Comparing Magnetic Bearings with Symmetry of 3

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Abstract

The stators of Active Magnetic Bearings (AMBs) usually show a symmetry of 4, because the magnetic forces applied to the rotor are generated in 4 points in a plane, two in the canonical x direction and two in the y direction. AMBs with symmetry of 3 are harder to find, either in real-world applications or in the specific literature. Their stators generate forces in 3 coplanar points; one of these points, at most, can lie in one of the canonical x,y directions. In this paper, basic aspects of the symmetry of 3 will be explained; they will help to develop the mathematical models of symmetry of 3 AMBs and to compare some situations in this world with the corresponding facts in the symmetry of 4 devices.

Keywords: *Symmetry of 4, Symmetry of 3, Coupled fluxes, Uncoupled fluxes.*

1. Introduction

The most usual AMBs [9, 2, 10] show the structure in the left side of Figure 1. There are two "U-shaped electromagnets" in the x or horizontal direction and two in the y or vertical direction, resulting in four independent magnetic flux loops. A different configuration for AMBs is possible [6, 5, 4, 3] with only four windings that lead to interconnected magnetic loops, as depicted in the right side of the same Figure. These structures are sometimes called 08 poles and 04 poles based on the number of stumps that leave the stator and not because of any magnetic property.

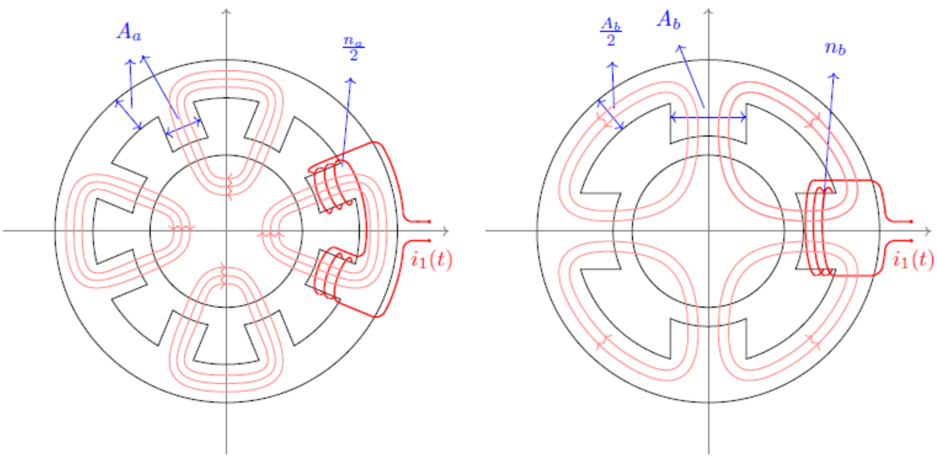

Figure 1: Windings, above, are shown for the positive x direction only; opposing pairs of windings along the x(y) direction control that position. In the traditional AMB, at the left, there are no connections among the flux paths; in the 04 poles case the flux paths are interconnected.

In both cases, the windings in the x,y directions are fed with currents $i_0 \pm i_{x,y}(t)$; the $i_{x,y}$ will control the rotor position. The resultant forces $f_{x,y}$ can be expressed in terms of these currents, the air magnetic permeability μ_0, the number of coils n, the cross-section areas A and the nominal length h of the air gaps [9, 2, 10]. After a standard linearization procedure [9] around the operating point $x = y = i_x = i_y = 0$, the forces generated by the traditional AMB are of the form $f_{x,y} = k_p x(y) + k_i i_x(i_y)$ where k_p and k_i are the position and current constants. Notice that the unconnected nature of the magnetic fluxes leads to uncoupled forces.

For the 04 poles stator the linearized expressions are the same, $f_{x,y} = k_p x(y) + k_i i_x(i_y)$, even though the fluxes are coupled; the important issue is that k_p and k_i have higher values than in the previous case and this allows AMBs to provide stiffer suspensions to rotors [6]. It seems safe to state that the 04 poles structure shows a cleaner and more compact design, which will probably result in more cost-effective manufacturing situations. It is also easy to accept that it offers more space for heat dissipation and that the flux losses in its coils are smaller. If all of these theoretical considerations turn out to be true, then the four poles geometry must be seen as a valid alternative to, if not as a better choice than, traditional AMBs [6].

As already mentioned, AMBs with 06 or 03 poles are harder to find, and the specific literature is not so vast [7, 1, 11, 8]. The main goal of this paper is to investigate whether the intriguing relations shown to exist between 08 and 04 poles AMBs do also appear in the symmetry of 3 geometries, between 06 and 03 poles AMBs. Is one of them "better" or stiffer than the other one?

2 Positioning a Body in a Plane with MDs

It is desired to drive a metallic particle to a point in the plane and to keep it there, with forces generated by Magnetic Devices (MDs). Such devices transform electric currents in forces that are applied to some mechanical load.

Planar positioning with 08 or 04 MDs is well known and provides basic examples of AMBs with symmetry of 4. Figure 2 depicts a particle at the origin of a coordinate system with 03 MDs symmetrically placed. An easy way to represent MDs is used in this figure; in real life, most MDs are either cylindrically or U-shaped pieces of a ferromagnetic material surrounded by coils of wire.

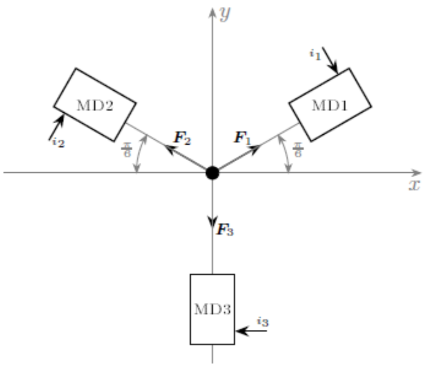

Figure 2: Three MDs are symmetrically placed around the centered particle.

The coordinates $x(t)$ and $y(t)$ (or the rotor position, in the AMBs case) are controlled by the forces $\boldsymbol{F}_k(t)$ for $k = 1,2,3$. The directions of these forces, see figure 2, can be considered fixed, for small displacements; projecting their absolute values in the x and y directions leads to

$$\sum_{k=1}^{3} F_k^x(t) = F_1(t)\cos\frac{\pi}{6} - F_2(t)\cos\frac{\pi}{6} + F_3(t)\cos\frac{3\pi}{2} = F^x(t) \tag{1}$$

$$\sum_{k=1}^{3} F_k^y(t) = F_1(t)\sin\frac{\pi}{6} + F_2(t)\sin\frac{\pi}{6} + F_3(t)\sin\frac{3\pi}{2} = F^y(t). \tag{2}$$

The dynamic characteristics of the particle (and also of the rotor) movements depend, basically, on the modules of the $\boldsymbol{F}_k(t)$. It is well known [9, 2, 10] that forces produced in MDs depend on the square of the magnetic flux and the area occupied by it, as in

$$F_k(t) = \frac{\phi_k^2(t)}{2\mu_0 A_k} \quad \text{for} \quad k = 1,2,3, \tag{3}$$

where μ_0 is the air magnetic permeability. For equal areas A_k each

force depends only on the magnetic flux $\phi_k(t)$. This flux is squared in equation (3) and thus MDs generate attractive forces only. In some cases, $\phi_k(t)$ is affected only by the current $i_k(t)$ and by the distance d_k between the body and a point in MDk, usually the closest and most internal one. This case is fairly easy to study but in other situations fluxes in each channel depend on the parameters of all MDs:

$$\phi_k(t) = \varphi_k\left(i_1(t), i_2(t), i_3(t), d_1(t), d_2(t), d_3(t)\right) \quad \forall k = 1, 2, 3.$$

Symmetry assures that the distance *h* between the origin and each actuator is the same. Let e_k measure the body's displacement in the *k* axis, considered positive when pointing towards MDk. Figure 3 shows the situation for channel 1. When the load dimensions are significant, as in AMBs, *h* is the nominal gap width. In the general case, it is clear that

$$d_k(t) = h - e_k(t) \quad \text{for} \quad k = 1, 2, 3. \tag{5}$$

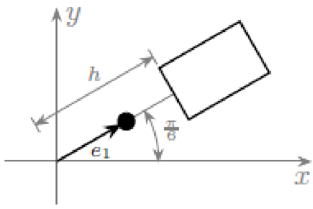

Figure 3: Particle's displacement towards MD1; if the real position is not as above, it should be projected on the line from the origin to the i actuator.

It is easier to measure the $e_k(t)$, the body's displacements from the origin than its distances $d_k(t)$ to the MDs. Therefore, equation (4) can be updated.

$$\phi_k(t) = \varphi_k\left(i_1(t), i_2(t), i_3(t), e_1(t), e_2(t), e_3(t)\right) \quad \forall k = 1, 2, 3. \tag{6}$$

The body's motion depends on 03 injected currents in the MDs, and on 03 positions to be measured. The displacements e_1,e_2,e_3 are redundant to determine the position in the plane, only two numbers are required for this. It is natural to assume that only two displacement sensors are installed in practical implementations, covering the canonical and orthogonal directions *x* and *y*.

When the particle is not located at a particular position in the plane, as in figure 3, the vectors ***x***(*t*) and ***y***(*t*) must be projected on the symmetry of 3 directions as shown in figure 4.

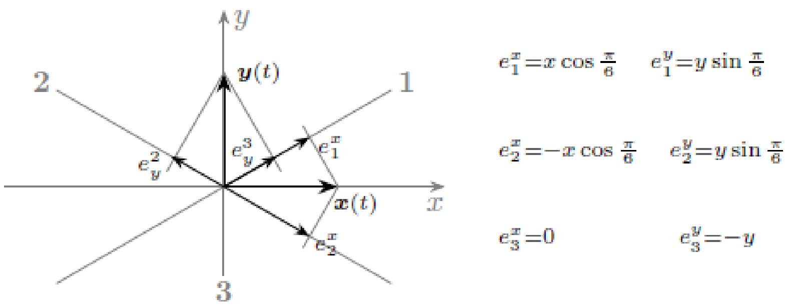

Figure 4: How displacements measured in the *x* and *y* directions are felt in the symmetrical axes 1, 2, 3 of the Mds.

The $e_k(t)$ in figure 4 are related to the measurable quantities *x*(*t*) and *y*(*t*) with the help of

$$e_1(t) = x(t)\cos\frac{\pi}{6} + y(t)\sin\frac{\pi}{6} \tag{7}$$

$$e_2(t) = -x(t)\cos\frac{\pi}{6} + y(t)\sin\frac{\pi}{6} \tag{8}$$

$$e_3(t) = x(t)\cos\frac{3\pi}{2} + y(t)\sin\frac{3\pi}{2} = -y(t) \tag{9}$$

The quantities $d_k = h - e_k$ using these values are approximations to the real distances between the body and the MDs. Since these displacements are small, the approximations are accepted and used without any care. There are, now, only 05 variables affecting the behavior of fluxes in MDs and the obtained resultant forces. Equations (4) and (6) can be further improved

$$\phi_k(t) = \varphi_k\left(i_1(t), i_2(t), i_3(t), x(t), y(t)\right) \quad \forall k = 1, 2, 3. \tag{10}$$

To analyze the action of the currents $i_k(t)$ and whether it is possible to position the particle with less than 3 control variables, particular cases will be considered in the next sections.

3 AMBs with uncoupled fluxes

A typical illustration is shown in figure 5. The stator is a cylindrical piece of ferromagnetic material in which three pairs of poles can be recognized. The word pole used here has no magnetic meaning, it indicates parts of the stator that point towards the rotor.

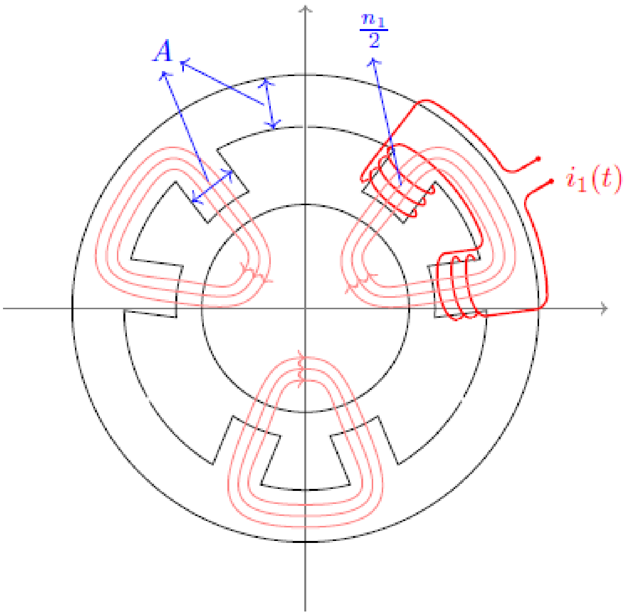

Figure 5: 06 poles configuration for AMBs; windings are shown for channel 1 only; there are no connections among the flux paths. All pairs of windings control the *x* and *y* positions.

Each pair of poles is surrounded by coils carrying currents and performs as an independent U-shaped electromagnet. The three MDs generating forces are autonomous because all the flux injected in the rotor by one of the poles is absorbed by the other pole in the same channel. Each of the fluxes ϕ_k follows a path in the plane that does not touch or intercept the fluxes by the other channels: they are said to be uncoupled, decoupled, or unconnected. Basic results on reluctance forces [9, 2, 10] can be used to find the magnitude of the force generated at each MD:

$$F_k(t) = \frac{\mu_0 A_k n_k^2}{4}\left(\frac{i_k(t)}{d_k(t)}\right)^2 \qquad \text{for} \qquad k = 1, 2, 3. \tag{11}$$

where μ_0 is the air magnetic permeability. Assuming a uniformly built stator, $A_1 = A_2 = A_3 = A$ and $n_1 = n_2 = n_3 = n$; using equation (5):

$$F_k(t) = K\left(\frac{i_k(t)}{h - e_k(t)}\right)^2 \quad \text{with} \quad K = \frac{\mu_0 A n^2}{4} \quad \text{for} \quad k = 1, 2, 3. \tag{12}$$

The forces on the rotor, in the *x* and *y* directions, come from equations (1) and (2); the result, omitting the (*t*) in the time-varying signals, is

$$F^x = (F_1 - F_2)\cos\frac{\pi}{6} = \frac{K\sqrt{3}}{2}\left[\left(\frac{i_1}{d_1}\right)^2 - \left(\frac{i_2}{d_2}\right)^2\right] \quad (13)$$

$$F^y = (F_1 + F_2)\sin\frac{\pi}{6} - F_3 = \frac{K}{2}\left[\left(\frac{i_1}{d_1}\right)^2 + \left(\frac{i_2}{d_2}\right)^2 - 2\left(\frac{i_3}{d_3}\right)^2\right]. \quad (14)$$

Recalling equations (7), (8) and (9) and since $d_k = h - e_k$, the three distances in the above formulas depend on the displacements *x* and *y* only:

$$d_1 = h - \frac{\sqrt{3}}{2}x - \frac{1}{2}y; \quad d_2 = h + \frac{\sqrt{3}}{2}x - \frac{1}{2}y; \quad d_3 = h + y. \quad (15)$$

AMBs operate close to a situation of no displacements and zero external inputs and so it is natural to linearize (13) and (14) around $(i_1^0, i_2^0, i_3^0, x^0, y^0) = (0,0,0,0,0)$. The result would be an uncontrollable model for the AMB. This suggests the use of procedures that work successfully in the symmetry of 04 world: base and differential currents. It is reasonable to consider a fixed base current i_0 and three differential currents $v_k(t)$ to be rigged as

$$i_1(t) = i_0 + v_1(t); \qquad i_2(t) = i_0 + v_2(t); \qquad i_3(t) = i_0 + v_3(t). \quad (16)$$

Equations (15) and (16) have similar structures; the first one shows how the 03 parameters d_k can be expressed in terms of *x* and *y*. What if the same thing could be done to the 03 $v_k(t)$ in the last equation? Let's postulate two differential currents $i_x(t)$ and $i_y(t)$ and, guided by (15), choose $v_1 = i_x + i_y$, $v_2 = -i_x + i_y$ and $v_3 = -i_y$. This leads to

$$i_1(t) = i_0 + i_x(t) + i_y(t), \quad i_2(t) = i_0 - i_x(t) + i_y(t), \quad i_3(t) = i_0 - i_y(t). \quad (17)$$

The actions of three differential currents would be done by only two… Will this scheme work? will i_x and i_y control the *x* and *y* directions? The (positive) answers to these questions depend on a detailed study of equation (16) that will not be done here.

3.1 Base and differential currents

A mathematical model for the force generation in 06 poles AMBs can be obtained by combining equations (13), (14), (15) and (16). The net result is

$$F^x = f(v_1, v_2, v_3, x, y) \qquad \text{and} \qquad F^y = g(v_1, v_2, v_3, x, y). \quad (18)$$

The traditional procedure for linearizing around the operating point OP = $(v_1^o, v_2^o, v_3^o, x^o, y^o) = (0,0,0,0,0)$ leads to

$$F^x - [F^x]_o = \sum_{k=1}^{3}\left[\frac{\partial f}{\partial v_k}\right]_o (v_k - v_k^o) + \left[\frac{\partial f}{\partial x}\right]_o (x - x^o) + \left[\frac{\partial f}{\partial y}\right]_o (y - y^o) \quad (19)$$

$$F^y - [F^y]_o = \sum_{k=1}^{3}\left[\frac{\partial g}{\partial v_k}\right]_o (v_k - v_k^o) + \left[\frac{\partial g}{\partial x}\right]_o (x - x^o) + \left[\frac{\partial g}{\partial y}\right]_o (y - y^o) \quad (20)$$

A thorough calculation of the parameters and derivatives results in

$$F^x = k_p x + \sqrt{3}k_v(v_1 - v_2) \quad \text{and} \quad F^y = k_p y + k_v(v_1 + v_2) - 2k_v v_3 \quad \text{where} \quad (21)$$

$$k_p = \frac{3Ki_0^2}{h^3} = 3\frac{\mu_0 A n^2 i_0^2}{4h^3} \qquad \text{and} \qquad k_v = \frac{Ki_0}{h^2} = \frac{\mu_0 A n^2 i_0}{4h^2}. \quad (22)$$

The wild guess of a previous paragraph, $v_1 = i_x + i_y$, $v_2 = -i_x + i_y$ and $v_3 = -i_y$ transforms the equations above to

$$F^x = k_p x + 2\sqrt{3}k_v i_x \quad \text{and} \quad F^y = k_p y + 4k_v i_y \quad (23)$$

showing a completely decoupled structure: this is a nice and desired fact. To understand how these magnetic forces depending on 03 (or 02) differential currents act on the rotor dynamics, a detailed mathematical model is needed; this will be not covered in this article. In the next section one of the possible symmetry of 3 correspondents of a 04 poles structure will be studied.

4 AMBs with 03 poles and coupled fluxes

A typical illustration is shown in figure 6. The stator is a cylindrical piece of ferromagnetic material from which three single poles point inwards, in the direction of the rotor. The same considerations made previously about the word pole are valid here and in the rest of the article.

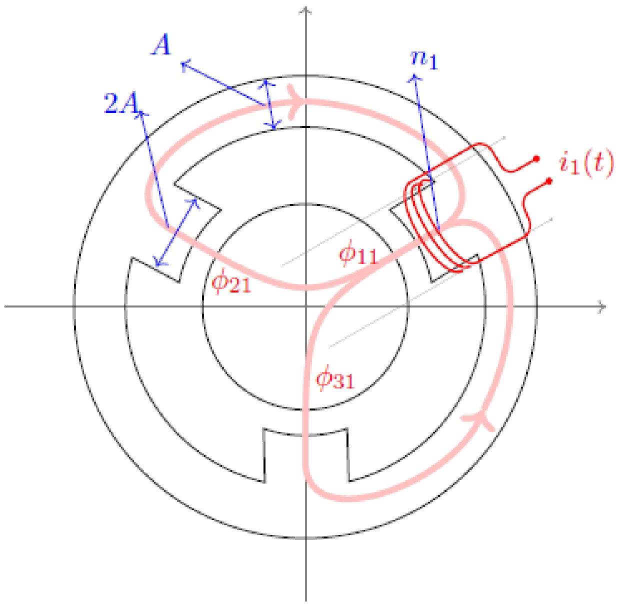

Figure 6: 03 poles configuration for AMBs. Windings are shown for channel 1 only; the flux generated in this channel enters the other poles. All windings control the *x* and *y* positions.

The ferromagnetic connections in a 03 poles structure allow a current injected in any winding to cause fluxes in all three air gaps; figure 6 illustrates the effects of i_1. If ϕ_{jk} denotes the flux in air gap *j* caused by a current in winding *k*, the total magnetic flux ϕ_1 in pole 1 depends on the fluxes ϕ_{11},ϕ_{12},ϕ_{13}. Assuming no air or ferromagnetic losses and positive signs for fluxes headed to the rotating center, the total magnetic fluxes in the poles are:

$$\phi_1 = \phi_{11} - \phi_{12} - \phi_{13}, \quad \phi_2 = -\phi_{21} + \phi_{22} - \phi_{23}, \quad \phi_3 = -\phi_{31} - \phi_{32} + \phi_{33}. \qquad (24)$$

For the determination of the ϕ_{jk}, consider the circuit in figure 7 that models the magnetic flux situation. The magneto-motive force generated by i_1 is denoted by 1 and the reluctance of the air gaps by 1, 2, and 3.

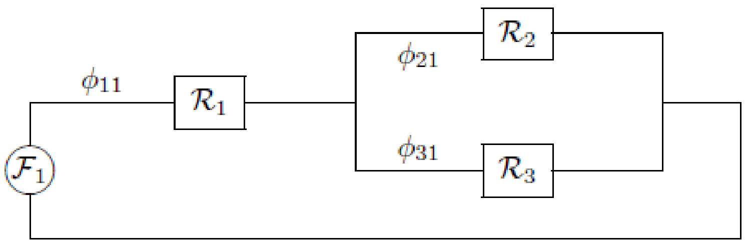

Figure 7: Magnetic flux equivalent circuit associated with current $i_1(t)$.

Recalling that 2*A* is the cross-section area of the poles and that the displacements $d_k = h - e_k$ are explained in figure 3, the reluctances are:

$$\mathcal{R}_1 = \frac{d_1}{\mu_0 2A}, \qquad \mathcal{R}_2 = \frac{d_2}{\mu_0 2A}, \qquad \mathcal{R}_3 = \frac{d_3}{\mu_0 2A}. \qquad (25)$$

A simple way to study magnetic circuits is by using the passive electric circuit analogy: fluxes are treated as currents, reluctances act as resistances and the usual operations allowed in Kirchoff's rules are valid. Reluctance 1 above is in series with the parallel combination of 2 and 3. If * denotes the equivalent reluctance of the parallel combination, then the magnetic circuit in figure 7 can be replaced by a single reluctance 1^e given by:

$$\mathcal{R}_1^e = \mathcal{R}_1 + \mathcal{R}_* = \mathcal{R}_1 + \frac{\mathcal{R}_2\mathcal{R}_3}{\mathcal{R}_2 + \mathcal{R}_3} = \frac{\mathcal{R}_1\mathcal{R}_2 + \mathcal{R}_1\mathcal{R}_3 + \mathcal{R}_2\mathcal{R}_3}{\mathcal{R}_2 + \mathcal{R}_3} \tag{26}$$

To avoid cumbersome formulas, some auxiliary variables are defined:

$$N = \mathcal{R}_1\mathcal{R}_2 + \mathcal{R}_1\mathcal{R}_3 + \mathcal{R}_2\mathcal{R}_3, \qquad D = d_1d_2 + d_1d_3 + d_2d_3, \tag{27}$$

$$D_1 = \mathcal{R}_2 + \mathcal{R}_3, \qquad D_2 = \mathcal{R}_1 + \mathcal{R}_3, \qquad D_3 = \mathcal{R}_1 + \mathcal{R}_2. \tag{28}$$

Assuming the same number of coils *n* in each pole and using again the imposed currents as in (16), algebraic operations lead to expressions for the fluxes generated by Pole 1 of figure 6:

$$\phi_{11} = ni_1\frac{D_1}{N}, \qquad \phi_{21} = ni_1\frac{\mathcal{R}_3}{N}, \qquad \phi_{31} = ni_1\frac{\mathcal{R}_2}{N}. \tag{29}$$

The same procedure, repeated for currents i_2, i_3 imposed at the windings in Poles 2, and 3 in figure 6 results in:

$$\phi_{12} = ni_2\frac{\mathcal{R}_3}{N}, \qquad \phi_{22} = ni_2\frac{D_2}{N}, \qquad \phi_{32} = ni_2\frac{\mathcal{R}_1}{N},$$

$$\phi_{13} = ni_3\frac{\mathcal{R}_2}{N}, \qquad \phi_{23} = ni_3\frac{\mathcal{R}_1}{N}, \qquad \phi_{33} = ni_3\frac{D_3}{N}.$$

Using these values of the partial fluxes ϕ_{jk} in equation (24) the total fluxes ϕ_k for $k = 1,2,3$ can be determined. Then, with the help of (3), the total reluctance forces generated in a 03 poles AMB can be expressed as:

$$F_k = \mu_0 A n^2 \left(\frac{N_k}{D}\right)^2 \quad \text{for} \quad k = 1, 2, 3. \tag{30}$$

where D comes from (27) and, using $I_{jk} = i_j - i_k$,

$$N_1 = d_2I_{13} + d_3I_{12} \qquad N_2 = d_1I_{23} + d_3I_{21} \qquad N_3 = d_1I_{32} + d_2I_{31}. \tag{31}$$

These last results together with equations (1) and (2) can be used to find the resultant forces in the *x* and *y* directions:

$$F_x = \frac{\sqrt{3}}{2}(F_1 - F_2) = \cdots \ = K_x\frac{N_1^2 - N_2^2}{D^2} \tag{32}$$

$$F_y = \frac{1}{2}(F_1 + F_2) - F_3 = \cdots \ = K_y\frac{N_1^2 + N_2^2 - 2N_3^2}{D^2} \tag{33}$$

where $K = \mu_0 A n^2/2$, K_x = Ksqrt{3}/2 and $K_y = K/2$. *It is clear that the forces in the canonical directions*

$$F_x = K_x q_x(N_1, N_2, N_3, D) \qquad \text{and} \qquad F_y = K_y q_y(N_1, N_2, N_3, D) \tag{34}$$

depend on geometric and excitation parameters defined on the symmetry of 3 axes: the positions d_1, d_2, d_3 and the currents i_1, i_2, i_3. Equations (7), (8), (9) together with (5) lead to

$$d_1 = h - \frac{\sqrt{3}}{2}x - \frac{1}{2}y, \qquad d_2 = h + \frac{\sqrt{3}}{2}x - \frac{1}{2}y, \qquad d_3 = h + y. \tag{35}$$

This means that $D = d_1d_2 + d_1d_3 + d_2d_3$ and N_k depend on the canonical parameters *x* and *y*: $D = D(x,y)$ and $N_k = N_k(i_1,i_2,i_3,x,y)$. The resultant forces can be expressed as

$$F_x = K_x q_x(i_1, i_2, i_3, x, y) = f(i_1, i_2, i_3, x, y) \tag{36}$$

$$F_y = K_y q_y(i_1, i_2, i_3, x, y) = g(i_1, i_2, i_3, x, y). \tag{37}$$

The next step, by the script used in the 06 poles case, would be to use the traditional base and differential currents idea, $i_k(t) = i_0 + v_k(t)$, and linearize the above expressions. This scheme fails, unfortunately, so a more detailed study of the problem will be made in the next section.

4.1 Generalized base and differential currents

The injected currents will be

$$i_1(t) = b_1 + v_i(t), \qquad i_2(t) = b_2 + v_2(t), \qquad i_3(t) = b_3 + v_3(t). \tag{38}$$

The base currents are still constant but not the same anymore. Different base currents can be used in horizontal rotors AMBs [1] but the idea here is to study how the 03 poles case linearization behaves in this general situation. Equations (32), (33) and (34) are now turned into

$$F_x = f(v_1, v_2, v_3, x, y), \qquad F_y = g(v_1, v_2, v_3, x, y). \tag{39}$$

The linearization around the operating point OP = ($v1^o$,$v2^o$,$v3^o$,x^o,y^o) = (0,0,0,0,0) relies on equations (19) and (20). A careful calculation of the parameters and derivatives, using B_{jk} to denote b_j - b_k and f_z^o to denote the partial derivative of *f* with respect to *z* evaluated at the operating point leads to

$$f_{v_1}^{\text{o}} = \frac{2K_x}{3h^2}B_{12}, \qquad f_{v_2}^{\text{o}} = \frac{-2K_x}{3h^2}B_{23}, \qquad f_{v_3}^{\text{o}} = \frac{2K_x}{3h^2}B_{21}, \tag{40}$$

$$f_x^{\text{o}} = \frac{K_x\sqrt{3}}{9h^3}\left[B_{12}^2 + B_{13}^2 + B_{23}^2\right], \qquad f_y^{\text{o}} = 0, \tag{41}$$

$$g_{v_1}^{\text{o}} = \frac{2K_y}{3h^2}(B_{12} + B_{32}), \quad g_{v_2}^{\text{o}} = \frac{2K_y}{3h^2}(B_{21} + B_{31}), \quad g_{v_3}^{\text{o}} = \frac{2K_y}{3h^2}(B_{13} + B_{23}), \tag{42}$$

$$g_x^{\text{o}} = 0, \qquad g_y^{\text{o}} = \frac{2K_y}{9h^3}\left[2B_{12}^2 + B_{13}^2 + B_{23}^2 + b_3(b_3 - b_1 - b_2) + b_1 b_2\right]. \tag{43}$$

Useful linearizations for AMBs require that Fx0=Fy0=0. The calculations for this case lead to the necessary conditions

$$[F_x]_{\text{o}} = 0 \Longleftrightarrow (b_1 + b_2 - 2b_3)(b_1 - b_2) = 0, \tag{44}$$

$$[F_x]_{\text{o}} = 0 \Longleftrightarrow (b_1 - 2b_2 + b_3)(b_1 - b_3) + (b_2 - 2b_1 + b_3)(b_2 - b_3) = 0. \tag{45}$$

The chosen values for the b_k must satisfy the above equations; in order to make (44) true there are two possibilities:

Choice 01: b_1 - b_2 = 0 or b_1 = b_2. Plugging this in (45) leads to b_3 = b_1 and the overall choice is b_1 = b_2 = b_3 = b_0 the same base current for all injected currents. Using this in equations (40), (41), (42) and (43) shows us that no linearization can be achieved, because

$$f_{v_1}^{\text{o}} = f_{v_2}^{\text{o}} = f_{v_3}^{\text{o}} = f_x^{\text{o}} = f_y^{\text{o}} = g_{v_1}^{\text{o}} = g_{v_2}^{\text{o}} = g_{v_3}^{\text{o}} = g_x^{\text{o}} = g_y^{\text{o}} = 0. \tag{46}$$

Choice 02: b_1 + b_2 - 2b_3 = 0 or b_3 = (b_1 + b_2)⁄2. Using this in (45) leads, after some manipulations, to … b_1 - b_2 = 0.

The sad conclusion is that no linearization is possible; the fluxes are so coupled and interconnected in the 03 poles geometry that any practical utilization of them is jeopardized. A parallel physical observation of the 03 poles structure can put reasonable doubts about its validity; the above mathematical procedures seems to give a definite answer to this question. The uncoupled/coupled comparisons between 08 and 04 poles can still be made in the symmetry of 3 world, because there is yet another possible structure for AMBs with magnetic forces separated by (2π⁄3)rd angles; it will be presented and partially studied in the next section.

5 AMBs with 06 poles and coupled fluxes

A typical illustration is shown in figure 8. The stator is a cylindrical piece of ferromagnetic material from which six equally spaced single poles point toward the rotor. Each pair of diametrically opposed poles forms a channel; poles 1 and 4, 2 and 5, 3 and 6 are responsible for channels 1, 2, 3. The poles in channel *k* are fed with currents $i_0 \pm v_k$.

The ferromagnetic structure in a 06 poles with coupled fluxes allows currents injected in any winding to cause fluxes in all air gaps; figure 8 illustrates the effects of $i_1(t) = i_0 + v_1(t)$. If ϕ_{jk} denotes the flux in air gap j caused by a current in winding k, the total magnetic flux ϕ_1 in pole 1 depends on the fluxes ϕ_{11},ϕ_{12} ϕ_{16}. Assuming no air or ferromagnetic losses and positive signs for fluxes headed to the rotating center, the total magnetic fluxes in the poles are:

$$\phi_1 = \phi_{11} - \phi_{12} - \phi_{13} - \phi_{14} - \phi_{15} - \phi_{16}, \ \phi_2 = -\phi_{21} + \phi_{22} - \phi_{23} - \phi_{24} - \phi_{25} - \phi_{26}, \tag{47}$$

$$\phi_3 = -\phi_{31} - \phi_{32} + \phi_{33} - \phi_{34} - \phi_{35} - \phi_{36}, \phi_4 = -\phi_{41} - \phi_{42} - \phi_{43} + \phi_{24} - \phi_{45} - \phi_{46}, \tag{48}$$

$$\phi_5 = -\phi_{51} - \phi_{52} - \phi_{53} - \phi_{54} + \phi_{35} - \phi_{56}, \phi_6 = -\phi_{61} - \phi_{62} - \phi_{63} - \phi_{64} - \phi_{65} + \phi_{66}. \tag{49}$$

More about this will be presented in a future article.

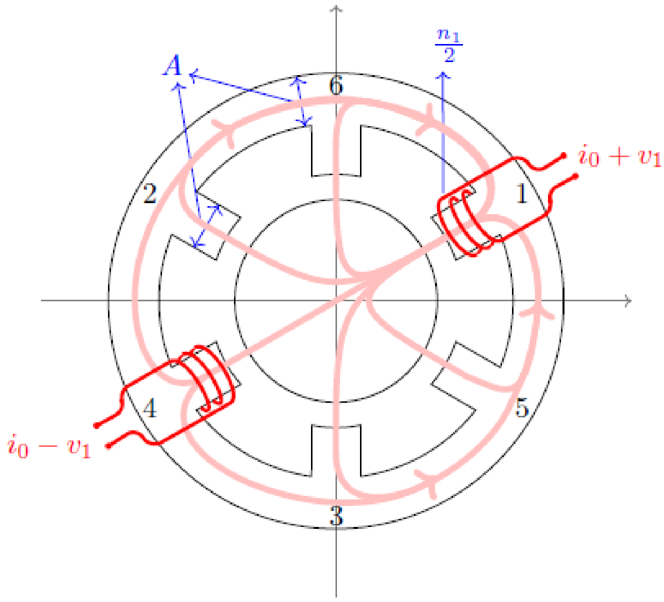

Figure 8: 06 poles and coupled fluxes configuration for AMBs. Windings are shown for channel 1 — poles 1 and 4 — only. The flux generated in any pole enters all the other poles; flux emanating from pole 1 is shown.

6 Conclusions

In the symmetry of 4 world 04 poles devices work as good as, if not better than, 08 poles ones. It was expected that the same would happen in the symmetry of 3 world with 06 and 03 poles, but the above paragraphs show that this is not the case. What remains to be investigated is whether the 03 poles with coupled fluxes structure plays in the symmetry of 3 world a role that corresponds to the 04 poles devices; and, in a case of a positive answer, can AMBs built based on this geometry be of any practical value?

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