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Compensation of Destabilizing Rotor Position Dependency in Asymmetric Radial Magnetic Bearings

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Abstract

Due to the nature of magnetic forces, the forces of a magnetic bearing depend not only on the coil currents but also on the position of the rotor with respect to the stator of the bearing. For conventional radial magnetic bearings with even number of poles, the destabilizing position dependency is modeled as a negative stiffness by linearizing the quadratic force-current relationship with respect to a constant bias current and zero displacements. Then, it is possible to apply a linear feedback control to stabilize it. Asymmetric bearings with odd number of poles have nonlinear and cross-coupled force-current relationships. In this paper, we use magnetic circuit theory to derive the force-current relationships of asymmetric bearings with odd number of poles. Through a linearization process, a new method of compensating the position dependency is proposed. A test rig consisting of a prototype compressor equipped with 9-pole radial bearings is constructed. Test results verify that the compensation is critical to achieve stable levitation. Only with the proposed compensation, the output sensitivities of the bearings satisfy the standard.

Keywords : Asymmetric Radial Bearing, Unbiased Control, Force-current Linearization

1. Introduction

Almost all heteropolar radial magnetic bearings are symmetric with respect to control axes. Typically, two adjacent poles are wired in series to form a horseshoe configuration. Since the force-current relation of symmetric bearings is quadratic, it is linearized by using bias currents. The electromagnetic forces are also related to the position of rotor with respect to the stator. This position dependency is destabilizing and must be taken care of by feedback control. For symmetric radial AMBs with even number of poles, the position dependency is often expressed as a (negative) stiffness after linearizing the quadratic force-current relationship around a fixed bias current.

Research efforts to use asymmetric bearings can be found in the literature, most notably bearings with three-poles (Chen and Hsu, 2002; Meeker and Maslen, 2006; Hemenway and Severson, 2020). Three-pole bearings are suitable only for very small rotors, because the shaft carrying the rotor laminations becomes increasingly small as the bearing size gets large. For the bearings of moderate size, other asymmetric bearings are possible: 5-pole, 7-pole, 9-pole, etc. Especially, 9-pole bearings with three adjacent poles wired in series are comparable to conventional 8-pole bearing in terms of size, and can be driven by a three-phase inverter.

In this paper, we describe the process of linearizing the force-current relationship of asymmetric bearings with odd number of poles and show how to model the position dependency as stiffnesses that take into account the cross-coupling between control axes. Also, a compensation scheme to eliminate or reduce the position dependency is proposed. Experimental tests are carried out to validate the compensation scheme. It is found that the proposed compensation is critical to satisfy stability requirements.

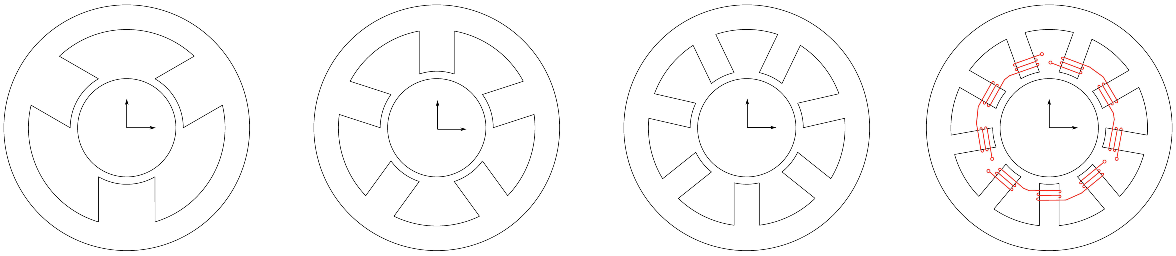

Fig. 1: Some examples of asymmetric radial bearings considered in this paper: 3-pole, 5-pole, 7-pole, and 9-pole bearing. Bearings are equipped with independent coils on each pole, except the 9-pole case. For the 9-pole bearing, three adjacent poles are wired in series.

2. Asymmetric Radial Magnetic Bearing

2.1. Asymmetric Bearings with Independent Coils

Some examples of asymmetric radial bearings are shown in Fig. 1. All bearings are equipped with independent coils (N_c turns) on each pole, except the 9-pole case. The magnetic field densities at each air gap, $\mathbf{b}$ are related to the coil current vector as

$$\mathcal{R}\mathbf{b} = \mathbf{N}\mathbf{i}\,. \tag{1}$$

The reluctance matrix is

$$\mathcal{R} = \frac{1}{\mu_0}\begin{bmatrix} g_1 & -g_2 & 0 & \cdots & 0 & 0 \\ 0 & g_2 & -g_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -g_{n_p-1} & 0 \\ 0 & 0 & 0 & \cdots & g_{n_p-1} & -g_{n_p} \\ g_0 & g_0 & g_0 & \cdots & g_0 & g_0 \end{bmatrix} \tag{2}$$

where

$$g_i = g_0 - x\cos\theta_i - y\sin\theta_i\,.$$

Here, θ_i is the angle of i-th pole with respect to x-axis. Using the reluctance and coil turn matrices, the magnetic forces are quadratically related to coil current vector as

$$f_q = \mathbf{i}^T\mathbf{M}_q\mathbf{i}, \quad q = x, y\,, \tag{3}$$

where $\mathbf{M}_q$ is the current mapping matrix determined from the reluctances of air gaps and the coil windings.

Since only two independent control currents are necessary, a current distribution matrix is introduced (Meeker, 2017; Hemenway and Severson, 2020)

$$\mathbf{i} = \begin{bmatrix} i_r \\ i_i \end{bmatrix} = \mathbf{W}\mathbf{i}_c = \mathbf{W}\begin{bmatrix} i_r \\ i_i \end{bmatrix} \tag{4}$$

For asymmetric bearings with odd number poles, a mapping matrix is derived if the poles are independently magnetized (Meeker, 2017)

$$\mathbf{W} = \sqrt{\frac{8}{n}}\begin{bmatrix} (-1)^0\cos\frac{1}{2}\theta_1 & (-1)^0\sin\frac{1}{2}\theta_1 \\ (-1)^1\cos\frac{1}{2}\theta_2 & (-1)^1\sin\frac{1}{2}\theta_2 \\ \vdots & \vdots \\ (-1)^{n_p-1}\cos\frac{1}{2}\theta_{n_p} & (-1)^{n_p-1}\sin\frac{1}{2}\theta_{n_p} \end{bmatrix} \tag{5}$$

This mapping matrix results in

$$\mathbf{W}^T\mathbf{M}_x\mathbf{W} = c_f\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} . \tag{6}$$

$$\mathbf{W}^T\mathbf{M}_y\mathbf{W} = c_f\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} \tag{7}$$

at the bearing center. Here, the constant is defined as

$$c_f = \frac{\mu_0 N^2 A_p}{g_0^2} .$$

A useful feature of the mappings of (6) and (7) is that the force-current relationships can be expressed in a complex form:

$$f_x + jf_y = c_f\,(i_r + ji_i)^2 \ , \tag{8}$$

so the control current can be obtained by simply inverting this quadratic equation if requested force commands are generated by the controller.

The force-current relationships of (3) are affected by the positions of the rotor since (6) and (7) only hold at the bearing center. We can linearize the force equation by defining the derivatives of the mapping matrices with respect to the displacements as

$$\mathbf{m}_{qr} = \frac{\partial \mathbf{M}_q}{\partial r}, \quad q, r = \bar{x}, \bar{y} \ , \tag{9}$$

where $\bar{x}$ and $\bar{y}$ are the displacements normalized by the bearing gap. Then, the linearized force in x-direction can be written as

$$f_x \approx \mathbf{i}_c^T\mathbf{W}^T\left(\mathbf{M}_x + \mathbf{m}_{xx}\bar{x} + \mathbf{m}_{xy}\bar{y}\right)\mathbf{W}\mathbf{i}_c \ . \tag{10}$$

The force in y-direction can be written similarly. The linearized force can also be expressed in a vector-matrix form as

$$\begin{bmatrix}f_x\\ f_y\end{bmatrix} = \begin{bmatrix}f_{x,0}\\ f_{y,0}\end{bmatrix} + \begin{bmatrix}K_{xx} & K_{xy}\\ K_{yx} & K_{yy}\end{bmatrix}\begin{bmatrix}\bar{x}\\ \bar{y}\end{bmatrix} \tag{11}$$

where the stiffnesses are defined as

$$K_{qr} = \mathbf{i}_c^T\mathbf{W}\mathbf{m}_{qr}\mathbf{W}\mathbf{i}_c \ . \tag{12}$$

Using (11), a compensation scheme is proposed to eliminate (or reduce) the dependency of force with respect to displacements. Denoting $\mathbf{i}_c^*$ as the requested control current at zero displacements obtained from (8) and $\mathbf{i}_{\text{comp}}$ as the small compensation current, we can write the linearized force equation such that

$$f_x = \left(\mathbf{i}_c^* + \mathbf{i}_{\text{comp}}\right)^T\mathbf{W}^T\left(\mathbf{M}_x + \mathbf{m}_{xx}\bar{x} + \mathbf{m}_{xy}\bar{y}\right)\mathbf{W}\left(\mathbf{i}_c^* + \mathbf{i}_{\text{comp}}\right) \tag{13}$$

Neglecting higher-order terms, (13) can be approximated as

$$f_x \approx \mathbf{i}_c^{*T}\mathbf{W}^T\mathbf{M}_x\mathbf{W}\mathbf{i}_c^* + 2\mathbf{i}_c^{*T}\mathbf{W}^T\mathbf{M}_x\mathbf{W}\mathbf{i}_{\text{comp}} + \mathbf{i}_c^{*T}\mathbf{W}^T\left(\mathbf{m}_{xx}\bar{x} + \mathbf{m}_{xy}\bar{y}\right)\mathbf{W}\mathbf{i}_c^* \tag{14}$$

Since the first term on the right-hand side is the requested force, the position dependency can be eliminated if the compensation current is determined so that the last two terms become zero. Combining the compensation in y axis, we can obtain the compensation scheme as

$$\mathbf{i}_{\text{comp}} = -\frac{1}{2}\begin{bmatrix}\mathbf{i}_c^{*T}\mathbf{W}^T\mathbf{M}_x\mathbf{W}\\ \mathbf{i}_c^{*T}\mathbf{W}^T\mathbf{M}_y\mathbf{W}\end{bmatrix}^{-1}\begin{bmatrix}K_{xx} & K_{xy}\\ K_{yx} & K_{yy}\end{bmatrix}\begin{bmatrix}\bar{x}\\ \bar{y}\end{bmatrix} \tag{15}$$

For the current distribution matrix of (5), the compensation current of (15) is simplified to

$$\mathbf{i}_{\text{comp}} = -\begin{bmatrix}i_r^* & i_i^*\\ -i_i^* & i_r^*\end{bmatrix}\begin{bmatrix}\bar{x}\\ \bar{y}\end{bmatrix} . \tag{16}$$

2.2. Nine-pole Bearing with Three-phase Drive

A variation of 3-pole bearings is 9-pole bearings as shown as the rightmost bearing in Fig. 1. Three adjacent poles are wired in series, forming one phase. With this winding pattern, it is possible to use a single three-phase inverter to drive the bearing. Also, the bearing size is comparable to the conventional 8-pole bearing in terms of the rotor shaft and the stator sizes (Noh and Jeong, 2023).

The force-current relationship of (3) can be obtained after defining the coil turn matrix as

$$\mathbf{N} = N_c \begin{bmatrix} -2 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 2 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -2 & 1 & 0 \end{bmatrix}^T \tag{17}$$

Also, the current distribution matrix is found by following the same optimization process proposed by Meeker (Meeker, 2017):

$$\mathbf{W} = \begin{bmatrix} 0.99126 & 0.26561 \\ -0.26561 & -0.99126 \\ -0.72565 & 0.72565 \end{bmatrix} \tag{18}$$

The force inversion of (8) and the compensation scheme of (15) produce a linear plant seen by the control. Thus, we can apply a linear control algorithm such as lead filters for stable suspensaion. The block diagram of suspension control is illustrated in Fig. 2.

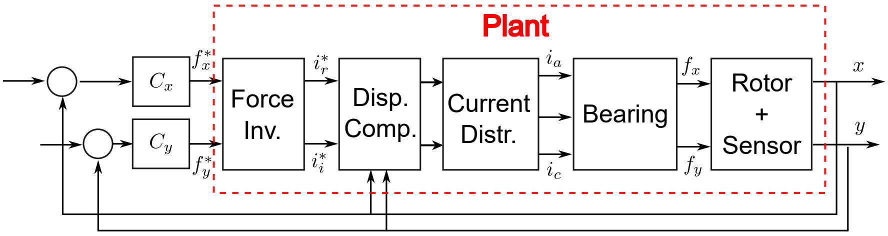

Fig. 2: Block diagram of the position-dependency compensation scheme

3. Experimental Results

In order to validate the compensation scheme, a test rig is built. Fig. 3 shows the picture of the 9-pole radial bearing and the prototype compressor equipped with two 9-pole radial bearings.

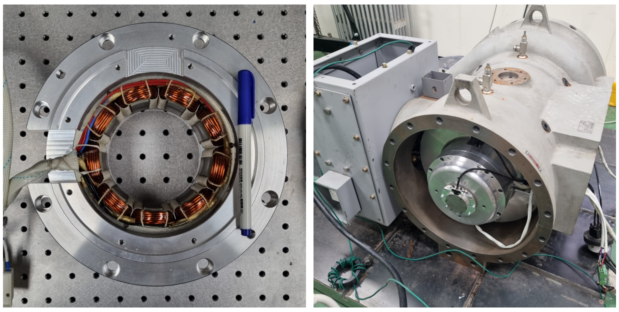

Fig. 3: Nine-pole radial bearing on the left, and the prototype compressor equipped with 9-pole bearings on the right

According to (15), no compensation is necessary if the rotor displacements are zero. Therefore, it is possible to levitate the rotor even without compensation when there is no disturbance. However, the effect of the compensation scheme become apparent if disturbance forces act on the rotor. Fig. 4 is the x displacement of the bearing 1 while the rotor is spinning at 300 rpm. The position compensation is turned off and on repeatedly.

If the position compensation is not applied, the rotor moves by about 70 μm. It decreases to about 40 μm, if the compensation scheme is turned on.

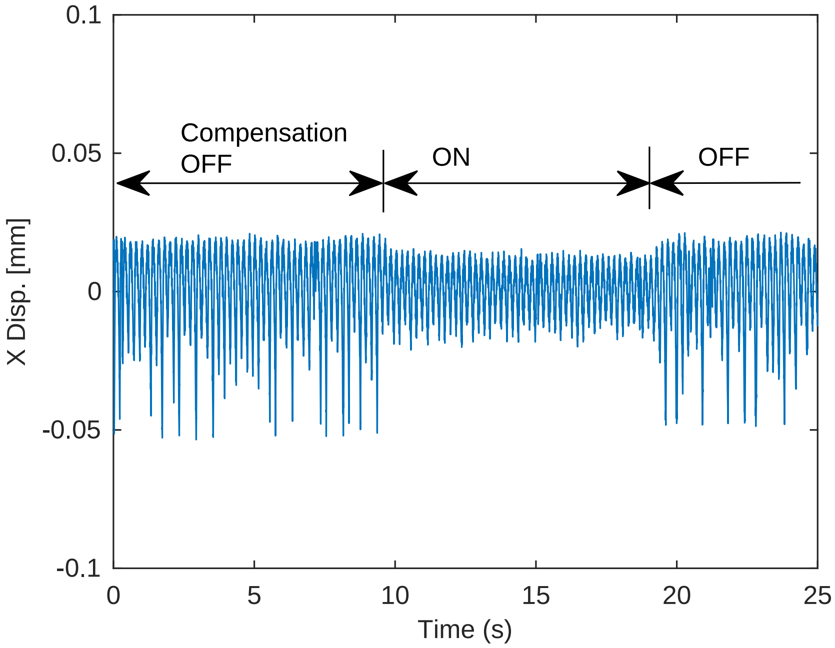

Fig. 4: Position compensation is turned off and on while the rotor is spinning at 300 rpm

4. Acknowledgments and Conflicts of Interest

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