2nd International Symposium on Magnetic Beaing
July 12-14, 1990, Tokyo, Japan

Application of Periodic Learning Control with Inverse Transfer Function Compensation in Totally Active Magnetic Bearings

T.HIGUCHI*, M.OTSUKA*, T.MIZUNO**, T.IDE***

*Institute of Industrial Science, University of Tokyo;
7-22-1, Roppongi, Minato-ku, Tokyo 106
**Faculty of Engineering, Saitama University; Shimo-Okubo Urawa 338
***Fuji Heavy Industries Ltd.; 3-9-6, Osawa, Mitaka, Tokyo 181, Japan

Abstract

Rotating machinery has two vibration problems caused by mass imbalance of a rigid rotor --- whirling motion of the rotor and vibration of the housing. In order to solve these problems in magnetic bearings, the authors propose to apply periodic learning control (**PLC**) with inverse transfer function compensation (**ITFC**), which can deal with periodic disturbances. In order to suppress the whirling motion, the PLC with the ITFC calculates compensatory inputs added to force control signals from the sampled rotor displacement. In order to reduce the vibration of the housing, the PLC with ITFC calculates compensatory inputs added to force control signals from the sampled force control signals. The implementation of the PLC with ITFC is easily accomplished by adding a digital controller, which consists of a personal computer, an A/D converter, and four D/A converters, to a conventional analog PID-controller. The authors have carried out several experiments by applying the PLC with ITFC to obtain the following results: (1) Translational and tilting whirling motions are reduced from about *±3μm* to *±1μm*, and from about *±80μrad* to *±20μrad*, respectively, at the rotor speed of *51,000rpm* (*850rps*); (2) Vibration of the housing of a magnetic bearing spindle is reduced from about *±0.6G* to *±0.1G* in average at the rotor speed of *57,000rpm* (*950rps*).

1. Introduction

Rotating machinery has two major vibration problems caused by mass imbalance of a rigid rotor. One is whirling motion of the rotor. The other is vibration transmitted to the housing of the machine. Magnetic bearings can solve these problems by controlling the forces of electromagnets.

The whirling motion of the rotor can be suppressed by providing the forces of electromagnets which counteract the disturbance forces arising from the imbalance. Several control laws based on this concept have been developed([1],[2], for example). Based on the same concept, the authors propose to apply periodic learning control (**PLC**) with inverse transfer function compensation (**ITFC**) in magnetic bearings to suppress the whirling motion.

The vibration of the housing can be reduced by making the axis of rotation coincide with the principal axis of inertia[3]. When the axis of rotation coincides with the principal axis of inertia, attractive forces of electromagnets do not fluctuate. The authors proposed control schemes to reduce those fluctuations of the forces of electromagnets which are synchronous with the rotation of the rotor[4]. However, it is generally difficult to measure the forces of electromagnets. On the other hand, it is easy to measure coil-currents of electromagnets. When the coil-currents do not fluctuate, the forces of electromagnets fluctuate very little, because the difference between the geometrical axis of the rotor and the principal axis of inertia is generally small. Thus, reducing the fluctuations of coil-currents have a good effect to reduce the vibration of the housing. The authors propose to apply the PLC with ITFC in magnetic bearings in order to reduce the fluctuations of coil-currents, aiming at the reduction of the vibration of the housing.

2. Periodic Learning Control (PLC) with Inverse Transfer Function Compensation (ITFC)

Disturbances to rotating machinery contain periodic and repetitive components. The period is the time for one rotation. Periodic learning control (PLC) can make periodic output errors converge to zero by repeatedly adding the errors in the last period into the control input in the next period[5][6][7].

Fig.1 shows a block diagram of the PLC with the ITFC we propose to apply[7][8]. Let us now explain the PLC with ITFC of one degree of freedom for the sake of brevity. While the filter F(s) is not always necessary for the basic form of the PLC with the ITFC, F(s) is employed here in order to deal only with the fundamental component of the output error E(s).

Symbol ^ added to $G^{-1}(s)$ denotes that $G^{-1}(s)$ may have some errors. Providing that $R(s)=0$, the output error of the n^{th} repetition is given by

$$E_n(s) = -G(s)\cdot\{1-\hat{G}^{-1}(s)F(s)G(s)\}^n\cdot W(s)\ . \quad (1)$$

The output error converges to zero when the following equation is satisfied:

$$|\ 1-\hat{G}^{-1}(s)F(s)G(s)\ | < 1\ . \quad (2)$$

Because the filter F(s) extracts only the component synchronous with the rotation, eq.(2) yields;

$$|\ 1-\hat{G}^{-1}(j\omega)G(j\omega)\ | < 1\ . \quad (3)$$

If eq.(3) is satisfied at the angular velocity ω of the rotor, the error converges to zero.

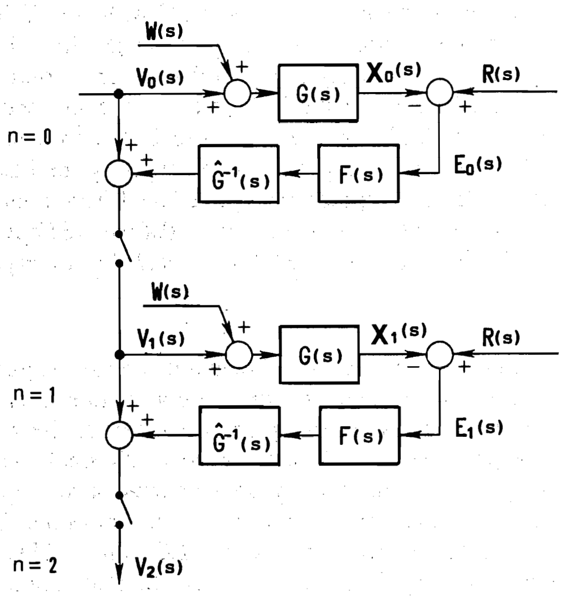


Fig.1 Block Diagram of Periodic Learning Control with Inverse Transfer Function Compensation

X(s): output of the controlled system

V(s): compensatory input ($V_0(s)=0$)

R(s): periodic reference input

E(s): output error of the controlled system; = R(s)-X(s)

G(s): closed loop transfer function of the controlled system, where $X(s)= G(s)V(s)$.

F(s): filter which extracts only the fundamental component of the system.

W(s): periodic disturbance

n: repetitive number of periodic learning control

(All subscripts denote the repetitive number.)

3. Application of PLC with ITFC in Magnetic Bearings

Fig.2 shows a structure of a control system. As mentioned in chapter 2, the PLC with the ITFC is applied in order to make the output errors of the controlled system converge to zero. The controlled system corresponds to a closed loop system which consists of magnetic bearings and a PID-controller in Fig.2. The PID-controller stabilizes translational motion (x,y,z) and tilting motion (θ_x,θ_y) of the rotor, respectively; Such a control scheme is called central control. The definition of coordinate is shown in Fig.3. A symbol u_k (current control signal) in Fig.2 represents a signal of control force in the *k* direction (k=x,y,z,θ_x,θ_y). Coil-current in each electromagnet is determined from these current control signals through geometrical conversion[1].

The **PL-controller** shown in Fig.2, which consists of a personal computer, an A/D converter, and four D/A converters, is added to the controlled system to carry out the PLC with the ITFC. The PL-controller samples the output errors of the controlled system, carries out the calculation of F(s) and $\hat{G}^{-1}(s)$ (See Fig.1), and adds the compensatory inputs to the current control signals. The reference inputs to the controlled system are treated as zero in this paper.

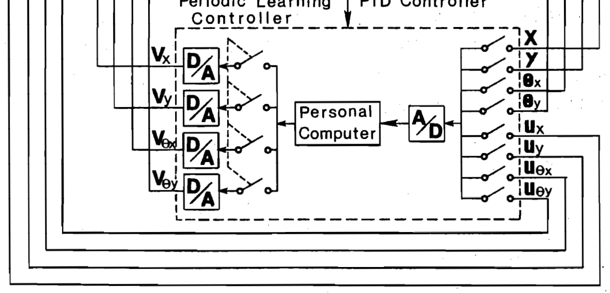


Fig.2 Diagram of Control System

u_k: current control signal in the *k* direction (k=x,y,z,θ_x,θ_y)

v_k: compensatory signal in the *k* direction (k=x,y,θ_x,θ_y)

Because the PLC with ITFC makes the output errors of the controlled system converge to zero, what is treated as the output determines the effect of the PLC with ITFC. When the rotor displacement is treated as the output, the PLC with ITFC suppresses the whirling motion. When the fluctuations of current control signals are treated as the output, the PLC with the ITFC reduces the fluctuations of coil-currents.

As the authors proposed in [4], the PLC with the ITFC can reduce the vibration of the housing by treating the fluctuations of the forces of electromagnets as the output.

Applying the PLC with the ITFC in magnetic bearings has some good properties[8]: 1)The PLC with the ITFC makes the output errors converge to zero even when parameters of the controlled system are not accurately identified. 2)The implementation of the PLC with the ITFC is easily accomplished by adding a digital controller to a conventional (analog) PID-controller. 3)The PLC with the ITFC does not affect the stability of magnetic bearings at all.

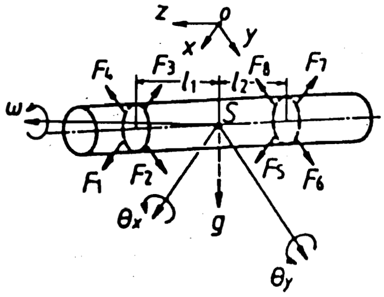


Fig.3 Coordinate System and Forces Acting on the Rotor

4. Suppression of Whirling Motion

The authors carried out an experiment to suppress the whirling motion at the rotor speed 850rps (51,000rpm). A section view of the magnetic bearing spindle used in the experiment is shown in Fig.4. It consists of eight radial electromagnets, four radial gap sensors, two axial electromagnets, an axial gap sensor, a motor-stator, and a rotor.

Here we define symbols used to explain a procedure of the PLC with the ITFC to suppress the whirling motion.

n : repetitive number of the PLC

$\mathbf{x}(t) = [x, y, \theta_x, \theta_y]^T$: rotor displacement

$\mathbf{x}^*(t) = [x^*, y^*, \theta_x^*, \theta_y^*]^T$: component of **x** with the same frequency as the rotor speed

$\mathbf{v}(t) = [v_x, v_y, v_{\theta x}, v_{\theta y}]^T$: compensatory input to suppress the whirling motion

T_f^{-1} : 4-4 complex matrix that represents characteristics of the inverse transfer function $G^{-1}(s)$ at the rotor speed, f(rps)

The structure of T_f^{-1} is as follows:

$$T_f^{-1} = (c_{LM}) \ , \ c_{LM} = a_{LM} \exp(j \cdot p_{LM}) \ (L, M = 1, .., 4).$$

T_f^{-1} is calculated from Tf obtained by another experiment, where each term of **x**(t) (the output of the controlled system) is measured when a sinusoidal test signal is given to each term of **v**(t) (the compensatory input)[9]. This experiment is carried out when the rotor is not rotating. (T_f^{-1} can also be calculated from parameters of the controlled system[8].)

The procedure of the PLC with the ITFC to suppress the whirling motion is as follows:

[n.1] Compensation with $\mathbf{v}_n$ begins. $\mathbf{v}_n$ is given to the controlled system through four D/A converters every 10μs.

[n.2] $\mathbf{x}_n$ is sampled by the A/D converter every 10μs throughout one rotation at least. [n.2] is carried out when steady-state response can be observed.

[n.3] $\mathbf{x}_n^*$ is calculated by means of Fourier series expansion. [n.3] corresponds to F(s) in Fig.1.

[n.4] $\mathbf{x}_n^*$ is multiplied by T_f^{-1} from the left side. [n.4] corresponds to $\hat{G}^{-1}(s)$ in Fig.1.

[n.5] The next compensatory input $\mathbf{v}_{n+1}$ is calculated by the following equation:

$$\mathbf{v}_{n+1} = - \sum_{k=0}^{n} T_f^{-1} \mathbf{x}_k^* \ .$$

A set of fluctuations of $x, .., \theta_y$ denoted with n=0 in Fig.5 (a) is whirling motion before the PL-controller works [0.1], where the brackets [.] correspond to those in the procedure of the PLC with the ITFC above. The personal computer calculates synchronous components of the sampled data [0.3]:

$$\mathbf{x}_0^* = \left\{ \begin{array}{c} 2.4\cos(2\pi ft + 162\mathrm{deg}) \ (\mu\mathrm{m}) \\ 1.7\cos(2\pi ft - 118\mathrm{deg}) \ (\mu\mathrm{m}) \\ 63\cos(2\pi ft - .49\mathrm{deg}) \ (\mu\mathrm{rad}) \\ 82\cos(2\pi ft - 137\mathrm{deg}) \ (\mu\mathrm{rad}) \end{array} \right\} .$$

Phase angle is defined as an angle from the rotor-fixed reference. The PL-controller obtains the position of this reference of angle as a pulse per rotation. Next, $\mathbf{v}_1$ is calculated according to [0.4] and [0.5].

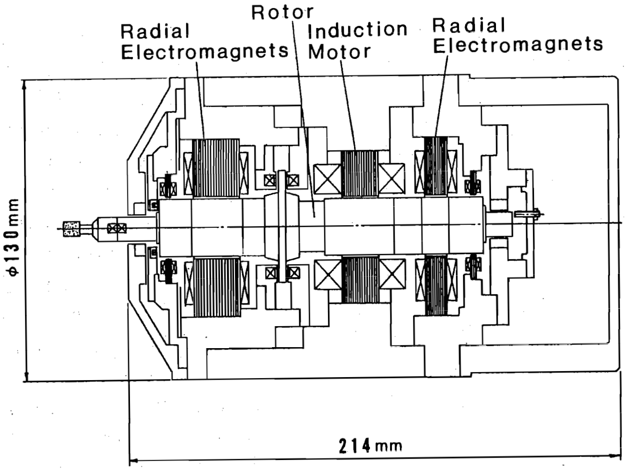
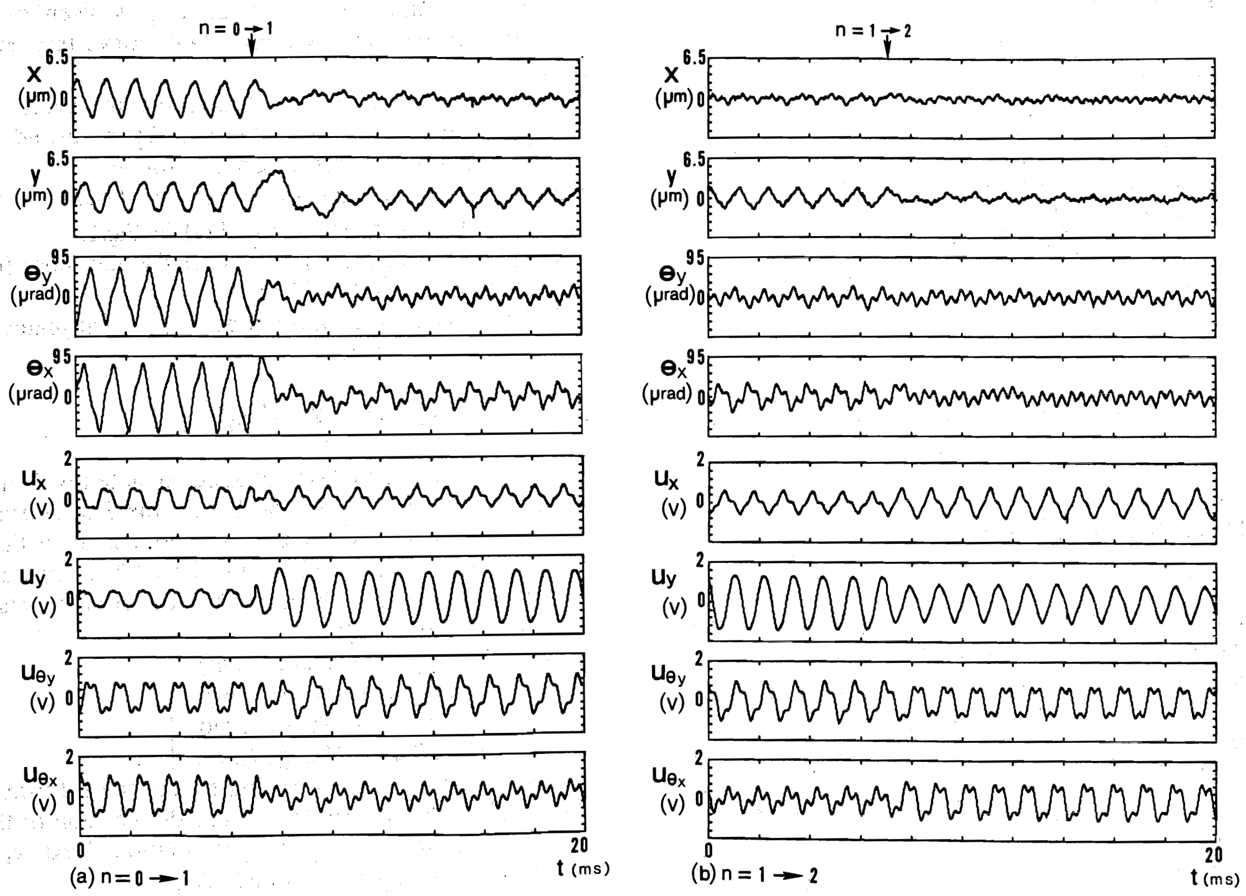


Fig.4 Cross Section of Magnetic Bearing Spindle
(Manufactured by Seiko Seiki Co., Ltd. , Japan)



(a) n=0 → 1

(b) n = 1 → 2

Fig.5 Suppression of Whirling Motion by PLC with ITFC (850rps).
$x,..,\theta_y$: rotor displacement , $u_x,..,u_{\theta y}$: force control signals , n: repetitive number of the PLC

"n=0→1" in Fig.5 (a) denotes the time when the compensation with $\mathbf{v}_1$ begins [1.1]. The $\mathbf{v}_1$ is given continuously through four D/A converters every 10μs. The figure shows that the residual whirling motion is not negligible. This estimation implies that the T_f^{-1} has some errors. In order to reduce this residual whirling motion, the PLC with the ITFC proceeds to [1.2],.,[1.5].

"n=1→2" in Fig.5 (b) denotes the time when compensatory input changes form $\mathbf{v}_1$ to $\mathbf{v}_2$ [2.1]. The figure shows that the whirling motion becomes smaller after this change. Fig.6 shows that the component of x at 850Hz is reduced very well. On the other hand, the other components are not affected because only the component synchronous with the rotation is extracted in the procedure [n.3]. Fig.7 shows that suppressing the whirling motion does not result in the reduction of the housing vibration. In the experiment, the vibrations are measured at a band which fixes the housing on the base.

In the next chapter, it is shown that the reduction of the housing vibration can be realized by another application of the PLC with the ITFC.

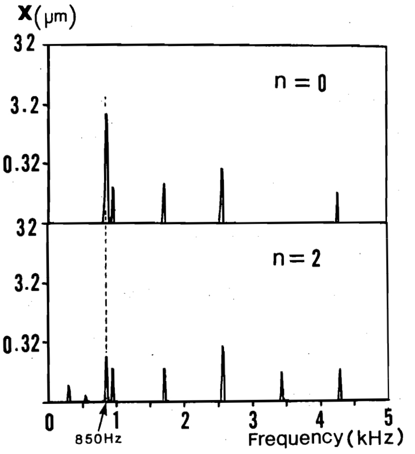


Fig.6 Comparison of Whirling Motions in Frequency Domain with and without Suppression of Whirling Motion
n : repetitive number of periodic learning control

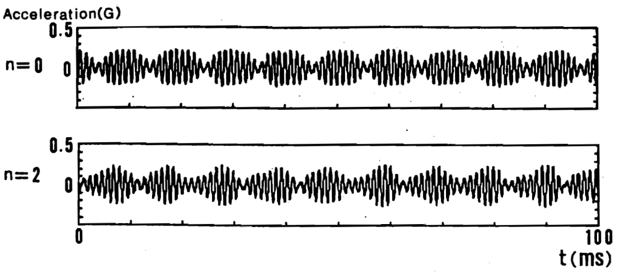


Fig.7 Comparison of Vibrations of Spindle Housing with and without Suppression of Whirling Motion
n : repetitive number of periodic learning control

5. Reduction of fluctuations of coil-currents in electromagnets

The vibration of the housing is usually undesirable because it is transmitted to the environment. Thus, to reduce the vibration of the housing is as important as to suppress the whirling motion.

The authors apply the PLC with the ITFC in order to reduce synchronous fluctuations of coil-currents of electromagnets, aiming at the reduction of the vibration of the housing.

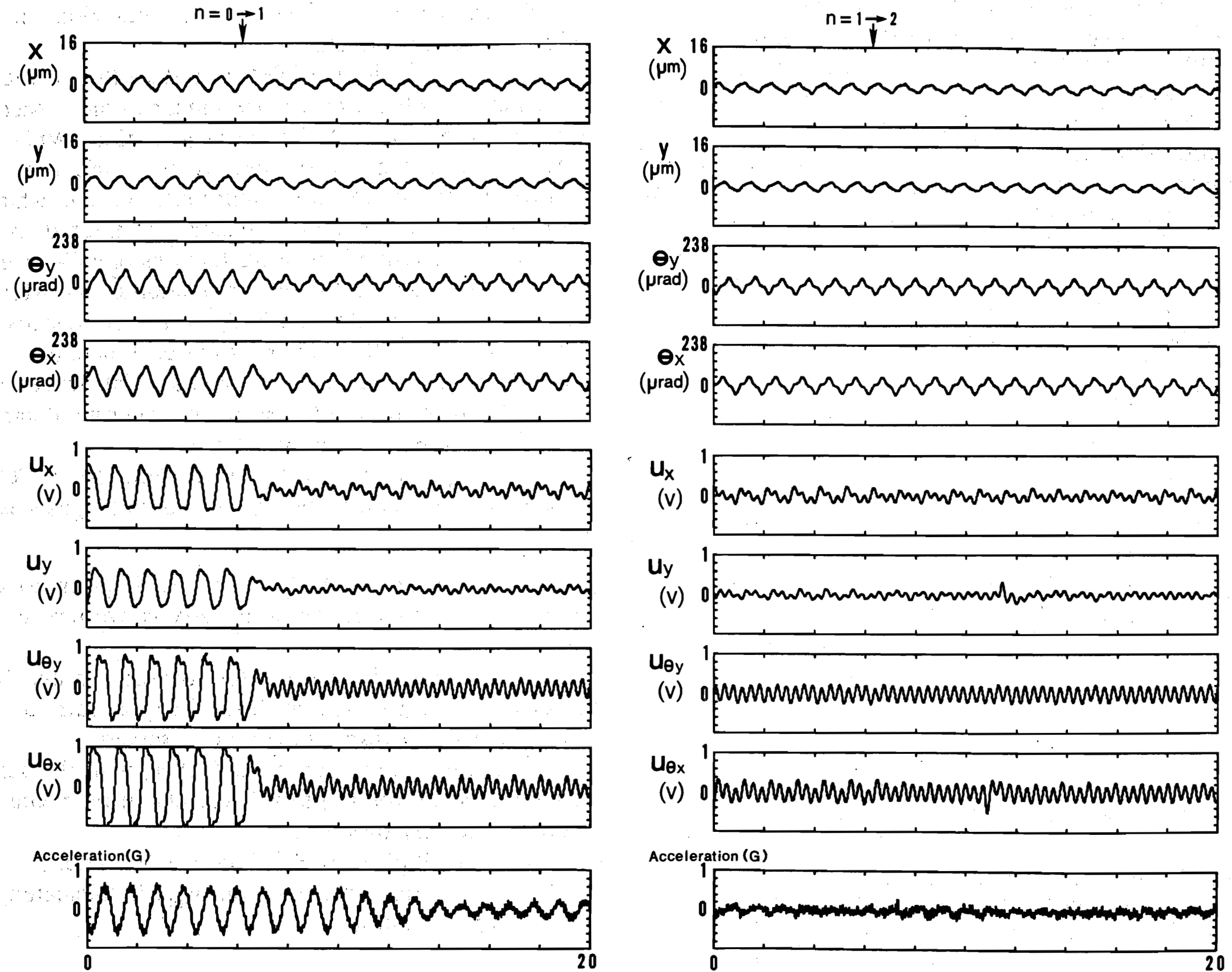
The PLC with the ITFC to reduce the fluctuation of coil-currents has a procedure similar to the procedure for the suppression of the whirling motion. While the rotor displacement is treated as the output of the controlled system for the suppression of the whirling motion, the current control signals are treated as the output for the reductions of fluctuations of coil-currents. As T_f^{-1} is introduced for the suppression of the whirling motion, a 4-4 complex matrix S_f^{-1} which represents the characteristics of the inverse transfer function of the controlled system at the rotor speed is introduced. S_f^{-1} is calculated from S_f obtained by an experiment similar to the experiment carried out for T_f.

The procedure of the PLC with the ITFC to reduce the fluctuations of current control signals is as follows:

[n.1] Compensation with $\mathbf{v}_n$ begins. $\mathbf{v}_n$ is given to the controlled system through four D/A converters every 10μs.
[n.2] $\mathbf{u}_n$ is sampled by the A/D converter every 10μs throughout one rotation at least. [n.2] is carried out when steady-state response can be observed.
[n.3] $\mathbf{u}_n^*$, which is the component of $\mathbf{u}_n$ with the same frequency as the rotor speed, is calculated by means of Fourier series expansion.
[n.4] $\mathbf{u}_n^*$ is multiplied by S_f^{-1} from the left side.
[n.5] The next compensatory input $\mathbf{v}_{n+1}$ is calculated by the following equation:

$$\mathbf{v}_{n+1} = - \sum_{k=0}^{n} S_f^{-1} \mathbf{u}_k^* \ .$$

The experiment was carried out at the rotor speed of 950rps (57,000rpm). Fig.8 (a) shows that the first trial (n=1) of the PLC with the ITFC reduces the fluctuations of the current control signals and the vibration of the housing. Transient vibration did not disappeared in Fig.8 (a). The steady-state vibration when n=1 is shown in Fig.8 (b). This figure shows that the PLC with the ITFC drastically reduces the vibration of the housing. Fig.9 shows that only the synchronous fluctuation of current control signal u_x and only the syñchronous vibration of the housing are reduced.



(a) n = 0 → 1

(b) n = 1 → 2

Fig.8 Reduction of Fluctuations of Current Control Signals, and Reduction of Vibration of Housing (950rps).
The accelerometer was attached to the metal band that fixes the spindle housing to the base.
$x,..,\theta_y$: rotor displacement , $u_x,..,u_{\theta y}$: force control signals , n: repetitive number of the PLC

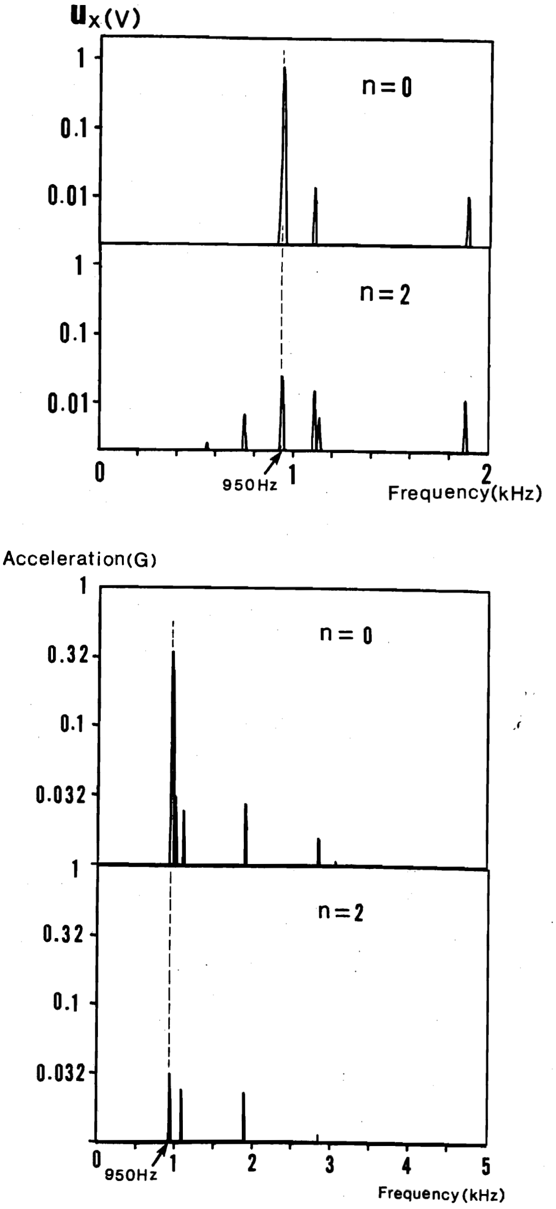


Fig.9 Fluctuation of Current Control Signal and Vibration of Spindle Housing in Frequency Domain

6, Applications and Future Prospects

In this paper, the authors have applied the PLC with the ITFC in the magnetic bearings stabilized by central control. The PLC with the ITFC can be applied also in the magnetic bearings stabilized by decentralized control in order to suppress the whirling motion and to reduce the vibration of the housing. The authors will report about this in [10].

Machine tools are subject to periodic disturbances such as cutting resistance. The PLC with ITFC can be applied in the magnetic bearings for machine tools because the PLC with ITFC can also compensate for periodic disturbances other than those caused by the mass imbalance.

The authors proposed the basic idea of two other applications of the PLC with ITFC to reduce the vibration of the housing. One is the PLC with ITFC which reduces the fluctuations of the forces of electromagnets[4]. This will effectively reduce the vibration of the housing even if the difference between the geometrical axis of the rotor and the principal axis of inertia is relatively large. The other is the PLC with ITFC in which compensatory inputs are added not to the current control signals but to sensor-outputs[11]. The amplitudes and the phase-angles of the compensatory inputs in this application vary very little even if the rotational speed changes. Therefore, the reduction of the vibration of the housing will be easily accomplished with this improved application when the rotor speed is increasing or decreasing.

The authors are aiming to carry out experiments to demonstrate the effects of the applications mentioned in this chapter.

7. Conclusion

The authors proposed to apply the PLC with the ITFC in magnetic bearings for solving the vibration problems. The implementation of the PLC with ITFC is easily accomplished by adding a digital controller to a conventional controller. The experiments were carried out to demonstrate that the PLC with the ITFC can suppress the whirling motion or can reduce the vibration of the housing. The results of the experiments are as follows: (1) Translational and tilting whirling motions are reduced from about ±*3μm* to ±*1μm*, and from about ±*80μrad* to ±*20μrad*, respectively, at the rotor speed of *51,000rpm* (*850rps*); (2) The vibration of the housing of the magnetic bearing spindle is reduced from about ±*0.6G* to ±*0.1G* in average at the rotor speed of *57,000rpm* (*950rps*).

Acknowledgment

We would like to thank Mr.M.Ota, Mr.S.Ando, and Mr.T.Kawashima of Seiko Seiki Co.,Ltd. for their continuous support and for allowing us to use their magnetic bearing spindle and their controller.

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