

# Implementation of the DSP-based Fractional Order PID Controller for a Centrifugal Compressor by AMBs

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## Abstract

Active magnetic bearings (AMBs) have many advantages compared with the traditional bearings, especially in the non-contact characteristic that improves applicability of AMBs supported rotor to high rotation speed applications such as the centrifugal compressor that has played a key role in high-end air-conditioning systems. However, precise positioning and high stability are indispensable in controller design for AMBs. Unavoidably, there are many harmful elements which affect the AMBs supported rotor in actual operations. The conventional PID controller has gained popularity in AMBs systems because of easily implementation and tuning intuitive. However, the PID controller may not be sufficient to stabilize complex dynamics of AMBs supported rotor. In order to enhance the stability of the AMBs supported rotor, this study proposed to apply the fractional order PID (FOPID) controller to the centrifugal compressor, in which the centrifugal compressor equipped with five-degree of freedom AMBs and the FOPID has been implemented by digital signal processor (DSP). The discretization implementations are described in detail, including the difficulties encountered and the corresponding solutions. Experimental results show the performance of FOPID compared with the traditional integer order PID (IOPID), and the FOPID can exhibit satisfactory performance, that included levitation and rotation.

**Keywords:** Magnetic bearings, Centrifugal compressor, Fractional order PID, DSP

## 1. Introduction

Centrifugal compressor is the key hardware components of the high efficiency chiller, which is used to the air conditioning systems for large buildings. The most critical element of centrifugal compressor is the bearings that support the shaft at rotation motion. However, the high coefficient of performance (COP) of the chiller is the indispensable for the energy saving strategy. Therefore, there are many technologies in improving COP of the chiller, in which the AMBs supported rotor that allows the centrifugal compressor to the higher rotation speed, and the compact designed compressor will be obtained. Active magnetic bearings have many advantages compared with the traditional bearings, especially in the non-contact characteristic. However, the mechanical stiffness of AMBs is negative and an open loop AMB is an unstable dynamic system, the task of stabilizing this unstable system by a suitable controller comes down to finding an appropriate current command signal [1]. Consequently, precise positioning and high stability are indispensable in controller design for AMBs.

In pace with advancement of semiconductor process technology, the advanced control theory can be more easily implemented in DSP. Chen et al. applied integral sliding mode control to the three-pole AMB system with assembly error and non-uniform flux distribution [2] Yoon et al. proposed to applied output regulation approach to deal with the rotor unbalance problem of AMB systems [3]. Yoon et al. implemented the LQG control and  $\mu$ -synthesis to the maglev centrifugal compressor [4]. Although the modern control theory has the ability to deal with the nonlinear phenomenon of maglev centrifugal compressor, however these advanced control approach requires the accurate system modeling technology, and their mathematical method is too complex, even if the advanced control approach has the strongly performance.

In actual industrial applications, the conventional PID controller has gained popularity in AMBs systems because of easily implementation and tuning intuitive. However, the PID controller may not be sufficient to stabilize complex dynamics of AMBs supported rotor. Recently, PID controller have been extended to another expressed form using the all-coefficient adaptive control approach (ACAC). Di et al. applied ACAC to a High-speed desorption pump supported by AMBs, the ACAC can guarantee the close-loop stability and minimize the vibration during the speed increase as well as at the operational speed [5]. Furthermore, fractional order PID Controller has been developed to the control engineering, FOPID is based on the PID generalized form using fractional calculus [6], Matignon proved the stability results as well as the theory of the controllability and observability of finite-dimensional linear fractional differential systems [7]. Oustaloup was the first to propose the concept of fractional PID controller and successfully used in the CRONE control [8]. Padula

proposed the tuning rules for FOPID and the tuning rules allow to minimize the integrated absolute error subject to a constraint on the maximum sensitivity [9]. In actually, a fractional-order system which includes a fractional-order controller, a fractional-order controlled object, and both in practical applications.

According the aforementioned literature reviews, the advanced control approach and real time adaptive control method are not suitable for AMB systems in industrial applications, the reasons included that the actual industrial control products require the strongly reliability and stability. Moreover, the easily maintenance is a critical point for the service engineers. In view of this, FOPID has the advantages that included easily implementation and tuning intuitive, and FOPID can be more closely fit the actual system dynamics. This study has been applied the FOPID to the centrifugal compressor by AMBs, and experimental results show that the FOPID exhibits satisfactory performance compared with the traditional PID.

The rest of this paper is organized as follows. Section 2 gives an introduction of the AMB system modeling. Section 3 introduces the fractional order PID Controller and its implementation. Experimental setup and results are included in Section 4, while conclusions are drawn in Section 5.

## 2. AMB system description and dynamics equation

The rotor-bearing model diagram of centrifugal compressor can be shown in Fig. 1 [1].

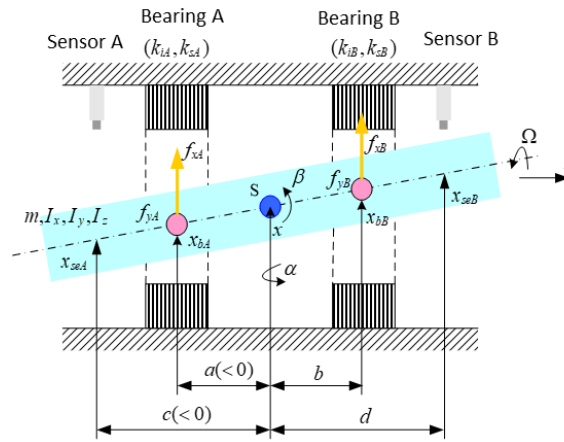


Figure 1 Rotor-bearing model diagram [1].

According to Fig. 1, the center of mass displacements  $x$  and  $y$  as well as the Euler angles  $\alpha$  and  $\beta$ , all combined into the vector  $q$ , the measured rotor displacements  $x_{seA}$ ,  $x_{seB}$ ,  $y_{seA}$ , and  $y_{seB}$ , are comprised in the output vector  $Y$ . the rotor-bearing dynamics equation can be written as below [1]

$$M\ddot{q} + G\dot{q} = Bu_f \quad (1)$$

$$Y = Cq \quad (2)$$

$$\text{where } M = \begin{bmatrix} I_y & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & I_x & 0 \\ 0 & 0 & 0 & m \end{bmatrix}, \quad B = \begin{bmatrix} a & b & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & 1 & 1 \end{bmatrix}, \quad G = \begin{bmatrix} 0 & 0 & I_z\Omega & 0 \\ 0 & 0 & 0 & 0 \\ -I_z\Omega & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} c & 1 & 0 & 0 \\ d & 1 & 0 & 0 \\ 0 & 0 & c & 1 \\ 0 & 0 & d & 1 \end{bmatrix},$$

$q = (\beta, x, -\alpha, y)^T$ ,  $u_f = (f_{xA}, f_{xB}, f_{yA}, f_{yB})^T$ ,  $Y = (x_{seA}, x_{seB}, y_{seA}, y_{seB})^T$ . the magnetic bearing force  $u_f$  can be described as

$$u_f = \begin{bmatrix} f_{xA} \\ f_{xB} \\ f_{yA} \\ f_{yB} \end{bmatrix} = - \begin{bmatrix} k_{sA} & 0 & 0 & 0 \\ 0 & k_{sB} & 0 & 0 \\ 0 & 0 & k_{sA} & 0 \\ 0 & 0 & 0 & k_{sB} \end{bmatrix} \begin{bmatrix} x_{bA} \\ x_{bB} \\ y_{bA} \\ y_{bB} \end{bmatrix} + \begin{bmatrix} k_{iA} & 0 & 0 & 0 \\ 0 & k_{iB} & 0 & 0 \\ 0 & 0 & k_{iA} & 0 \\ 0 & 0 & 0 & k_{iB} \end{bmatrix} \begin{bmatrix} i_{xA} \\ i_{xB} \\ i_{yA} \\ i_{yB} \end{bmatrix} \\ = -K_s q_b + K_i i \quad (3)$$

where  $k_s$  is the displacement stiffness and  $k_i$  is the current stiffness.

### 3. Fractional Order PID Controller and its implementation

#### 3.1 A brief introduction to fractional order calculus

Fractional, or non-integer order fundamental operator of differentiation and integration is denoted by

$${}_a D_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha}, & \alpha > 0, \\ 1, & \alpha = 0, \\ \int_a^t (d\tau)^{-\alpha}, & \alpha < 0, \end{cases} \quad (4)$$

Where  $\alpha$  is the fractional order of the differentiation or integration, and typically  $\alpha \in \Re$  but it also can be complex number [10]. This article discusses the case where the fractional order is a real number. And the Riemann-Liouville definition is used to define fractional calculus as follows.

$${}_a D_t^\alpha f(t) = \lim_{h \rightarrow 0} \frac{1}{\Gamma(m-\alpha)} \left( \frac{d}{dt} \right)^m \int_0^t \frac{f(\tau)}{(t-\tau)^{1-m+\alpha}} d\tau \quad (5)$$

where  $m-1 < \alpha < m$  and  $\Gamma(m-\alpha) = \int_0^\infty e^{-t} t^{(m-\alpha)-1} dt$  is Euler's gamma function. After establishing the definitions of fractional order calculus, the formula for the Laplace transform of the derivative of order  $\alpha$  has been discussed. It is shown in [11] that the Laplace transform of an  $\alpha$ , the derivative of signal  $x(t)$  is given by

$$L\{D^\alpha x(t)\} = \int_0^\infty e^{-st} {}_0 D_t^\alpha x(t) dt = s^\alpha X(s) - \sum_{k=0}^{m-1} s^k {}_0 D_t^{\alpha-k-1} x(t) \Big|_{t=0} \quad (6)$$

A fractional order differential equation can be expressed in a transfer function form which it provided both the input and output signal as follow

$$G(s) = \frac{a_1 s^{\alpha_1} + a_2 s^{\alpha_2} + \dots + a_m s^{\alpha_m}}{b_1 s^{\beta_1} + b_2 s^{\beta_2} + \dots + b_m s^{\beta_m}} \quad (7)$$

#### 3.2 Fractional order transfer function approximation algorithm and its implementation

Oustaloup [8] presented an approximation algorithm which can fit the fractional order operator to a bank of integer order filters in the specified frequency band. The fractional order  $\alpha$  in the specified frequency band  $[\omega_l, \omega_h]$  is given as follows

$$s^\alpha = (\omega_h)^\alpha \prod_{k=1}^N \frac{s + \omega_k'}{s + \omega_k}, \quad 0 < \alpha < 1 \quad (8)$$

Where  $N$  is the number of poles and zeros which are evaluated as  $\omega_k' = \omega_l (\omega_h / \omega_l)^{2k-1-\alpha/2N}$  and  $\omega_k = \omega_l (\omega_h / \omega_l)^{2k-1+\alpha/2N}$ . For the case  $\alpha < 0$ , the Eq. (8) would be inverted. For the case  $|\alpha| > 1$ ,  $s^\alpha$  should be rearranged to  $s^\alpha = s^n s^\sigma$ . Where  $n$  is an integer number and  $\sigma \in [0, 1]$ ,  $s^\sigma$  term needs to be approximated by Eq. (8).

Bilinear transformation is the IIR filter design technology used in this article. The s-domain transfer function, H(s), is mapped to the z-domain through Eq. (9) to become the transfer function, H(z). For implementation, The H(z) needs to be sorted into Eq. (10). There will be no spectrum aliasing problem regardless of the original s-domain transfer function and bandwidth. After bilinear transformation, the amplitude frequency response range will not exceed half of the sampling frequency, fs. Even if there is no spectral aliasing problem in bilinear transformation, the input data must satisfy the sampling theorem. If the original input data is under sampled due to a low sampling frequency and violates the sampling theorem, then any filter cannot eliminate the errors caused by under sampling. However, it must be noted that there will be nonlinear distortion with the original s-domain frequency after bilinear transformation conversion, and the frequency distortion effect will be more apparent as it approaches half of the sampling frequency.

$$s = \frac{2}{T_s} \left( \frac{1-z^{-1}}{1+z^{-1}} \right) \quad (9)$$

where  $T_s$  is the sampling period and  $z$  is a complex number.

$$H(z) = \frac{\sum_{k=0}^N b(k)z^{-k}}{1 - \sum_{k=1}^M a(k)z^{-k}} \quad (10)$$

In order to implement the differential equation in digital signal processor, the digital filter uses Direct-Form I [12]. While there are many variants of this type of filter structure, the Direct-Form I method is considered to have the best performance against filter coefficient quantization errors and stability problems, and is also the method used in this article. To balance the performance of the fractional order PID with the difficulty of DSP implementation, the number of poles and zeros  $N$  is set to 5, and the frequency limits  $\omega_l$  and  $\omega_h$  to 0.1 and 50,000 respectively. However, since the system order is still high, the filter poles and zeros in the direct structure are very sensitive to various errors in the filter coefficients. For instance, quantization errors in the coefficients can cause the filter poles and zeros to move on the  $z$  plane, leading to amplitude frequency response that fails to meet the requirements. Additionally, rounding errors may accumulate with the number of operations, leading to filter instability, while overflow errors can cause the filter to oscillate. Fortunately, JTAG (Joint Test Action Group) can confirm the DSP variables of the embedded system in real-time, allowing for the detection of overflow errors quickly.

During the practice, it was found that the simulated filter coefficients were stable during computer analysis but unstable during DSP implementation. The instability of the control system caused by such filter coefficients was resolved by changing single-precision floating-point numbers to double-precision floating-point numbers. At the same time, the possibility of overflow errors in numerical calculations is reduced. In addition to consuming more memory space, changing from single-precision floating-point numbers to double-precision floating-point numbers posed a main difficulty, which was that the DSP floating-point arithmetic unit in used was FPU32, which could only calculate 32-bit floating-point numbers. Therefore, the operation of 64-bit double-precision floating-point numbers could not be effectively accelerated. The AMB control involves the control of five degrees of freedom. With the decoupling architecture, it includes the translational, inclination displacements of the radial X-Y axis, and axial position control. Each degree of freedom has a corresponding fractional order PID controller. In addition to the AMB controller, the control system also includes several filters for suppressing position and current signal noise and AMB system resonance. Besides, a serial communication interface program is in operation to monitor the AMB trajectory on a personal computer. Due to limited DSP resources, the AMB position control loop in this article is choose as 3kHz. To fully utilize the performance of fractional order PID controller in five degrees of freedom AMB control system, a DSP with a 64-bit double-precision floating-point arithmetic unit is necessary and crucial.

## 4. Experimental setup and results

### 4.1 Experimental Setup

The hardware of the experimental system, which is shown in Fig. 2, consists of a centrifugal compressor equipped with five degrees of freedom AMBs. Its composition includes an impeller, an induction motor, a rotor, two radial AMBs, one axial AMB, inductive-type sensors, a variable-frequency drive, and a magnetic bearing drive controller. The magnetic bearing drive controller, which includes a drive circuit, a DSP-based control circuit, and a sensor circuit, is embedded on the side of the centrifugal compressor. The sensor circuit provides five-degree of freedom position feedback and pulse signal of rotor rotation. The magnetic centrifugal compressor is used to evaluate the performance of FOPID controller, which had implemented by digital signal processor, TI-TMS320F28335. The fractional order setting of FOPID derivative term is 0.97 and  $N=5$ .

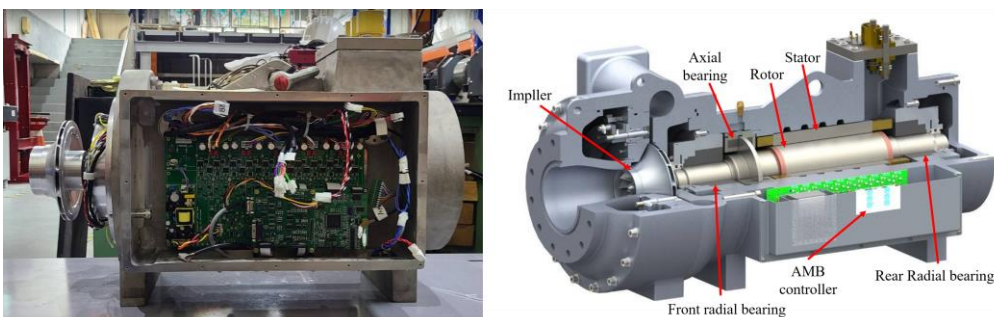


Figure 2 Maglev centrifugal compressor equipped with five-degree of freedom AMBs.

### 4.2 Experimental results

#### Experiment #1: Rotor levitation for evaluating the performance of FOPID

Figure 3 shows the rotor levitation result using FOPID, the black circle is the backup bearing clearance, the red dashed circle is the 30% of the backup bearing clearance from international standard (ISO-14839-2) [13]. According to the results of Fig. 3, both the front radial orbit and the rear radial orbit can be precisely controlled in the center of the backup bearing clearance, the maximum vibration percentages are 0.339% and 0.579%, the RMS percentages of orbit are 0.13% and 0.199%. From the vibration of the trajectory, FOPID demonstrates sufficient stability.

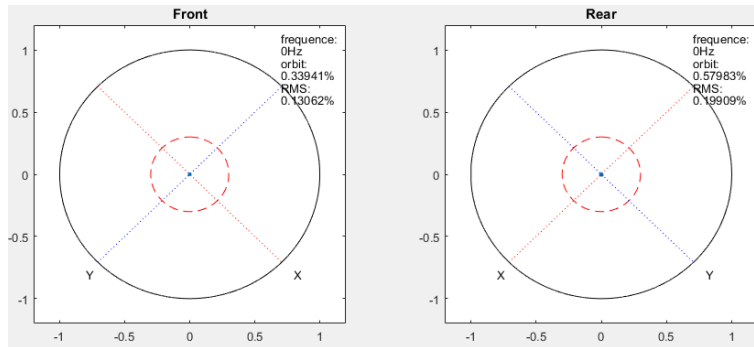


Figure 3 The measured rotor orbit of the levitation verification.

#### Experiment #2: Rotor rotated motion for evaluating the performance compared with the FOPID and IOPID

Figures 4 and 5 show the measured rotor orbit of the centrifugal compressor at 9,000rpm (150Hz), in which the IOPID represents the traditional integer order PID, the black circle is the backup bearing clearance, the red dashed circle is the 30% of the backup bearing clearance (ISO-14839-2). In the experimental result of using IOPID, the maximum vibration percentages are 6.915% and 6.236%. In the experimental result of using FOPID, the maximum vibration percentages are 5.034% and 4.85%. Clearly, The FOPID exhibits satisfactory performance compared with the IOPID. Which means FOPID can be more closely fit the actual system dynamics.

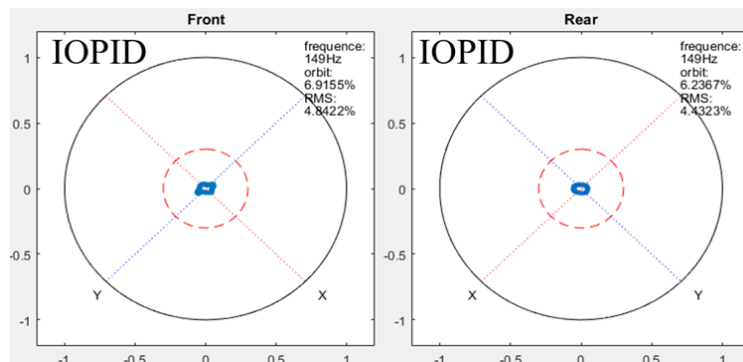


Figure 4 The measured rotor orbit of the centrifugal compressor at 9,000rpm using IOPID.

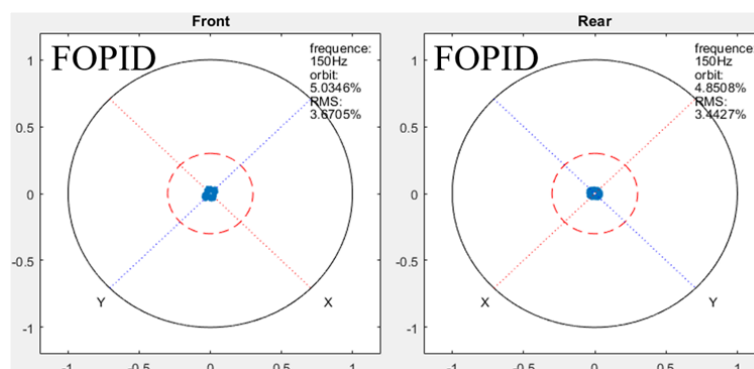


Figure 5 The measured rotor orbit of the centrifugal compressor at 9,000rpm using FOPID.

## 5. Conclusions

In the actual industrial applications of the AMBs system, the requirements of controller design include the strongly reliability, stability, and easily maintenance. However, the advanced control approach requires the accurate system modeling technology, and the maintenance is extremely difficult for the service engineers. Therefore, this article applied FOPID to the magnetic centrifugal compressor, the advantages of FOPID are that the controller can be more closely fit the actual system dynamics, easily implementation and tuning intuitive. The fractional order transfer function approximation algorithm and its discretization implementation are described in detail, including the difficulties encountered and the corresponding solutions. The experimental results show that the rotor can be precisely positioned at the center of the backup bearing. In addition, the FOPID controller exhibits superior performance compared to the IOPID controller at high rotation speeds.

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