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A DIGITAL TIME DELAY CONTROLLER FOR ACTIVE MAGNETIC BEARINGS

K. Youcef-Toumi, Associate Professor, S. Reddy, Graduate Student
and I. Vithiananthan, Graduate Student

Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, MA 02139
U.S.A.

Abstract

The successful design of actively controlled magnetic bearings depends greatly on the design of the control system. The function of the controller is to maintain bearing performance in the face of large system dynamic variations and unpredictable disturbances. This paper discusses issues associated with the design and implementation of a digital time delay controller for active magnetic bearings.

The control system evaluation conducted in this research consists of several tests. Various components of the system are identified and their corresponding theoretical models are then validated experimentally. The effectiveness of the digital control algorithm was then validated using several simulations which are based on linear and nonlinear models for the bearing including bending mode effects. Several experiments were conducted for spinning and nonspinning conditions. These include time responses, closed loop frequency responses and disturbance rejection frequency response data. Evaluations were performed at bearing rotational speeds of 10,900 rpm, 20,100 rpm, 30,400 rpm and 34,800 rpm with Time Delay Controller bandwidths of 100, 200 and 400 rad/sec. The digital controller presented shows an extremely high performance for the prototype considered by maintaining almost the same desirable dynamic behavior over the whole range of speeds.

1 Introduction

The use of magnetic bearings to support rotating structures without contact has received considerable attention in the last forty years. There are two main areas of advantages of using magnetic bearings over conventional bearings. The first is due to their contactless nature which eliminates friction which is inherent in conventional bearings. The absence of friction in magnetic bearings not only contribute to efficiency in energy but also to longer life and elimination of mechanical maintenance of the bearing. The contactless nature also eliminates the use of lubrication in the bearing. Lubrication poses problems because its presence precludes operation in many environments. Absence of lubrication in magnetic bearing makes them compatible to environments such as vacuum and other hostile environments. Another advantage is that the magnetic bearings can operate in a wide range of temperature ranging from −250°C to 450°C [20].

The other advantages are due to the closed loop control of the bearing such as the elimination of vibrations due to unbalance [20].

The characteristics of active magnetic bearings are inherently nonlinear due to the nonlinearities of electromagnetic fields. The nonlinear nature requires an increase in the modeling complexity, estimation and control of the bearings. Furthermore there exist unpredictable disturbances due to mass unbalances of the rotor, and uncertainty of the model due to changes of the rotor speed. For many applications, the systems are assumed to be linear with no parameter uncertainties. The linear controllers are then designed based on the linearized time invariant model. Additional background and historical review can be found in [16].

The most recent controller design approaches use full-state feedback. Hubbard and McDonald [5] used linear-quadratic design in their pendulous supported flywheel. Stanway and O'Reilly [9] used eigenstructure assignment for the control of suspension systems for rotating machinery. Salm and Schwietzer [7] demonstrated modal control of a flexible rotor. These approaches use linear controllers for linearized models, and therefore ignore all model uncertainties and unexpected disturbances. Additional references are found in [16].

Most of the well-developed control theory, either in the frequency domain or in the time domain, deals with systems whose mathematical representations are completely known. However, in many practical situations, the parameters of the system are either poorly known or operate in environments where unpredictable large parameter variations and unexpected disturbances exist. In such applications, the usual fixed-gain linear controllers will not be adequate to achieve satisfactory performance in the entire range over which the characteristics of the system may vary.

Recently several advanced control techniques have been developed for such systems. One of the primary method is Adaptive Control [1,3,6,10] where the structure of the controller, usually a PD or PID type controller is first selected. The controller gains are then updated so that the plant output closely follows the desired response [2,4]. This method considers slowly varying plant parameters, linear equations and/or bounded uncertainty.

Variable Structure Control (VSC) is another very powerful method to deal with nonlinear systems with uncertain dynamics. VSC was first proposed by Utkin in 1972 [11], and its early application was made by Young in 1978 [18] to robot manipulators. Based on Lyapunov's second method, this control scheme is characterized by a discontinuous function with high frequency chattering, which forces the system to follow the reference signal quickly without the parameter identification process used in adaptive control. This infinite frequency chattering, however, is undesirable in many practical applications such as mechanical systems. Young proposed a linear region to eliminate chattering [19]. In 1983, Slotine and Sastry [8] proposed a modified VSC method to eliminate chattering by introducing a sliding zone. If the uncertainty is large, the sliding zone must be wide enough to cover the uncertainty. In this case the controller tends to be a regular PID controller for the operating range, thus losing the advantages of the VSC approach. That is, SMC is useful only for systems where the plant parameter variations and disturbances are bounded and small.

Learning control is another approach which is based on trial and error. Arimoto and Miyazaki extended it to Multi-Input Multi-Output nonlinear systems. In Learning Control, the generated tracking errors are stored during every iteration. After each iteration, the controller adds a signal, which is proportional to the stored error and/or its derivative, to the previous control input such that the tracking error will decrease during the next iteration. By repeating this process several times, betterment in performance is obtained. Therefore this approach is restricted to repetitive tasks only.

In 1986, Youcef-Toumi and Ito proposed a novel method, called Time Delay Control (TDC), which is applicable to nonlinear plants with unknown dynamics and unexpected disturbances [12,13,14]. TDC law depends on neither estimation of specific parameters or repetitive actions, nor does it generate a discontinuous signal. Rather, it depends on the direct estimation of the effect of uncertainties. This is accomplished using time delay. The gathered information is used to cancel the unknown dynamics and the unexpected disturbances simultaneously. Then the controller inserts the desired dynamics into the plant. In other words, the TDC uses past observation of the system's response and the control inputs to modify the control actions directly rather than adjusting the controller gains. Thus this algorithm can deal with large unpredictable system parameter variations and disturbances.

In this paper, we first present models of an eight pole attractive magnetic bearing as used in a High Speed turbo molecular pump. The Time Delay Controller for magnetic bearings is briefly presented in Section 3. Section 4 deals with the implementation and evaluation of the Time Delay Controller.

2 System Identification and Model Validation

In this section, the system parameters are identified and the models are experimentally validated. The nonlinear and linearized equations of motion for the bearings are also presented. The theoretical and experimental frequency responses of the system are also presented.

2.1 Plant Dynamics

The dynamic behavior of a rotor under the action of an eight pole attractive magnetic bearing in the radial direction can be modelled by the following equation:

$$m\frac{d^2x}{dt^2} = -f_\ell + f_r + d \qquad (1)$$

where m is half of the rotor mass (kg) (since the rotor is controlled by two radial bearings) and the left and right magnet forces (N) are given by

$$f_\ell = \frac{\mu_0 A_g N^2 I_\ell^2}{h_\ell^2} \text{ and } f_r = \frac{\mu_0 A_g N^2 I_r^2}{h_r^2} \qquad (2)$$

μ_0 is the permeability of free space and A_g is the air gap area of one pole. N is the number of turns. The left and right bearing clearances h_ℓ and h_r respectively are

$$h_\ell = h_0 + x \text{ and } h_r = h_0 - x \qquad (3)$$

h_0 is the nominal bearing radial clearance and x is the deviation of the shaft from the bearing center. d is a disturbance force.

A schematic of the radial bearing is shown in Figure 1. The variable u indicates the control signal. Using Eqn. (2) and Eqn. (3) and introducing a bias current I_0, Eqn. (1) becomes

$$m\frac{d^2x}{dt^2} = -\frac{K(I_0 - 0.5u)^2}{m(h_0 + x)^2} + \frac{K(I_0 + 0.5u)^2}{m(h_0 - x)^2} + \frac{d}{m} \qquad (4)$$

At steady state, the current in the left and right electromagnets is I_0, therefore $u = 0$, $x = 0$, and $\dot{x} = 0$. Thus the linearized model becomes:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{4KI_0^2}{mh_0^3}x_1 + \frac{2KI_0}{mh_0^2}u \qquad (5)$$

where x_1 and x_2 are the position and velocity. The transfer function of the actuator and plant can be given by the following expression:

$$\frac{X_1(s)}{U(s)} = \left(\frac{\frac{2KI_0}{mh_0^2}}{s^2 - \frac{4KI_0^2}{mh_0^3}}\right) \qquad (6)$$

The system parameters are summarized in Table 1.

The bending modes were included in the plant equation to evaluate their effect on the control performance. The first bending mode of the plant can be modelled as a second order single degree of freedom system. Figure 2 shows a single degree of freedom system, where the massless dashpot of damping coefficient D and a spring of stiffness k are mounted between the mass m and the fixed wall. The transfer function between the displacement x and the force F can be written as:

$$\frac{X(s)}{F(s)} = \left(\frac{1}{M}\right)\left(\frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right)$$

where ω_n^2 is equal to K/M, and ζ is equal to $D/2\sqrt{M/K}$. Since there are more than one bending mode of the rotor, the mass m of the rotor has to be distributed among these modes. Therefore the mass for the first bending mode can be thought of as the effective mass for the first bending mode. The value of stiffness K is very high and the damping D is very low for metals. Therefore the damping ratio ζ of the bending mode is small. The effective mass of the bending mode has to be less than the actual mass of the rotor.

The system was modelled such that the perturbations caused by the input force due to the bending mode was then added to the position of the rigid plant affected by the same force. The block diagram of the modelled system is presented in Figure 3

2.2 System Identification and Model Validation of an Eight Pole Magnetic Bearing

The turbo molecular pump has two radial bearings and a thrust bear-

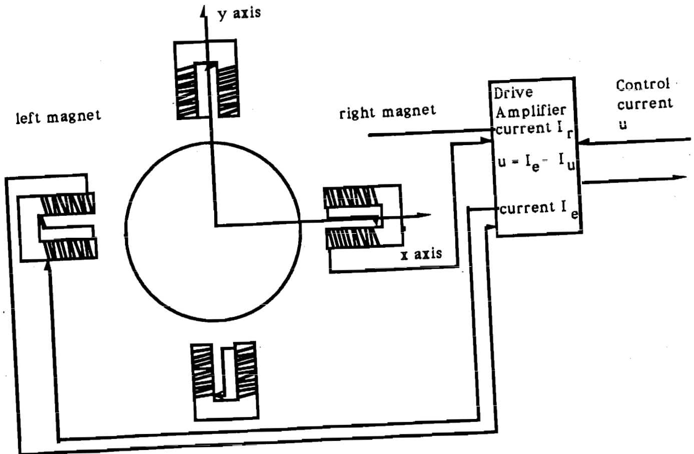

Figure 1: The Control Current Setup for the Bearing in the Radial Direction

K	$= \mu_0 A_g N^2 = 1.23 \times 10^{-6} Nm^2/A^2$
I_0	$= 0.36$ A
m	$= 1.1$ kg
h_0	$= 2.5 \times 10^{-4}$ m
A_g	$= 9.75 \times 10^{-5}$ m^2
N	$= 100$ turns

Table 1: System Parameters

ing. Each radial bearing has two degrees of freedom in the horizontal plane and the thrust bearing has one degree of freedom in the vertical plane. It is assumed that the bearing forces are uncoupled in the five axes ($\mathbf{x_1}, \mathbf{y_1}, \mathbf{x_2}, \mathbf{y_2}$, and $\mathbf{z}$). The controller is similar to a regulator where the system output follows a desired position indicated by the reference input.

Several frequency responses were obtained for comparison with theoretical models. These frequency responses were obtained for both the thrust and the radial bearings. A HP dynamic signal analyzer was used to send a swept sine signal with a frequency range of .1 Hz to 10 kHz. The experimental transfer function is between the output position measured in terms of voltage and the input current measured in terms of voltage. The sensor gain for the radial bearing is 25000 V/m. The analytical transfer function obtained is:

$$\frac{X_1(s)}{U(s)} = \frac{10472}{(s+15700)} \cdot \frac{12.88 \times 25000}{s^2 - 37100} \left(\frac{V}{V}\right) \tag{7}$$

The plant is an unstable plant with two poles at +192.6 rad/sec (30.7 Hz) and -192.6 rad/sec (-30.7 Hz). The experimental plant frequency response is presented in Figure 4. The break frequency of the experimental plant transfer function is 21.8 Hz. The theoretical break frequency is very close to the experimental break frequency. The estimated experimental transfer function is

$$\frac{X_1(s)}{U(s)} = \frac{9 \times 25000}{s^2 - 24670} \cdot \frac{10472}{s + 15700}$$

The experimental frequency response portray the bending mode characteristics of the rotor. The bending mode affects the gain at about 900Hz and at about 2 kHz. The phase response of the plant also shows the effect of the bending mode of the plant.

3 Time Delay Control

This section deals with the Time Delay Controller for magnetic bearings. This controller estimates a feedforward action in order to cancel the unknown dynamics and disturbances present in the system and introduce some desired dynamics. This method is discussed in detail [17].

The nonlinear equations of motion for a given axis of the Active Magnetic Bearing can be written as,

$$\begin{aligned} \frac{dx_1}{dt} &= x_2 \\ \frac{dx_2}{dt} &= g(x_1, u) + d(t) \end{aligned} \tag{8}$$

where $g(x_1, u)$ is a nonlinear function relating $\dot{x}_2$ to x_1, and u; and $d(t)$ is the disturbance force. Time Delay Control law assumes that both the function g and disturbance scheme are unknown. The control as discussed in [17] assumes a reference model which generates the desired trajectory. A second order reference model is chosen as

$$\begin{aligned} \frac{dx_{1m}}{dt} &= x_{2m} \\ \frac{dx_{2m}}{dt} &= -a_{1m}x_{1m} - a_{2m}x_{2m} + b_m r \end{aligned} \tag{9}$$

The values for a_{1m} and a_{2m} are chosen as to satisfy the requirement of the natural frequency, and damping ratio of the second order reference model. The TDC control law derived in the previous section was chosen to satisfy this requirement and therefore given by the equation:

$$\begin{aligned} u(t) = \; & \tfrac{1}{b}[-\hat{\dot{x}}_2(t-1) + bu(t-1) - a_{1m}x_1(t) \\ & -a_{2m}(t)x_2(t) + b_m r + k_1e_1 + k_2e_2] \end{aligned} \tag{10}$$

where $\hat{\dot{x}}_2$ is an estimate of the acceleration of the rotor, x_2 is the velocity of the rotor, and x_1 is the position of the rotor, $e_1 = x_{1m} - x_1$, and $e_2 = x_{2m} - x_2$. The error gains k_1 and k_2 are appropriately chosen. While this control law requires an estimate of the acceleration, a new method has been developed in [17] which does not need such signals.

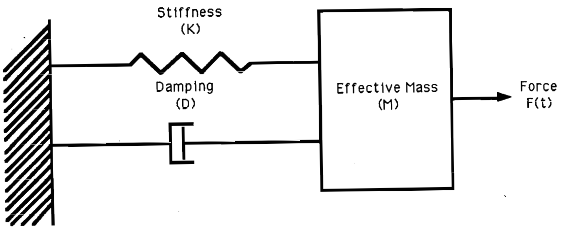

Figure 2: A Single Degree of Freedom System

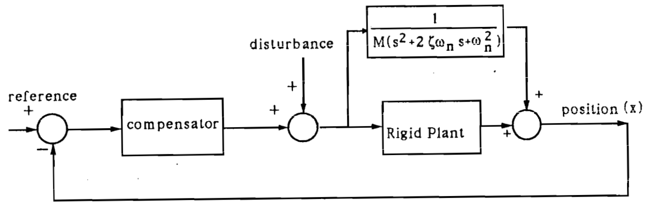

Figure 3: Block Diagram of the Plant including the First Bending Mode

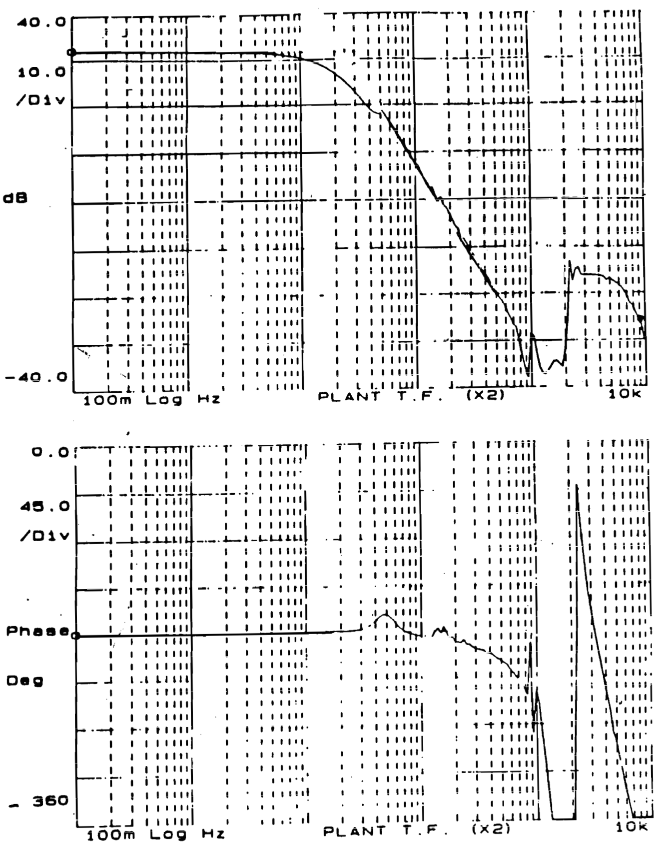

Figure 4: Experimental Frequency Response of the Plant Transfer Function ($\mathbf{x_2}$ axes)

4 Controller Implementation and Evaluation

4.1 Simulation Results

The Time Delay Controller law for the radial bearing was designed under the assumption that the rotor was rigid. The rotor has the first bending mode at frequencies between 800 Hz and 1000 Hz and the second bending mode between 2 kHz and 2.5 kHz. The bending mode natural frequencies were experimentally determined by using an impulse hammer, accelerometer and a signal analyzer. The damping ratio and the effective mass were estimated based on the theory of vibrations discussed in [16]. In the following simulations, $b = 100$, $a_{1m} = 4.0 \times 10^4$, and $a_{2m} = 282.8$ were selected for controlling the thrust bearing (z-axis). $b = 100$, $a_{1m} = 4.0 \times 10^4$ and $a_{2m} = 400$ were used in the case for the radial bearing.

4.1.1 Rigid Model

Simulations were performed for the radial bearing under Time Delay Control with the assumption that the rotor was a rigid body. Figures 5 a, b and c contain the position, the position error, and the control current responses for the radial bearing controller. The specifications for the simulations are as follows: second order reference model with a bandwidth of 200 rad/sec, damping ratio of 1, sampling rate T_s of 4 KHz. The time response of the above mentioned second order reference model should have no overshoot and a settling time of .02 sec. The system time response in Figure 5 behaves as that of a second order system as expected. The settling time of .025 sec obtained from Figure 5 a for the simulation is very close to the expected value of .02 sec. The time response has very negligible or no overshoot as expected. The control current increases in magnitude from 0 A to a maximum value of -0.44 A and then decreases to settle at a steady state value of about -0.01 A.

4.1.2 Flexible Model

The simulations were performed for the case where the closed loop bandwidth of the Time Delay Controller was 200 rad/sec. Figures 6 a, b and c contain the time response, the position error, and the control current of the rotor for the $\mathbf{x_2}$ axis. The specifications for the simulations are as follows: second order reference model with a bandwidth of 200 rad/sec, damping ratio of 1, sampling rate of 4 kHz. The effective mass m_{eff} of the bending mode was .5 kg, the damping ratio was .001, and the natural frequency is 875 Hz. The total rotor mass is 2.2 kg. The time response presented in Figure 6 a looks like the response of a second order system with a settling time of .03 sec and an overshoot of 5%. The time response of the rigid body without the bending mode for the same specifications was similar except that it had a negligible overshoot. The control current of the system does not show any significant change from the rigid body mode. Therefore for the bending mode with the above mentioned specification does not affect the system time response significantly.

4.2 Experimental Implementation and Results

4.2.1 Implementation of the Time Delay Controller

A detailed block diagram of the system is presented in Figure 7. In this experimental setup, we have the option of controlling the system using either a linear analog controller which resides in the compensation block, or a time delay controller implemented digitally in the DSP board.

The position signal for the Time Delay controller is obtained through the test points TP2 and/or TP3 and this signal is then sent into an analog to digital converter (A/D converter) which is linked to the DSP board. The A/D board has an adjustable built in low pass filter where the position signal can be filtered. The control voltage signal is sent out through the D/A converter which has a low pass filter with adjustable cutoff frequency. A switch for implementing the Time Delay Controller was installed in the control board. The system can be switched either to analog or digital from this switch.

4.2.2 Experimental Results

The Time Delay Controller was evaluated for its closed loop responses, disturbance rejection properties, and time responses for both the thrust (z axis) and the radial ($\mathbf{x_2}$ axis) bearing. The closed loop and disturbance rejection properties of the radial bearing were also analyzed while the rotor was spinning at speeds of 10,900 rpm, 20,100 rpm, 30,400 rpm, and 34,800 rpm.

The model reference for the thrust bearing was chosen to have a natural frequency of 200 rad/sec and a damping ratio of .707. The ex-

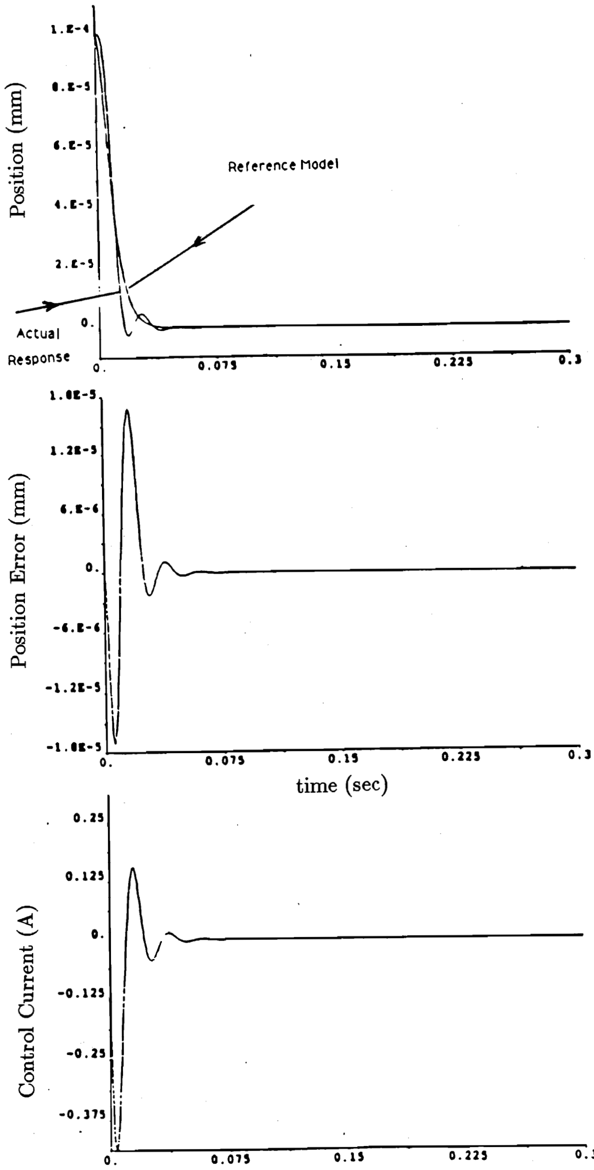

Figure 5: Simulated Rigid Body Model for x_2 Axis. Reference Model: $\omega_n = 200$ rad/sec, $\zeta = 1$, $T_s = 4$ KHz.

perimental data shown in Figure 8 indicate that the actual response tracks the reference model response very closely. In this case, the position moves from 200μm to 0μm which corresponds to the suspended configuration. The error is shown in the same figure and has about less than 10% maximum error. The control current necessary to produce this response is also shown in that same figure with a maximum current of about 1.75 amps. This is an excellent performance considering that the controller has no detailled information about the system. Figure 9 shows the closed-loop frequency response of the thrust bearing. In this case it is clear that the magnitude and phase characteristics are very close to those of the reference model selected. The disturbance rejection properties of the thrust bearing are shown in Figure 10. The controller rejects disturbances up to the bandwidth which is again around 200 rad/sec. The static stiffness is about 100 MN/m and the minimum stiffness is about 300 KN/m at the frequency of 200 rad/sec.

Figure 11 shows the closed loop frequency response for a radial bearing. This is very similar to that of the reference model. Figures 12 shows the disturbance rejection of the radial bearing when the rotor is at rest and while it is spinning at speeds of 10,900 rpm, 20,100 rpm, 30,400 rpm and 34,800 rpm. When the rotor is at rest the stator stiffness is about 200 MN/M and the minimum stiffness is about 500 kN/m at 100 Hz. It is clear that the disturbance rejection properties are almost the same for these different operating conditions. The peaks 800 Hz and 2.2 kHz correspond to the first two bending modes of the rotor. The peaks corresponding to the first bending mode are much more significant when the rotor is spinning at 34,800 rpm. Figure 13 shows the disturbance rejection when the rotor is at rest for reference model bandwidths of 100 and 200 rad/sec. The improvement in disturbance rejection properties with increase in reference model bandwidth is evident in this figure.

5 Conclusions

This paper has presented simple model for active magnetic bearings. The key point of the paper was the implementation of the Time Delay Control to such systems. The effectiveness of the digital control algorithm was first validated using several simulations which are based on linear and nonlinear models for the bearing including bending mode effects. Several experiments were conducted for spinning and nonspinning conditions. These include time responses, closed loop frequency responses and disturbance rejection responses. Evaluations were performed at bearing rotational speeds of 10,900 rpm, 20,100 rpm, 30,400 rpm and 34,800 rpm with Time Delay Controller bandwidths of 100 and 200 rad/sec. The digital controller presented shows an extremely high performance for the prototype considered by maintaining almost the same desirable dynamic behavior over the whole range of speeds.

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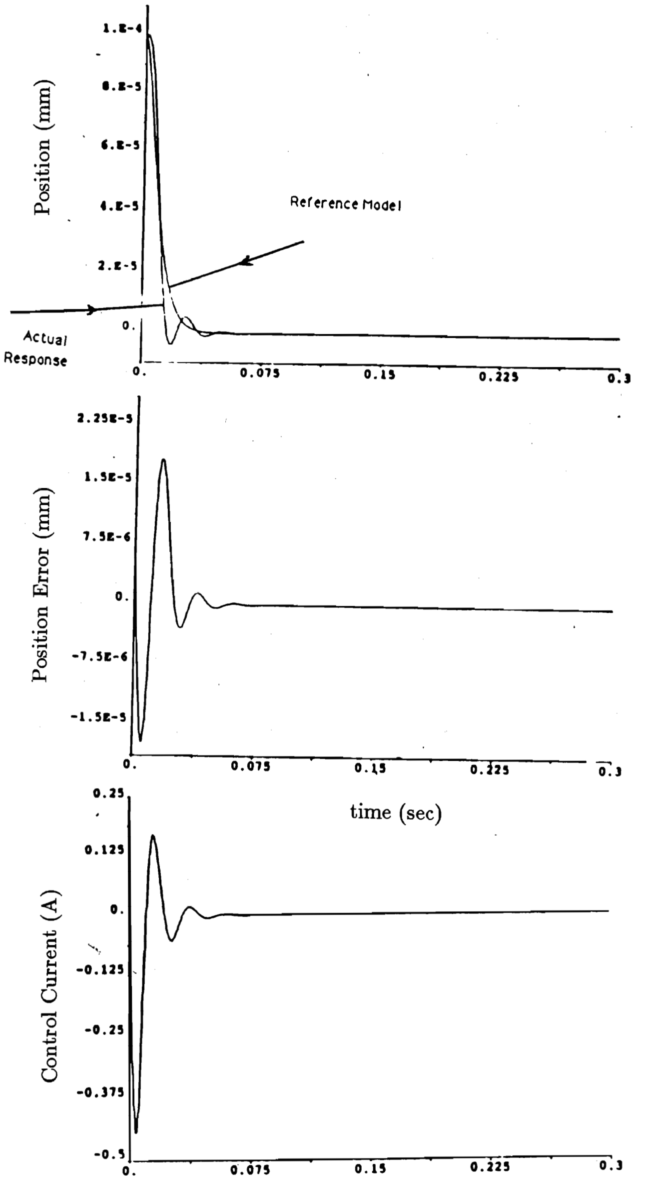

Figure 6: Simulated Flexible Body Model for x_2 Axis. Reference Model: $\omega_n = 200$ rad/sec, $\zeta = 1$, $T_s = 4$ kHz. Flexible Mode: $f_n =$ 875 Hz, $\zeta = 0.001$, $m_{eff} = 0.5$ Kg

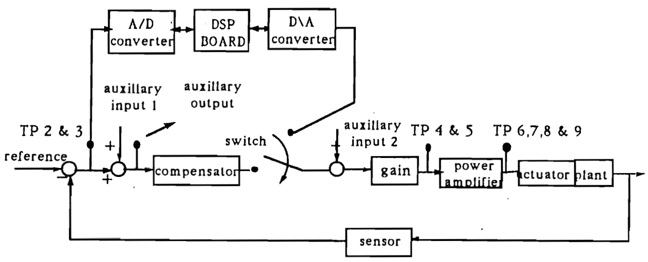

Figure 7: Block Diagram for the Analog and Digital Control

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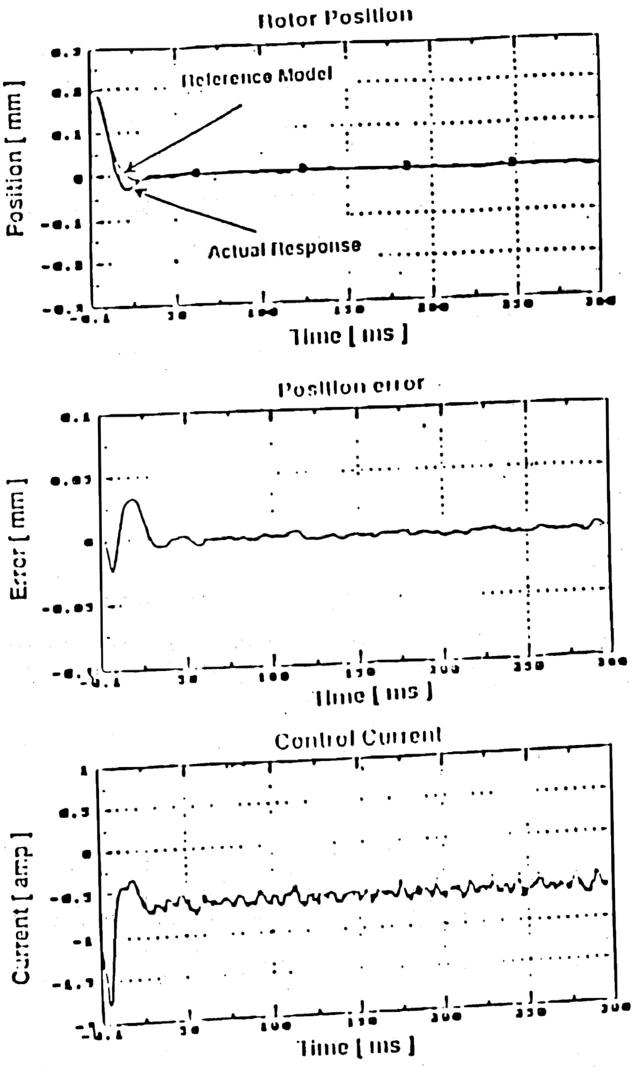

Figure 8: Experimental Time Response Data of the Thrust Bearing

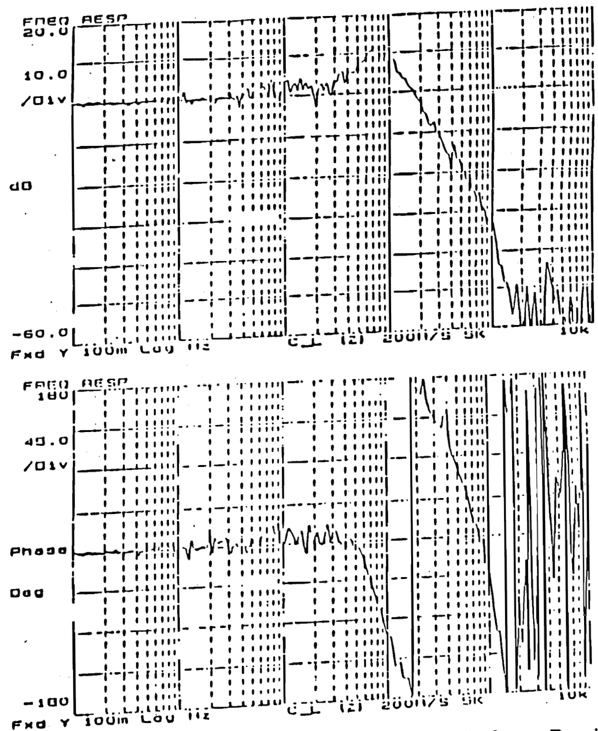

Figure 9: Closed Loop Frequency Response of Thrust Bearing

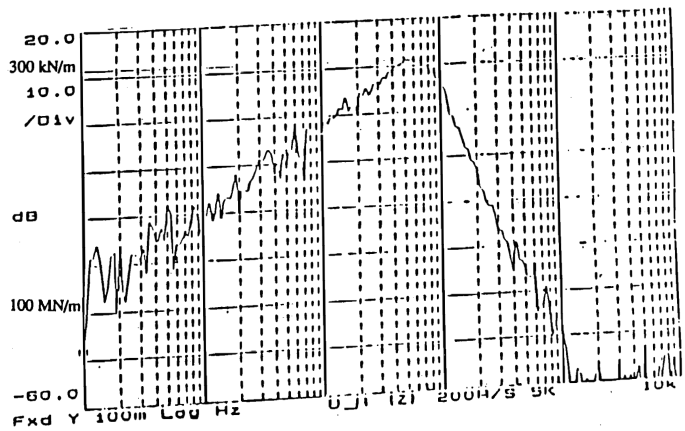

Figure 10: Disturbance Rejection of Thrust Bearing

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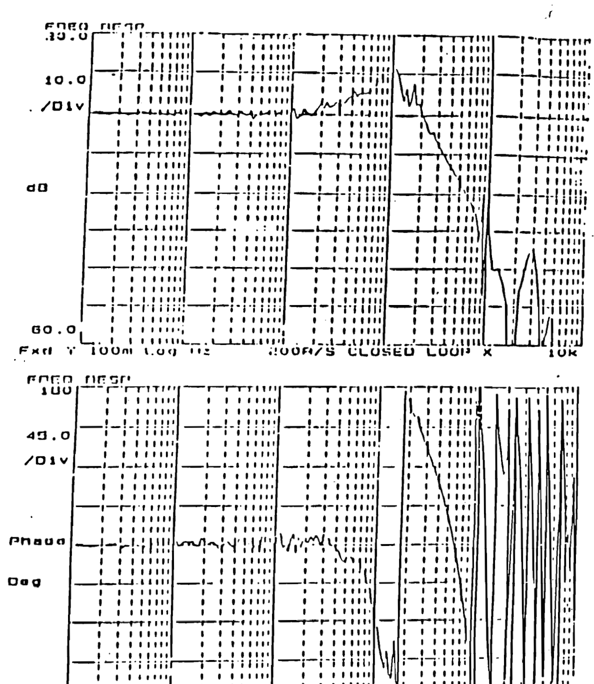

Figure 11: Closed Loop Frequency Response of Radial Bearing

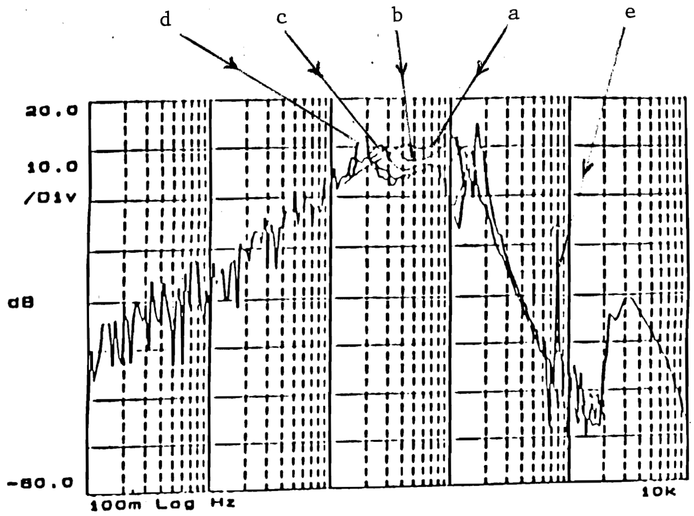

Figure 12: Disturbance Rejection of Radial Bearing: (a) at rest, (b) 10,900 rpm, (c) 20,100 rpm, (d) 30,400 rpm, (e) 34,800 rpm

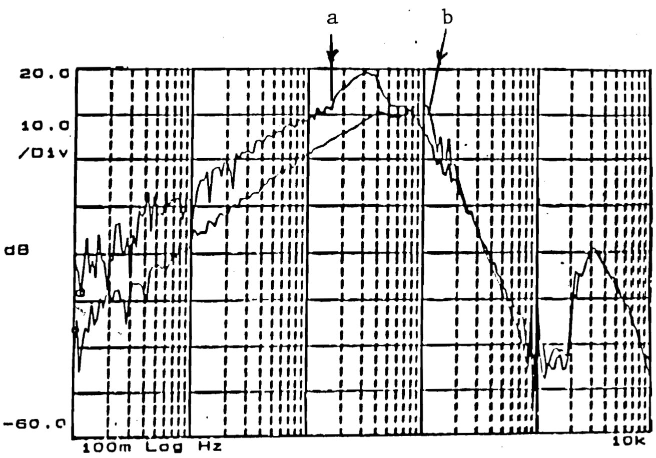

Figure 13: Disturbance Rejection of Radial Bearing with Rotor at Rest (a) bandwidth = 100 rad/sec (b) bandwidth = 200 rad/sec