearing system in which the rotation of the rotor is on its inertial axis with the following assumptions:

(1) The rotor is rigid body.

(2) The unbalance is in the radial direction only.

(3) The unbalance is arbitraryly small such that the rotor rotates around the axis of inertia within a region that nonlinearity is negligible.

In this study, our interest is concerned in the relatively small motion about the equilibrium point and in the control problem of the radial direction.

Unbalance

The coordinates of the rotor X_r, Y_r, Z_r , called inertial axes, are set as shown in Fig. 1. ε is the distance between the center and the center of mass of the rotor, and τ is the angle of the inertial axis to the geometrical axis X_f. Fig. 2 shows the relationship between the coordinates of the stator and the rotor. The position of the center and the center of mass of the rotor are given as $F(x_f, y_f, z_f)$ and $G(x_r, y_r, z_r)$, respectively. When the rotor rotates around the inertial axis inclining at an angle of τ, the following equations hold in the relationship between the center and the center of mass and in the relationship between the geometric axis and the inertial axis, respectively.

$$\begin{bmatrix} y_f \\ z_f \end{bmatrix} = \begin{bmatrix} y_r \\ z_r \end{bmatrix} + \begin{bmatrix} \varepsilon\cos(\phi+\kappa) \\ \varepsilon\sin(\phi+\kappa) \end{bmatrix} \qquad (1)$$

$$\begin{bmatrix} \theta_f \\ \psi_f \end{bmatrix} = \begin{bmatrix} \theta_r \\ \psi_r \end{bmatrix} + \begin{bmatrix} \tau\cos(\phi+\lambda) \\ \tau\sin(\phi+\lambda) \end{bmatrix} , \qquad (2)$$

where κ and λ are initial value. These two equa-

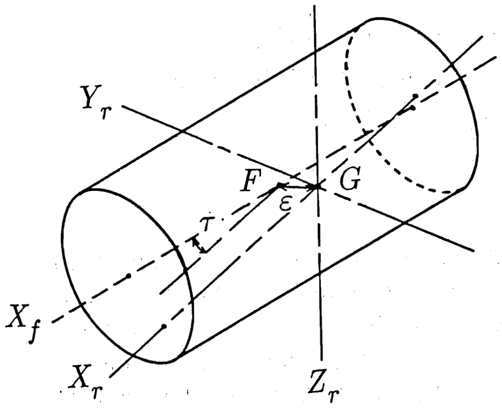

Fig.1 Coordinates of rotor

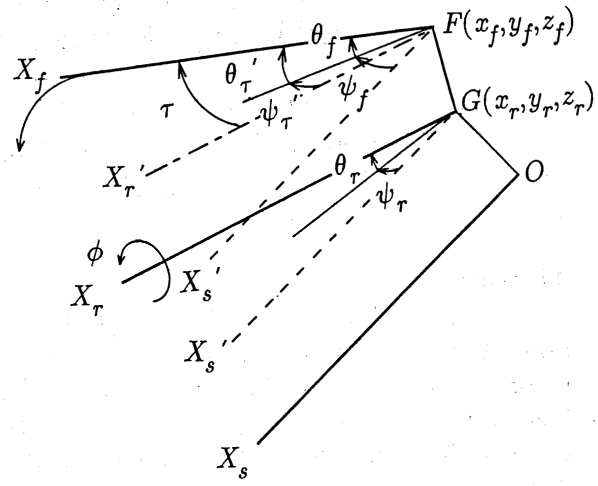

Fig.2 Relation between coordinates of stator and rotor

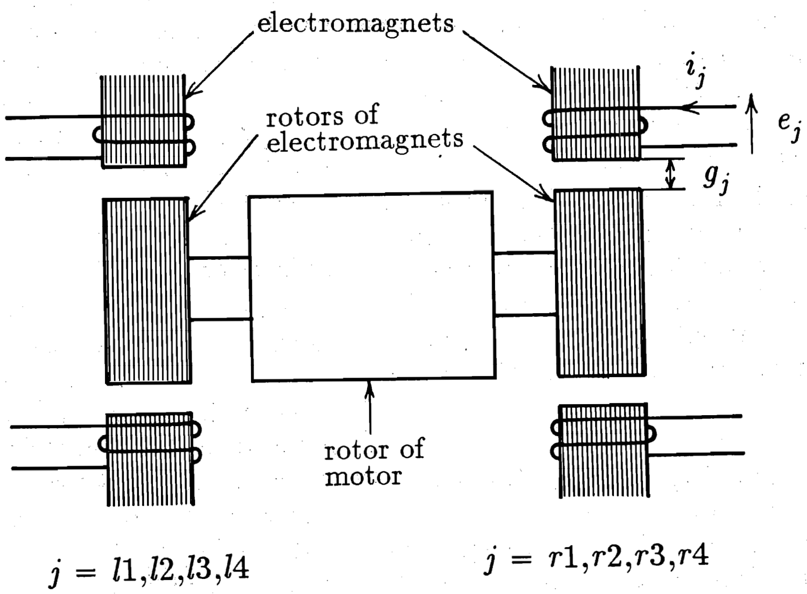

Fig.3 Structure and variables of magnetic bearing systems

tions are arranged in a vector form by writing

$$x_f = x_r + w. \tag{3}$$

where

$$x_f = \begin{bmatrix} y_f \\ z_f \\ \theta_f \\ \psi_f \end{bmatrix}, \quad x_r = \begin{bmatrix} y_r \\ z_r \\ \theta_r \\ \psi_r \end{bmatrix}, \quad w = \begin{bmatrix} \varepsilon\cos(pt+\kappa) \\ \varepsilon\sin(pt+\kappa) \\ \tau\cos(pt+\lambda) \\ \tau\sin(pt+\lambda) \end{bmatrix}.$$

Equations of Motion

The dynamical equations of the magnetic bearing systems in which the unbalance of the rotor is taken into consideration are given as follows

$$m\begin{bmatrix}\ddot{y}_r\\ \ddot{z}_r\end{bmatrix} = \alpha\begin{bmatrix}y_f\\ z_f\end{bmatrix}+\begin{bmatrix}f_{l3}-f_{l4}+f_{r3}-f_{r4}\\ f_{l2}-f_{l1}+f_{r2}-f_{r1}+mg\end{bmatrix}+\begin{bmatrix}f_{dy}\\ f_{dz}\end{bmatrix}+\alpha l_m\begin{bmatrix}-\psi_f\\ \theta_f\end{bmatrix} \quad (4)$$

$$J_y\begin{bmatrix}\ddot{\theta}_r\\ \ddot{\psi}_r\end{bmatrix} = pJ_x\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}\dot{\theta}_r\\ \dot{\psi}_r\end{bmatrix}+\begin{bmatrix}(f_{l1}-f_{l2})\,l_l-(f_{r1}-f_{r2})\,l_r\\ (f_{l3}-f_{l4})\,l_l-(f_{r3}-f_{r4})\,l_r\end{bmatrix}$$

$$+l_f\begin{bmatrix}-f_{dz}\\ f_{dy}\end{bmatrix}+\alpha l_m\begin{bmatrix}z_f\\ -y_f\end{bmatrix}+\alpha l_m^2\begin{bmatrix}\theta_f\\ \psi_f\end{bmatrix} \quad (5)$$

m : mass of the rotor
p : rotational speed of the rotor
J_x : moment of inertia about X_r
J_y : moment of inertia about Y_r or Z_r
l_r, l_l : length between the center of mass and the point of the force by the radial electromagnet of the right or left hand
f_{ri}, f_{li} : attractive force by each electromagnet of the right or left hand, i = 1 ~ 4
α : coefficient of the attractive force by the induction motor due to the shift of the rotor in the radial direction

Dynamical Equations in Electromagnet

Fig.3 shows the principle of the magnetic bearing systems. We use subscripts ri (i=1 ~ 4) , li (i=1 ~ 4) to denote each electromagnet. Letter e_j ($j=ri$ or li) represents the voltage applied to each electromagnet. Similarly, i_j and g_j denote the current flowing into each electromagnet coil and the gap length between the rotor and each electromagnet, respectively. Using a linearization, the forces applied on the rotor by each electromagnet are written as follows in a matrix form in terms of the vector g_f and the vector i.

$$f = C_2g_f+C_3i = C_2C_1x_f+C_3i \quad (6)$$

where $g_f = [\,g_{l1}'\ g_{r1}'\ g_{l3}'\ g_{r3}'\,]^T$, $i = [\,i_{l1}'\ i_{r1}'\ i_{l3}'\ i_{r3}'\,]^T$, where letters with a prime denote a variation about the equilibrium point (cf.appendix). Substituting (3) into (6) yields

$$f = C_2C_1x_r+C_3i+C_2C_1w. \quad (7)$$

In an electrical circuit including the electromagnet coil, the following dynamical equation holds.

$$\frac{d}{dt}i = -\frac{R}{L}Ii+\frac{1}{L}Ie \quad (8)$$

where

$$e = [\,e_{l1}'\ e_{r1}'\ e_{l3}'\ e_{r3}'\,]^T,$$

R :resistance of the electromagnet coil
L :inductance of the electromagnet coil

State Equation

Substituting (3), (7) and (8) into (4), we obtain

$$\begin{bmatrix}\dot{x}_r\\ \ddot{x}_r\\ \dot{i}\end{bmatrix} = \begin{bmatrix}0 & I & 0\\ A_1+B_1C_2C_1 & A_2P_1 & B_1C_3\\ 0 & 0 & (-R/L)I\end{bmatrix}\begin{bmatrix}x_r\\ \dot{x}_r\\ i\end{bmatrix}$$

$$+\begin{bmatrix}0\\ 0\\ (1/L)I\end{bmatrix}e+\begin{bmatrix}0\\ A_1+B_1C_2C_1\\ 0\end{bmatrix}w \quad (9)$$

(cf.appendix). Applying the coordinate transformation

$$g_r = C_1x_r \quad (10)$$

to the system (9) leads to

$$\dot{x} = \begin{bmatrix}0 & I & 0\\ C_1(A_1+B_1C_2C_1)C_1^{-1} & C_1A_2P_1C_1^{-1} & C_1B_1C_3\\ 0 & 0 & (-R/L)I\end{bmatrix}x$$

$$+\begin{bmatrix}0\\ 0\\ (1/L)I\end{bmatrix}e+\begin{bmatrix}0\\ C_1(A_1+B_1C_2C_1)\\ 0\end{bmatrix}w_1. \quad (11)$$

Since variables $g_f, \dot{g}_f, i$ can be measured, the output equation is given as follows.

$$y = \begin{bmatrix}I & 0 & 0\\ 0 & I & 0\\ 0 & 0 & I\end{bmatrix}\begin{bmatrix}g_r\\ \dot{g}_r\\ i\end{bmatrix}+\begin{bmatrix}C_{10}\\ C_{10}P_1\\ 0\end{bmatrix}w \quad (12)$$

These equations show that sinusoidal disturbances w_1 due to (caused by) the unbalance of the rotor affect the system dynamics and measured outputs. These disturbances can be described as the output of the autonomous system,

$$\dot{w}_1 = P_1w = \begin{bmatrix}0 & -p & 0 & 0\\ p & 0 & 0 & 0\\ 0 & 0 & 0 & -p\\ 0 & 0 & p & 0\end{bmatrix}w, \quad w(0) = \begin{bmatrix}\varepsilon\cos\kappa\\ \varepsilon\sin\kappa\\ \tau\cos\lambda\\ \tau\sin\lambda\end{bmatrix}. \quad (13)$$

3. Problem Statement

The system is described as the following equations.

$$\dot{x} = A(p)x+Bu+E_1w \quad (14)$$

$$\dot{w} = P_1(p)w_1 \quad (15)$$

$$y = C_1x + C_2(p)w \tag{16}$$

$$z = D_1x \tag{17}$$

where x is the state variable, u is the control input,
w is the disturbance, y is the measured output, and z is the controlled output. Matrices $A(p), P_1(p), C_2(p)$ have terms which increase in proportion to the angular velocity p. Our objective is to construct a controller which preserve internal stability and reject the disturbances ($z(t) \to 0$ as $t \to 0$) at any rotational speed in the presence of uncertain initial conditions. To solve this problem, we introduce so-called " output regulator with internal stability " theory [1].

4. Design of Control System

Output Regulator with Internal Stability

The control system has the feedback configuration and the feedforward configuration. The control input is given by

$$u = K_1x + K_2w \ . \tag{18}$$

The feedback input K_1x provides an internal stability and the feedforward input K_2w provides an output regulation. Design procedure of this control law is composed of following three steps [2].
<step1>The system is stabilized by the state feedback. <step2>A feedforward input which cancels
the influence from the disturbance to the state to be controlled is computed. <step3>An observer is introduced to estimate unmeasurable state variables and disturbances. Then x, w in (18) are replaced by output of the observer.

$$u = K_1\hat{x} + K_2\hat{w} \tag{19}$$

Implementation

We construct a controller according to above-mentioned procedure. <step1>At any rotational speed the system (14) is stabilized by using the quadratic stabilization technique. This technique is effective to the variation due to gyroscopic effect.

$A(p)+K_1B$ is stable.

<step2> The feedforward input to achieve a asymptotic disturbance rejection at any rotational speed is computed as follows.
The eigenvalue of $P_1(p)$ and the eigenvector corresponding to each eigenvalue are computed.

$$\lambda_1 = \lambda_3 = jp \ , \ \lambda_2 = \lambda_4 = -jp \ ,$$

$$t_1 = \begin{bmatrix} 1 \\ j \\ 0 \\ 0 \end{bmatrix}, \ t_2 = \begin{bmatrix} 1 \\ -j \\ 0 \\ 0 \end{bmatrix}, \ t_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ j \end{bmatrix}, \ t_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -j \end{bmatrix}.$$

For each λ_i, t_i , let (f_i, g_i) be a pair of vectors satisfying

$$\begin{bmatrix} \lambda_i I - A & B \\ D_1 & 0 \end{bmatrix} \begin{bmatrix} f_i \\ g_i \end{bmatrix} = \begin{bmatrix} E_1 \\ 0 \end{bmatrix} t_i \tag{20}$$

Let

$$K_2 = -UT^{-1} \ .$$

where

$$U = [\, g_1 \ g_2 \ g_3 \ g_4 \,]$$

Calculating K_2 for the variation of the rotational speed, let K_2 be a function of p. Resulting feedforward gain matrix K_2 makes the transfer function from $w(0)$ to z zero.

$$T(s,p) = D_1\{sI-(A(p)+BK_1)\}^{-1}(E_1+BK_2)(sI-P_1(p))^{-1} \tag{21}$$
$$= 0$$

Minimal Order Observer

<step3> In this system, a part of the state variables of the system and the disturbances to the system can't be measured. Therefore, we employ a minimal oder observer to estimate these variables. The observer is designed for the following augmented system.

$$\begin{bmatrix} \dot{x} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} A(p) & E_1 \\ 0 & P_1(p) \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u = A_c(p)x_c + B_cu \tag{22}$$

$$y = \begin{bmatrix} C_1 & C_2(p) \end{bmatrix} \begin{bmatrix} x \\ w_1 \end{bmatrix} = C_c(p)x_c \tag{23}$$

Observability of the pair $(C_c(p), A_c(p))$ is guaranteed except for the case $p=0$. Hence, we restrict the rotational speed within a reasonable range. The minimal order observer is given,

$$\xi = A(p)\xi + K_o(p)y \ , \tag{24}$$

$$\hat{x}_c = D(p)\xi + H(p)y \ , \tag{25}$$

where

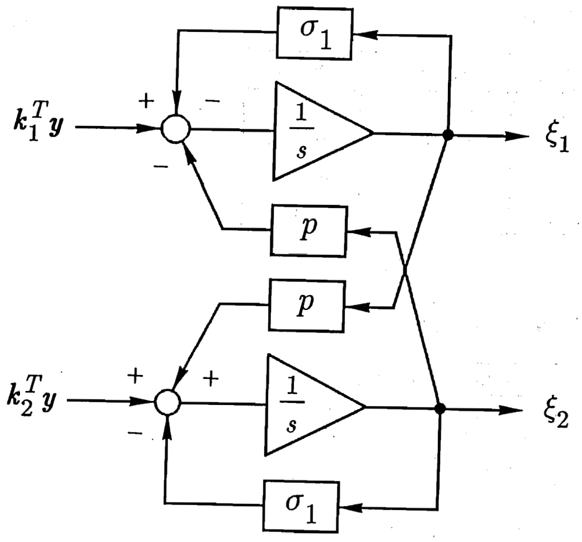

Fig.4 Internal model of sinusoidal disturbances

$$A(p) = P_1(p) - LE_3(p) \ ,$$

$$K_o(p) = A(p)L - LA(p) \ , \quad B = -LB = 0$$

$$D(p) = \begin{bmatrix} C_1 \\ C_1P_1(p) \\ 0 \\ I_4 \end{bmatrix} , \quad H(p) = \begin{bmatrix} I_4 + C_1L \\ I_4 + C_1P_1(p)L \\ I_4 \\ L \end{bmatrix} .$$

The observer is determined by specifying the observer gain matrix L . In specifying L , we use the turn over method to stabilize the observer at any rotational speed (except for $p=0$).

$A(p)$ is stable.

In this case, the resultant observer has an internal model of the disturbances as shown in Fig. 4. With the output of the above-designed observer, the control input is given as

$$u = K_1\hat{x} + K_2(p)\hat{w} \tag{26}$$

From (24),(25) and (26) such synthesis is achieved by the dynamical output feedback configuration.

5. Property of Composite System

We show that the proposed control system (24),(25) and (26) indeed preserves the internal stability and rejects the disturbances. Arrangement of (14),(24),(25) and (26) gives the following composite system.

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A(p)+BK_1 & -BK_1C_2(p)+BK_2(p)C_1 \\ 0 & A(p) \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} E_1+BK_2(p) \\ 0 \end{bmatrix} w_1 \tag{27}$$

$$\dot{w} = P_1(p)w \tag{28}$$

where e is an error vector of the observer. If $w_1 = 0$ in (27), the composite system is asymptotically stable since $A(p)+BK_1$ and $A(p)$ are stable at any rotational speed.

Laplace transforms of (17),(27) and (28) lead the following equation.

$$\begin{aligned} Z(s) &= D_1X(s) \\ &= D_1\{sI-(A+BK_1)\}^{-1}x(0) \\ &\quad + D_1\{sI-(A+BK_1)\}^{-1}(-BK_1C_2+BK_2(p))(sI-A)^{-1}e(0) \\ &\quad + D_1\{sI-(A+BK_1)\}^{-1}(E_1+BK_2(p))(sI-P_1)^{-1}w(0) \end{aligned} \tag{29}$$

where $x(0)$, $e(0)$, $w(0)$ express the initial value of each variable. Since $A(p)+BK_1$ and $A(p)$ are stable at any rotational speed, the first term and the second term of right side asymptotically approach to zero. Moreover, the third term is zero (see <step2>). Hence,

$$z(\infty) = 0$$

Therefore, effectiveness of the proposed controller is confirmed.

6. Numerical Example

Above-developed control scheme is applied for 4-axis active control type magnetic bearing system. Parameters of this system are given in Table 1,2. The system behavior is discussed for the variation of the rotational speed from 0rpm to 10000rpm.

<step1> The system is stabilized within 10000rpm by the quadratic stabilization method. Then,

$$K_1 = \begin{bmatrix} 1.635e5 & -9.474e2 & 0 & 0 & 6.752e2 & -5.29 & 0 & 0 & -1.42e2 & 9.953e{-1} & 0 & 0 \\ -9.474e2 & 1.635e5 & 0 & 0 & -5.29 & 6.752e3 & 0 & 0 & 9.953e{-1} & -1.42e2 & 0 & 0 \\ 0 & 0 & 6.735e4 & -6.526e2 & 0 & 0 & 4.474e2 & -7.805 & 0 & 0 & -8.945e1 & 1.152 \\ 0 & 0 & -6.526e2 & 6.735e4 & 0 & 0 & -7.805 & 4.474e2 & 0 & 0 & 1.152 & -8.945e1 \end{bmatrix}$$

Table 1 Parameters of rotor

item	letter	value	unit
Mass of rotor	m	1.39×10^{1}	kg
Moment of inertia about X_r	J_x	1.348×10^{-2}	kg ·m²
Moment of inertia about Y_r	J_y	2.326×10^{-1}	kg ·m²
Length	$l_r(l_l)$	1.3×10^{-1}	m

Table 2 Parameter of electromagnet

item	letter	value	unit
attractive force	F_{l1}	9.09×10	N
	F_{l2}	2.20×10	N
	F_{l3}	2.20×10	N
	F_{l4}	2.20×10	N
	F_{r1}	9.09×10	N
	F_{r2}	2.20×10	N
	F_{r3}	2.20×10	N
	F_{r4}	2.20×10	N
coil current	I_{l1}	6.3×10^{-1}	A
	I_{l2}	3.1×10^{-1}	A
	I_{l3}	3.1×10^{-1}	A
	I_{l4}	3.1×10^{-1}	A
	I_{r1}	6.3×10^{-1}	A
	I_{r2}	3.1×10^{-1}	A
	I_{r3}	3.1×10^{-1}	A
	I_{r4}	3.1×10^{-1}	A
gap length	W	5.5×10^{-4}	m
Resistance of coil	R	1.47×10	Ω
Inductance of coil	L	2.85×10^{-1}	H

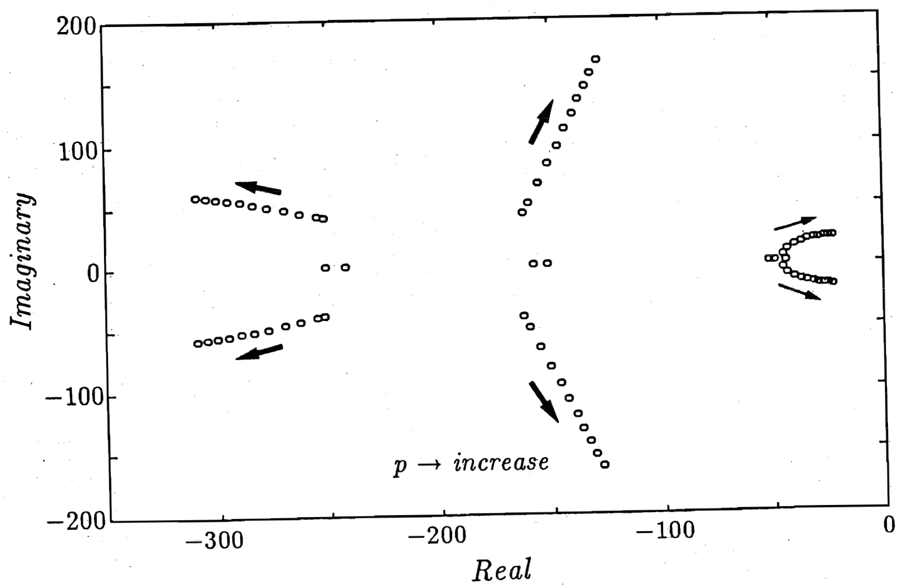

Fig.5 Root locus of closed loop system

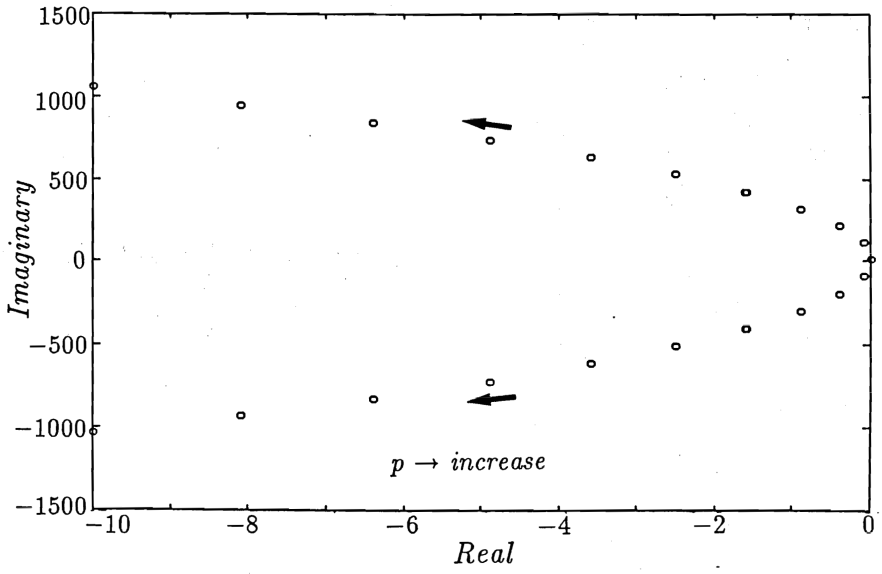

Fig.6 Root locus of observer

Fig. 5 shows the root locus of closed-loop system for the change of the rotational speed. Obviously, closed-loop system is stable at any rotational speed. <step2> Computation result of the feedforward gain is as follows.

$$\boldsymbol{K}_2(p) = \begin{bmatrix} 2.718e2\times p & 1.504e5 & -1.93e4 & 3.535e1\times p \\ 2.718e2\times p & 1.485e5 & 1.955e4 & -3.535e1\times p \\ -5.22e4 & 1.606e2\times p & -2.089e1\times p & -8.476e3 \\ -5.22e4 & 1.606e2\times p & 2.089e1\times p & 8.476e3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2.718e2 & 3.535e1 & 0 \\ 0 & 2.718e2 & -3.535e1 & 0 \\ -1.606e2 & 0 & 0 & -2.089e1 \\ -1.606e2 & 0 & 0 & 2.089e1 \end{bmatrix} \begin{bmatrix} 0 & -p & 0 & 0 \\ p & 0 & 0 & 0 \\ 0 & 0 & 0 & -p \\ 0 & 0 & p & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 1.504e5 & -1.93e4 & 0 \\ 0 & 1.485e5 & 1.955e4 & 0 \\ -5.22e4 & 0 & 0 & -8.476e3 \\ -5.22e4 & 0 & 0 & 8.476e3 \end{bmatrix}$$

The gain matrix $\boldsymbol{K}_2(p)$ has elements which are in proportion to the rotational speed.

<step3> Fig. 6 shows the root locus of the observer corresponding to the change of the rotational speed. The observer is stabilized at 10000rpm. In this case, poles of P is turned over against the axis $\mathrm{Re}\lambda=-5$. It can be seen that the system is stable except for $p=0$.

7. Conclusion

The modeling of a magnetic bearing system to achieve the rotation around its axis of inertia was developed. Then, based on the state-space approach, we proposed an effective control scheme for this problem. This problem was solved by using output regulator with internal stability theory. Constructing the dynamical output feedback controller, it has been shown that the unbalance compensation was achieved in a wide range of rotational speed.

8. References

[1]O.A.Sebakhy and W.M.Whonham:"A Design Procedure for Linear Multivariable Regulators" Automatica,12,467(1976)

[2]H.Kimura:"Observer-based Regulator Synthesis" Osaka University Technical Report, 85-4 (1985)

appendix

$$A_1 = \begin{bmatrix} \alpha/m & 0 & 0 & -\alpha l_m/m \\ 0 & \alpha/m & \alpha l_m/m & 0 \\ 0 & \alpha l_m/J_y & \alpha l_m^2/J_y & 0 \\ -\alpha l_m/J_y & 0 & 0 & \alpha l_m^2/J_y \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & J_x/J_y & 0 \\ 0 & 0 & 0 & J_x/J_y \end{bmatrix}, \quad P_1 = \begin{bmatrix} 0 & -p & 0 & 0 \\ p & 0 & 0 & 0 \\ 0 & 0 & 0 & -p \\ 0 & 0 & p & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 & 1/m & 1/m \\ -1/m & -1/m & 0 & 0 \\ l_l/J_y & -l_r/J_y & 0 & 0 \\ 0 & 0 & l_l/J_y & -l_r/J_y \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0 & 1 & -l_l & 0 \\ 0 & 1 & l_r & 0 \\ -1 & 0 & 0 & -l_l \\ -1 & 0 & 0 & l_r \end{bmatrix}$$

$$C_2 = -\frac{2}{W} diag\left[F_{l1}+F_{l2},\ F_{r1}+F_{r2},\ F_{l3}+F_{l4},\ F_{r3}+F_{r4} \right]$$

$$C_3 = 2\,diag\left[\left(\frac{F_{l1}}{I_{l1}}+\frac{F_{l2}}{I_{l2}}\right),\ \left(\frac{F_{r1}}{I_{r1}}+\frac{F_{r2}}{I_{r2}}\right),\ \left(\frac{F_{l3}}{I_{l3}}+\frac{F_{l4}}{I_{l4}}\right),\ \left(\frac{F_{r3}}{I_{r3}}+\frac{F_{r4}}{I_{r4}}\right) \right]$$