Contribution to Design of Hybrid magnetic bearings

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***Abstract*— Basic formulas for determining the magnetic force in the air gap of the electromagnet are derived at the beginning of this article. The application of these formulas for the heteropolar type and homopolar type AMB is described in the next sections of this article. The analysis of the magnetic circuit of the homopolar AMB with PM and three-phase excitation winding is given in Part IV. The formulas and recommendations for designing main dimensions of the AMB with PM are given in Part V.**

I. INTRODUCTION

The use of active magnetic bearings in technical practice began in the middle of the 20th century when a suitable control technology based on power electronic valves and microprocessors was available. The advantages of magnetic bearings compared to conventional bearings are an ability to run by higher speeds, to operate with lower losses, and the possibility of their work in vacuum, in explosive atmospheres or in high temperature. The issue of the active magnetic bearings has become so important that a working group ISO-TC2-SC4-WG7 called "Vibration of machines with active magnetic bearings" was founded. This WG7 prepared for publication the standard ISO 14839-1 "Vocabulary" in the year 2002. Similarly this WG7 prepared for publication the standard ISO 14839-2 "Evaluation of vibration" in the year 2004 , the standard ISO 14839-3 "Evaluation of stability margin" in the year 2006 and the standard ISO 14839-4 "Technical guidelines" in the year 2012.

During the use of active magnetic bearings, their different configurations appeared. The standard ISO 14839 - 1 - Vocabulary - distinguishes the following types of magnetic bearings from the point of view of the principle that bearings form a bearing force on the rotor: passive magnetic bearings (PMB), active magnetic bearings (AMB) and hybrid magnetic bearings (HMB). HMBs are a combination of a part of the active magnetic bearing and the passive magnetic bearing.

The standard ISO 14839 - 1 - Vocabulary defines also the special HMB type called Permanent-Magnet-based AMB - as the active magnetic bearing in which bias air gap fluxes are established by one or more permanent magnets. This paper deals in more detail with the design of the radial Permanent-Magnet-based AMB. Relations for determining the main dimensions of this bearing type are derived in it.

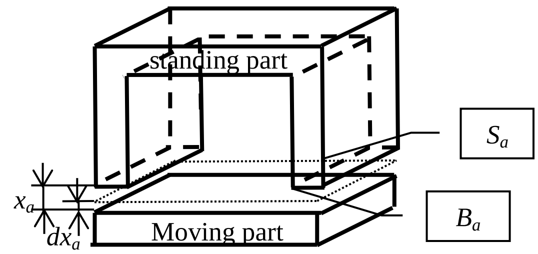

Figure 1 Magnetic circuit with two air gaps

II. FORCE AT THE AIR GAP BOUNDARIES

The closed magnetic circuit in Figure 1 contains two air gaps, each with the length x_a and area S_a.

It is known from the theory of magnetism that the magnetic energy W_a, stored in the volume of these two air gaps is determined by the relationship

$$W_a = \frac{1}{2} * B_a * H_a * 2 * S_a * x_a = \frac{1}{\mu_0} * B_a^2 * S_a * x_a \qquad (1)$$

where
- μ_0 is permeability of vacuum
- B_a is magnetic flux density in the air gap [T]
- S_a is the area of one air gap [m^2]
- x_a is the length of one air gap [m]

The attractive force F_m between the edge regions of the air gaps is determined from changing the length of the air gap dx_a

$$\frac{dW_a}{dx_a} = F_m = \frac{B_a^2}{\mu_0} * S_a \qquad (2)$$

The magnetic reluctance of the closed magnetic circuit in Figure 1 consists of two parts: the magnetic reluctances of the two air gaps $2*R_{mx}$ and the reluctance of the remaining ferromagnetic part R_{mFe}. It is possible to write

$$\begin{aligned} R_{mclosed} &= 2 * R_{mx} + R_{mFe} = \\ &= 2 * \frac{x_a}{\mu_0 * S_a} + \frac{l_{Fe}}{\mu_{Fe} * \mu_0 * S_{Fe}} = \\ &= 2 * \frac{x_a}{\mu_0 * S_a}\left(1 + \frac{\frac{l_{Fe}}{2 * x_a}}{\mu_{Fe} * \frac{S_{Fe}}{S_a}}\right) = \\ &= 2 * R_{mx} * k_{Rm} \end{aligned} \qquad (3)$$

where k_{Rm} is the coefficient of magnetic reluctance increase R_{mx}

The value of the increasing coefficient of magnetic reluctance k_{Rm} is around *1,1.*

$$F_{mmfx} = 2 * R_{mx} * k_{Rm} * \Phi_a = \\ = \frac{2 * x_a}{\mu_0 * S_a} * k_{Rm} * B_a * S_a = \\ = \frac{2 * x_a}{\mu_0} * k_{Rm} * B_a \quad (4)$$

where F_{mmfx} is magneto-motoric force (mmf) for closed magnetic circuit

The source of this mmf F_{mmfx} can be an electric winding with $2*N$ turns and with current I ($F_{mmfx} = 2 * N * I$) or a permanent magnet with the same mmf. We obtain from Eqs. (2) and (4)

$$F_m = \frac{S_a}{\mu_0} * \left(\frac{2 * N * I * \mu_0}{2 * x_a * k_{Rm}}\right)^2 = S_a * \mu_0 * \frac{N^2}{k_{Rm}^2} * \left(\frac{I}{x_a}\right)^2 \quad (5)$$

The resulting magnetic force F_m is proportional to the quadrate of the current I and inversely proportional to the quadrate of the air gap size x_a and this force is always attractive.

III. TYPES OF RADIAL AMB

A. Heteropolar Type Radial Active Magnetic Bearing

In Figure 2 a cross-section of the heteropolar type AMB is drawn. It is seen that the four electromagnets are symmetrically distributed around the rotor.

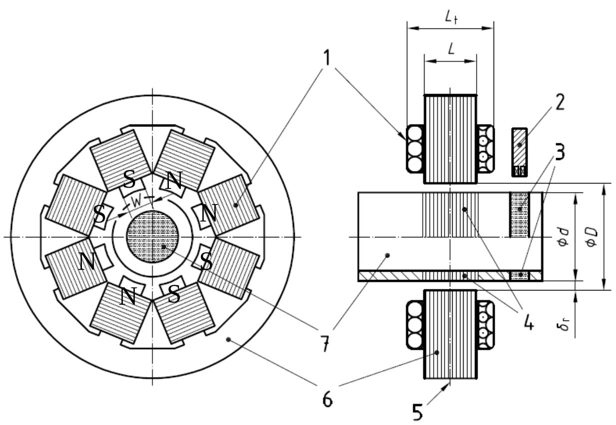

Figure 2 Assembly of a heteropolar type radial AMB 1-Radial core, 2-Radial sensor, 3-Radial sensor target, 4-Radial rotor core, 5-Axial centre of radial AMB, 6-Radial stator core, 7-shaft

Two opposite located electromagnets form one couple of electromagnets. The current of one electromagnet in this couple is I_1 and the current in opposite located electromagnet is I_2. If $I_1 = I_0 + \Delta I$, $I_2 = I_0 - \Delta I$, $x_{a1} = x_{a0} - \Delta x_a$, $x_{a2} = x_{a0} + \Delta x_a$, then the resulting magnetic force of this electromagnet couple ΔF_{mres} is

$$\Delta F_{mres} = S_a * \mu_0 * \frac{N^2}{k_{Rm}^2} * \\ * \left(\left(\frac{I_0 + \Delta I}{x_{a0} - \Delta x_a}\right)^2 - \left(\frac{I_0 - \Delta I}{x_{a0} + \Delta x_a}\right)^2\right) \\ = S_a * \mu_0 * \frac{8 * N^2}{k_{Rm}^2} * \frac{I_0^2}{x_{a0}^2}\left(\frac{\Delta x_a}{x_{a0}} + \frac{\Delta I}{I_0}\right) \quad (6)$$

It is seen that the resulting force of the electromagnet couple ΔF_{mres} is linearly proportional to ΔI and also to Δx_a. Its direction can be on both sides. The current I_0 is called as the bias current.

The polarity of the magnetic poles changes around the rotor in the order of N-S-S-N-N-S-S-N. This order removes a magnetic coupling between adjacent electromagnets. As a result of the above-mentioned pole magnetic polarity order, the magnetic flux in the rotor core varies with rotation of the rotor and therefore the rotor part has to be laminated.

The poles of one electromagnet are shifted about each other by 45^0 (see Figure 3) and their resulting force has to be reduced due to this shift.

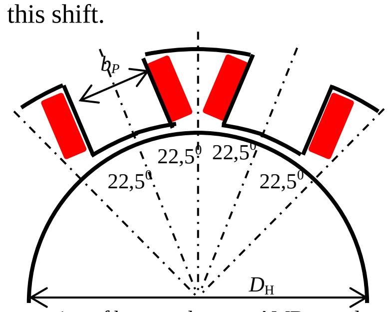

Figure 3 Cross-section of heteropolar type AMB one electromagnet, DH – diameter of the rotor, bP – pole width

The pole pitch of one pole is $\tau_P = \pi * \frac{D_H}{8}$, the pole cover $k_P = {b_P}/{\tau_P}$. The area S_a in Eq. (6) must be replaced by

$$S_a = \frac{D_H}{2} * l_P * \int_{\left(1-\frac{k_P}{2}\right)*\frac{\pi}{8}}^{\left(1+\frac{k_P}{2}\right)*\frac{\pi}{8}} \cos\varphi * d\varphi = \\ = \frac{D_H}{2} * l_P \\ * \left(\sin\left(\left(1 + \frac{k_P}{2}\right) * \frac{\pi}{8}\right) \\ - \left(\sin\left(1 - \frac{k_P}{2}\right) * \frac{\pi}{8}\right)\right) \quad (7)$$

Typical values are $\Delta I = I_0, \Delta x_a = 0$, $k_P = 0{,}6$, $k_{Rm} = 1{,}1$. Then resulting area S_a of one pole can be calculated from (7)

$$S_a = \frac{D_H}{2} * l_P * \left(\sin 1{,}3 * \frac{\pi}{8} - \sin 0{,}7 * \frac{\pi}{8}\right) = \\ = \frac{D_H}{2} * l_P * 0{,}271 \quad (8)$$

The resulting force of one electromagnet couple is

$$\Delta F_{mres} = \frac{D_H}{2} * l_P * 0{,}271 * \mu_0 * \frac{8 * N^2}{1{,}1^2} * \frac{I_0^2}{x_{a0}^2}(0+1) = 0{,}718 * D_H * l_P * \mu_0 * \frac{(NI_0)^2}{x_{a0}^2} \qquad (9)$$

B. Homopolar Type Radial Active Magnetic Bearing

In Figure 4 there is seen a cross-section of the homopolar type AMB. Its four electromagnets are also symmetrically distributed around the rotor, but the electromagnets are located in the axial direction. Magnetic polarity of all electromagnets on the same side in the axial direction is identical. This has the advantage that the magnetic flux in the rotor is practically constant and therefore the rotor can be without a lamination.

The two opposite electromagnets again form a couple of electromagnets and the same Eq. (6) is correct for the determination of the resulting magnetic force ΔF_{mres} of this couple.

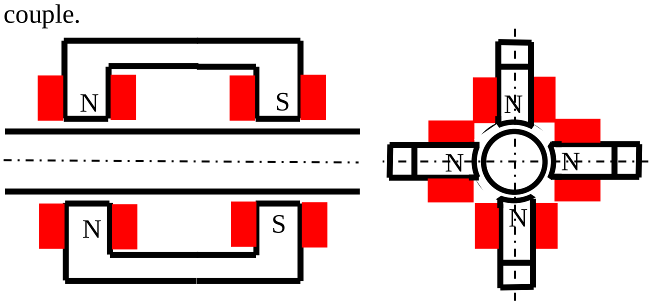

Figure 4 Cross-sections of homopolar type radial AMB

The pole pitch of one pole is now $\tau_P = \pi * \frac{D_H}{4}$ and the following Eq. (10) allows to calculate the pole area (see Figure 5)

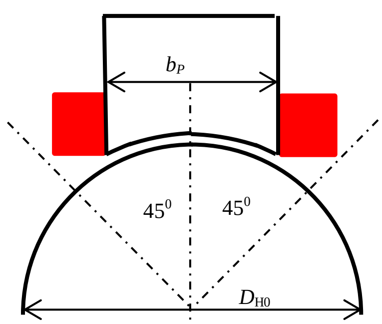

Figure 5 Homopolar type AMB one electromagnet cross-section, D_H – rotor diameter, b_P – pole width

$$S_a = \frac{D_H}{2} * l_P * \int_{-\left(1-\frac{k_P}{2}\right)*\frac{\pi}{4}}^{\left(1-\frac{k_P}{2}\right)*\frac{\pi}{4}} \cos\varphi * d\varphi = \frac{D_H}{2} * l_P * \left(\sin\left(1-\frac{k_P}{2}\right)*\frac{\pi}{4} - \sin\left(-\left(1-\frac{k_P}{2}\right)*\frac{\pi}{4}\right)\right) \qquad (10)$$

Typical values are again $\Delta I = I_0, \Delta x_a = 0$, $k_P = 0{,}6$, $k_{Rm} = 1{,}1$. Then resulting area S_a of one pole can be calculated from Eq. (11)

$$S_a = \frac{D_H}{2} * l_P * \left(\sin 0{,}7 * \frac{\pi}{4} - \sin\left(-0{,}7 * \frac{\pi}{4}\right)\right) = \frac{D_H}{2} * l_P * 1{,}044 \qquad (11)$$

The resulting force of one electromagnet couple is

$$\Delta F_{mres} = \frac{D_H}{2} * l_P * 1{,}044 * \mu_0 * \frac{8 * N^2}{1{,}1^2} * \frac{I_0^2}{x_{a0}^2}(0+1) = 3{,}455 * D_H * l_P * \mu_0 * \frac{N^2 I_0^2}{x_{a0}^2} \qquad (12)$$

C. Replacement of bias currents by permanent magnets in homopolar type radial AMB

If the radial homopolar type AMB is used, the effect of the bias currents I_0 can be replaced by permanent magnets placed in yokes of electromagnets (see Figure 6).

The replacement of the bias current I_0 by a permanent magnet allows to reduce losses in coils of the active magnetic bearing to one half or even more.

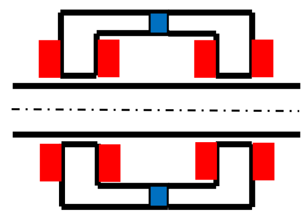

Figure 6 Cross-sections of homopolar type radial AMB with permanent magnets (blue)

A bias current I_0 that flows in two magnet windings each with N turns produces mmf F_{mmf0} that is equal to $2*N*I_0$. This mmf F_{mmf0} is the cause of the magnetic flux Φ_0. The value of this magnetic flux Φ_0 can be calculated from Eq. (13)

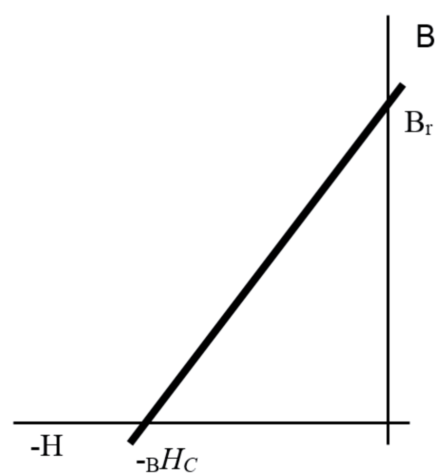

Figure 7 Permanent magnet magnetizing curve

$$\Phi_0 = \frac{2 * N * I_0}{2 * R_{mx0}} = \frac{N * I_0}{R_{mx0}} \tag{13}$$

Simplified magnetizing characteristic shape of a quality permanent magnet material is in Figure 7. A prism from the permanent magnet material with the length l_{PM} and with the cross-section area S_{PM} can be replaced by the mmf F_{PM} that is equal to $\left({}_{B}H_C * l_{PM} \right)$ and by the equivalent permanent magnet inner reluctance R_{mPM} according to Eq. (14)

$$R_{mPM} = \frac{{}_{B}H_C * l_{PM}}{B_r * S_{PM}} = \frac{l_{PM}}{\mu_{PM} * S_{PM}} \tag{14}$$

where μ_{PM} is permeability of the permanent magnet material

$$\mu_{PM} = {}^{\mathrm{B_r}}\!/_{{}_{B}H_C}$$

The equivalent diagram of a closed magnetic circuit is drawn in Figure 8. The magnetic flux Φ_0 can be calculated on the base of this equivalent circuit as shown in Eq. (15).

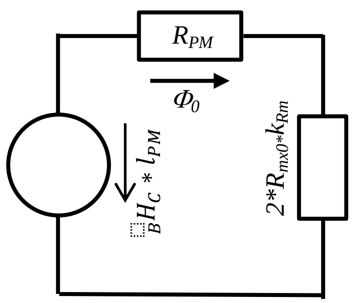

Figure 8 Equivalent diagram of closed magnetic circuit with PM

$$\Phi_0 = \frac{{}_{B}H_C * l_{PM}}{R_{PM} + 2 * R_{mx0} * k_{Rm}} \tag{15}$$

D. Replacement of electromagnet windings by threphase winding

The radial AMB contains two couples of electromagnets that are situated in axes α, β. These axes are perpendicular one to other. The magnetic forces $\Delta F_{m\alpha}$, $\Delta F_{m\beta}$ of their currents ΔI_α, ΔI_β represent two components of the resulting magnetic force vector $\widehat{\Delta F_m}$. If a position of the AMB rotor is in the center of the air gap ($\Delta x_a = 0$) then currents ΔI_α, ΔI_β represent components of the current vector $\widehat{\Delta I}$ in axes α, β.

$$\widehat{\Delta I} = \Delta I_\alpha + j * \Delta I_\beta \tag{16}$$

It is known from the theory of electric machines that three currents I_U, I_V, I_W in a three-phase winding of an AC machine produce one final vector of the mmf $\hat{F}_{mmf\delta}$. Therefore, eight windings in Figure 3 or in Figure 4 can be replaced by two three-phase windings in Figure 9.

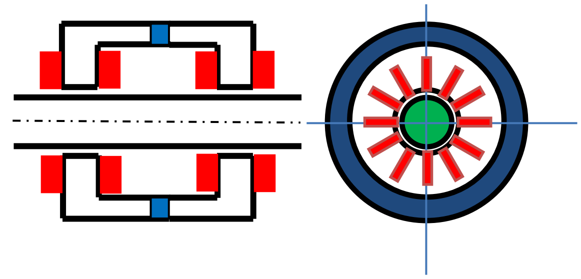

Figure 9 Cross-sections of homopolar type radial AMB with permanent magnet ring (blue) and two three phase windings (red)

Three-phase windings are located in multiple slots on both ends of the hollow cylinder. The bias magnetic flux Φ_0 is excited by the ring from a material for permanent magnets. This flux flows through the surface of a ferromagnetic material cylinder, through the air gap, through the surface of the rotor shaft and it returns back through the air gap and surface of a ferromagnetic material cylinder.

Slots at both ends of the hollow cylinder increase the air gap. This influence is respected in the theory of electric machines by the so-called Carter factor k_C

$$k_C = \frac{\tau_d}{\tau_d - \gamma * \delta} \quad \text{where} \quad \gamma = \frac{\left({}^{b}\!/_{\delta}\right)^2}{5 + {}^{b}\!/_{\delta}} \tag{17}$$

where τ_d is the slot pitch

b is the slot width

The working flux $\widehat{\Delta\Phi}$ is excited by the mmf $2{*}\hat{F}_{mmfx}$ of the three phase winding that is located in one end of the bearing cylinder. This F_{mmfx} is result of the current vector $\widehat{\Delta I}$ of one pole. The working flux $\widehat{\Delta\Phi}$ flows perpendicularly to the rotor through the shaft, through both air gaps, through stator teeth and through stator yokes. This working flux does not flow through the permanent magnet ring.

This design allows for better use of the AMB space and to separate the path of the bias magnetic flux Φ_0 from the path of the working flux $\widehat{\Delta\Phi}$.

Following mathematical equations allow to transform two current components ΔI_α, ΔI_β that flow through windings in perpendicular axes α, β to three currents I_U, I_V, I_W that flow through three windings in three axes U, V, W. Axes α and U have the same position. Axes U, V, W are shifted by 120^0 each to other.

$$\begin{aligned} I_U &= \frac{2}{3} * \Delta I_\alpha \\ I_V &= -\frac{1}{3} * \Delta I_\alpha - \frac{1}{\sqrt{3}} * \Delta I_\beta \\ I_W &= -\frac{1}{3} * \Delta I_\alpha + \frac{1}{\sqrt{3}} * \Delta I_\beta \\ I_U + I_V + I_W &= 0 \end{aligned} \tag{18}$$

It is known from theory of electric machine that the resulted mmf of the three phase winding for one pole is possible to calculate from following equation

$$F_{mmfmax} = \frac{m * \sqrt{2}}{\pi} * \frac{N_{phase} * I_{RMS} * k_w}{p} \qquad (19)$$

where m is number of phases

N_{phase} is number of turns in one phase

I_{RMS} is RMS value of the current in one phase

k_w is winding factor

p is number of pole pairs

Needed three phase winding for AMB must have m = 3, p = 1 and we estimate its winding factor $k_W = 0{,}8$. Then

$$F_{mmfmax} = 0{,}764 * N_{phase} * I_{max} \qquad (20)$$

where $I_{max} = \sqrt{2} * I_{RMS}$

E. Magnetic force of homopolar type radial AMB with permanent magnet ring

The three-phase winding for one pole creates a sine mmf wave along its air gap with the amplitude F_{mmfmax}. This mmf produces a magnetic flux density $B_w(\varphi)$ in a constant air gap $x_0 * k_C$. Magnetic flux density $B_w(\varphi)$ is added to the constant magnetic flux density B_0 from F_{PM} to the resulting magnetic flux density $B_{res}(\varphi)$. It is possible to write for one pole

$$B_{res+}(\varphi) = B_0 * \left(1 + \frac{B_w}{B_0} * cos\varphi\right) \qquad (21)$$

and for the opposite pole

$$B_{res-}(\varphi) = B_0 * \left(1 - \frac{B_w}{B_0} * cos\varphi\right) \qquad (22)$$

The differential of the magnetic force dF_m can be calculated from Eq. (23)

$$dF_m = \frac{1}{2} * \frac{B_{res}^2(\varphi) * l_P}{\mu_0} * \frac{D_H}{2} * d\varphi \qquad (23)$$

The direction of this force differential is perpendicular to the rotor axis. The resulting force F_m is given by the sum of the projections of these force differentials dF_m into the axis in the direction of maximum mmf F_{mmfmax}. We obtain for one pole

$$F_{m+} = \frac{D_H * l_P}{4 * \mu_0} * B_0^2 * \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 + \mathrm{k}_I * \cos\varphi)^2 * \cos\varphi * d\varphi \qquad (24)$$

We obtain after mathematical elaboration following result

$$F_{m+} = \frac{D_H * l_P * \mu_0}{4} * \frac{(N * I_0)^2}{x_0^2 * k_{Rm}^2 * k_C^2} * \left(\pi * k_I + 2 + \frac{4 * k_I^2}{3}\right) \qquad (25)$$

Similarly we obtain for the opposite pole

$$F_{m-} = \frac{D_H * l}{4 * \mu_0} * B_0^2 * \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 - \mathrm{k}_I * \cos\varphi)^2 * \cos\varphi \, d\varphi \qquad (26)$$

and after mathematical elaboration

$$F_{m-} = \frac{D_H * l_P * \mu_0}{4} * \frac{(N * I_0)^2}{x_0^2 * k_{Rm}^2 * k_C^2} * \left(-\pi * k_I + 2 + \frac{4 * k_I^2}{3}\right) \qquad (27)$$

The resulting magnetic force of both poles on one end of hollow cylinder is equal to

$$\Delta F_{mreshalf} = F_{m+} - F_{m-} = = \frac{D_H * l_P * \mu_0}{2} * \frac{(N * I_0)^2}{x_0^2 * k_{Rm}^2 * k_C^2} * \pi * k_I \qquad (28)$$

and finally the resulting magnetic force of both ends of hollow cylinder is equal to

$$\Delta F_{mres} = D_H * l_P * \mu_0 * \frac{(N * I_0)^2}{x_0^2 * k_{Rm}^2 * k_C^2} * \pi * k_I \qquad (29)$$

F. Magnetic forces comparison of various bearings types

We compare lengths of different AMB types with the same rotor diameter D_H, the same width of the air gap x_0, the same mmf (NI_0), the same $k_{Rm} = 1{,}1$ and the same $k_I = 1$. We obtain from Eqs. (9), (12) and (29)

$$\frac{F_{mres} * x_0^2 * k_{Rm}^2 * k_C^2}{D_H * \mu_0 * (N * I_0)^2} = 2{,}59 * l_{3ph} = 0{,}72 * l_{het} = = 3{,}46 * l_{hom} \qquad (30)$$

The heteropolar type of the AMB requires only one l_{het} in the axial direction. The homopolar type and the three phase type AMB require two l_{hom} or l_{3ph} in the axial direction. Therefore it will be better to compare the required lengths in the axial direction

Relations (30) can be rewritten to following relations

$$\frac{2{,}59}{2} * l_{3phax} = 0{,}72 * l_{het} = \frac{3{,}46}{2} * l_{homax} = \\ = 1{,}3 * l_{3phax} = 0{,}72 * l_{het} = 1{,}72 * l_{homax} \qquad (31)$$

We obtain for comparison lengths of two different types

$$l_{3phax} = \frac{0{,}718}{1{,}3} * l_{hetax} = 0{,}55 * l_{hetax} \qquad (32)$$

$l_{3phax} = \frac{1{,}72}{1{,}3} * l_{homax} = 1{,}33 * l_{homax}$	(33)
$l_{homax} = \frac{0{,}718}{1{,}72} * l_{hetax} = 0{,}42 * l_{hetax}$	(34)

IV. Solution of magnetic fluxes distribution in magnetic circuit of AMB with permanent magnets

In chapters C and D there is explained that two different mmf F_{mmfPM} and F_{mmfw} excite in magnetic circuit of the AMB with permanent magnets the magnetic fluxes that flow through different parts of this magnetic circuit. An equivalent circuit of the AMB with permanent magnets is drawn in Figure 10.

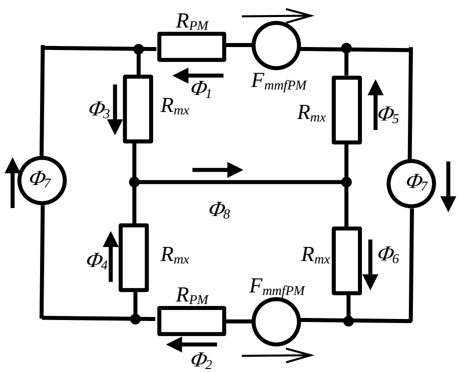

Figure 10 Equivalent circuit of AMB with permanent magnets

The equivalent circuit in Figure 10 can be described by the following 7 equations:

$$\begin{aligned} F_{mmfPM} &= R_{PM} * \Phi_1 + R_{mx} * (\Phi_3 + \Phi_5) \\ F_{mmfPM} &= R_{PM} * \Phi_2 + R_{mx} * (\Phi_4 + \Phi_6) \\ \Phi_3 &= \Phi_7 + \Phi_1 \\ \Phi_4 &= \Phi_2 - \Phi_7 \\ \Phi_5 &= \Phi_1 + \Phi_7 \\ \Phi_6 &= \Phi_2 - \Phi_8 \\ \Phi_8 &= \Phi_3 + \Phi_4 \end{aligned} \qquad (35)$$

The system of 7 equations (35) has 9 variables: Φ_1, Φ_2, Φ_3, Φ_4, Φ_5, Φ_6, Φ_7, Φ_8, F_{mmfPM},. For the unambiguous solution, two variables have to be chosen as independent variables. We chose F_{mmfPM} and Φ_7. Then the solution of equation system (35) is:

$$\begin{aligned} \Phi_1 &= \frac{F_{mmfPM} - 2 * (R_{PM} + R_{mx}) * \Phi_7}{R_{PM} + 2 * R_{mx}} \\ \Phi_2 &= \frac{F_{mmfPM} + 2 * (R_{PM} + R_{mx}) * \Phi_7}{R_{PM} + 2 * R_{mx}} \\ \Phi_3 &= \frac{F_{mmfPM} - R_{PM} * \Phi_7}{R_{PM} + 2 * R_{mx}} \\ \Phi_4 &= \frac{F_{mmfPM} + R_{PM} * \Phi_7}{R_{PM} + 2 * R_{mx}} \\ \Phi_5 &= \Phi_3) \\ \Phi_6 &= \Phi_4 \\ \Phi_8 &= \frac{2 * F_{mmfPM}}{R_{PM} + 2 * R_{mx}} \end{aligned} \qquad (36)$$

V. Determining the dimensions of the permanent magnet ring

A. Determination of permanent magnet ring lenght

When the magnetic flux Φ_7 that is caused by a current in the winding is equal to zero then we obtain from (36)

$$\Phi_3 = \Phi_4 = \Phi_5 = \Phi_6 = \frac{F_{mmfPM}}{R_{PM} + 2 * R_{mx0}} \qquad (37)$$

When magnetic reluctance of the magnetic flux Φ_7 in path of the ferromagnetic part of the magnetic circuit and an influence of slots is respected, then Eq. (37) changes to Eq. (38)

$$\Phi_3 = \Phi_4 = \Phi_5 = \Phi_6 = \frac{H_C * l_{PM}}{R_{PM} + 2 * R_{mx0} * k_{Rm} * k_C} \qquad (38)$$

Magnetic reluctances R_{PM} and R_{mx0} are given by relationships

$$R_{PM} = \frac{l_{PM}}{\mu_{PM} * S_{PM}} \qquad R_{mx0} = \frac{x_0}{\mu_0 * S_x} \qquad (39)$$

We obtain from Eqs. (38) and (39)

$$\begin{aligned} H_C * l_{PM} &= \Phi_3 * \left(\frac{l_{PM}}{\mu_{PM} * S_{PM}} + 2 * \frac{x_0}{\mu_0 * S_x} * k_{Rm} * k_C\right) = \\ &= \left(\frac{l_{PM}}{\mu_{PM}} * \frac{\Phi_3}{S_{PM}} + 2 * \frac{x_0}{\mu_0} * \frac{\Phi_3}{S_x} * k_{Rm} * k_C\right) = \\ &= \left(\frac{l_{PM}}{\mu_{PM}} * B_{PM} + 2 * \frac{x_0}{\mu_0} * B_{\delta 0} * k_{Rm} * k_C\right) \end{aligned} \qquad (40)$$

The relationship applies

$$\Phi_0 = B_{PM} * S_{PM} = B_{x0} * S_x \to \frac{B_{x0}}{B_{PM}} = \frac{S_{PM}}{S_x} \qquad (41)$$

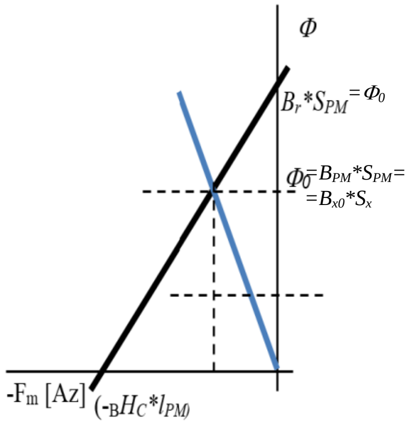

Figure 11 Determination of the magnetic flux density BPM

We can write following relation on the base of Figure 11:

$$\frac{\boldsymbol{\Phi_0}}{\boldsymbol{S_{PM}}} * \frac{\mathbf{1}}{\boldsymbol{B_r}} = \frac{\boldsymbol{B_{PM}}}{\boldsymbol{B_r}} = \frac{\boldsymbol{R_{PM}}}{(\boldsymbol{R_{PM}} + \mathbf{2} * \boldsymbol{R_{mx0}} * \boldsymbol{k_{Rm}} * \boldsymbol{k_C})} = \frac{1}{1 + 2 * \frac{R_{mx0} * k_{Rm} * k_C}{R_{PM}}} = \frac{1}{1 + 2 * \frac{\frac{x_0}{\mu_0 * S_x}}{\frac{l_{PM}}{\mu_{PM} * S_{PM}}} * k_{Rm} * k_C} \qquad (42)$$

Equation (42) can be rewritten to

$$\frac{B_{PM}}{B_r} = \frac{1}{1 + 2 * \frac{\frac{x_0}{\mu_0 * S_x}}{\frac{l_{PM}}{\mu_{PM} * S_{PM}}} * k_{Rm} * k_C} = \frac{\mathbf{1}}{1 + 2 * \frac{x_0}{l_{PM}} * \frac{\mu_{PM}}{\mu_0} * \frac{S_{PM}}{S_x} * k_{Rm} * k_C} = \frac{1}{1 + 2 * \frac{x_0}{l_{PM}} * \frac{\mu_{PM}}{\mu_0} * \frac{B_{\delta 0}}{B_{PM}} * k_{Rm} * k_C} \qquad (43)$$

Finally we obtain

$$\frac{B_{PM}}{B_r} = 1 - 2 * \frac{x_0}{l_{PM}} * \frac{\mu_{PM}}{\mu_0} * \frac{B_{\delta 0}}{B_r} * k_{Rm} * k_C \qquad (44)$$

We assume that all values in Eq. (44) are constant except for B_{PM} and l_{PM} variables. If the value of one variable B_{PM} or l_{PM} is known, then Eq. (44) allows to calculate the value of the second variable. As can be seen from Figure 11 it must be B_{PM} value less than the Br value. Typical values of constants in Eq. (44) are:

$B_{\delta 0} = 0{,}5\ T$, $x_0 = 0{,}5\ mm$, $\mu_0 = 4\pi*10^{-7}\ H/m$, $k_{Rm} = 1{,}1$, $k_C = 1{,}4$.

The permanent magnet material Ferroxdure FXD 400 has $B_r = 0{,}41\ T$, ${}_BH_C = 2{,}65*10^5\ A/m$. Then

$$\mu_{PM} = {B_r}\Big/{{}_BH_c} = {0{,}41}\big/{2{,}65 * 10^5} = 1{,}55 * 10^{-6}[Hm^{-1}]$$

The permanent magnet material Rare Earth RES 305 has $B_r = 1{,}15\ T$, $H_c = 8{,}5*10^5\ A/m$. Then

$$\mu_{PM} = {B_r}/{H_c} = {1{,}15}/{8{,}5 * 10^5} = 1{,}35 * 10^{-6}[Hm^{-1}]$$

The curves $B_{PM} / B_r = f(l_{PM})$ for typical values of constants in Eq. (44) and for two different materials of permanent magnets were calculated and they are drawn in Figure 12. The red curve is for a the material RareEarth RES 400 and blue curve is for the material Ferroxdure FXD400 .

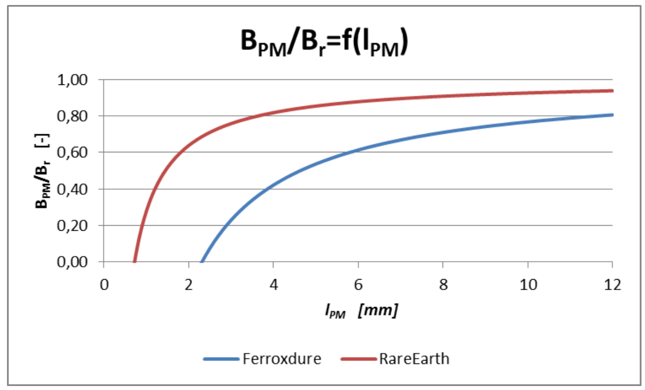

Figure 12 Functions BPM / Br = f(lPM) for typical values of constants in Eq. (44) and for a RareEarth RES 400 (red) or Ferroxdure FXD400 (blue)

A recommended value of the ratio B_{PM} / B_r is about 0,6. The length of the Ferroxdure permanent magnet ring was determined from Figure 12 to 5,8 mm and from RareEarth was determined to 1.8 mm.

B. Determination of PM ring diameters

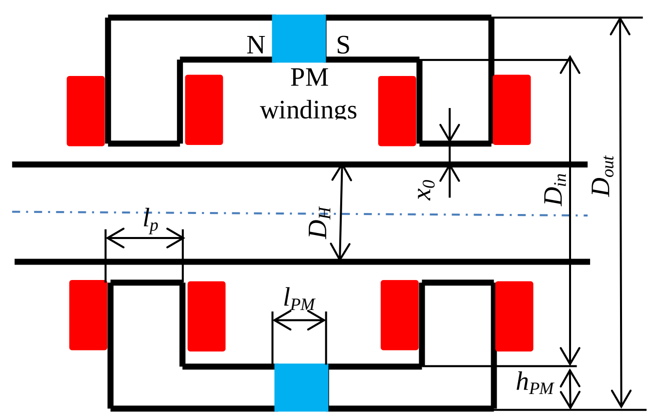

Figure 13 Longitudinal section of AMB with permanent magnet ring

It is assumed that the rotor diameter D_H of the shaft on which the radial magnetic bearing will be located is known and that the length of one air gap l_P is determined from Eq. (28) for the known values $(NI)_0$, x_0, k_{Rm}, k_C, k_I. It is also assumed that the inner diameter D_{in} of the PM ring is known.

It must be determined with respect to the space required for the three-phase winding.

The following Eq. (45) applies for the PM ring

$$S_{PM} = \frac{\pi}{4} * (D_{out}^2 - D_{in}^2) = \pi * h_{PM} * (D_{in} + h_{PM}) \qquad (45)$$

where $h_{PM} = \frac{D_{out} - D_{in}}{2}$

The following Eq. (46) applies for the air gap

$$S_x = \pi * D_H * l_P * k_P \qquad (46)$$

We obtain from Eq. (45), (46) and (41)

$$\frac{S_{PM}}{S_x} = \frac{\pi * h_{PM} * (D_{in} + h_{PM})}{\pi * D_H * l_P * k_P} = \frac{B_{x0}}{B_{PM}} \qquad (47)$$

$$h_{PM}^2 + D_{in} * h_{PM} - D_H * l_p * k_p * \frac{B_{x0}}{B_{PM}} = 0 \qquad (48)$$

and finally

$$h_{PM} = -\frac{D_{in}}{2} + \sqrt{\left(\frac{D_{in}}{2}\right)^2 + D_H * l_p * k_p * \frac{B_{x0}}{B_{PM}}} \qquad (49)$$

We obtain from Eq. (49) as an example for *D_{in} = 165 mm, D_{in} =165 mm, l_p = 20 mm, k_p = 0,6, B_{x0}= 0,5 T, B_r = 0,41 T, B_{x0}/ B_{PM} = B_{x0}/ B_r * B_r/ B_{PM} =0,5 / 0,41 / 0,6 = 2,03*

$$h_{PM} = -\frac{165}{2} + \sqrt{\left(\frac{165}{2}\right)^2 + 80 * 20 * 0{,}6 * 2{,}03} = 11{,}08 \;\; mm \qquad (50)$$

In Figure 14, there is a view of the AMB with permanent magnets in the CTU in Prague – FEE laboratory. The ring from a material for permanent magnet is replaced by prisms with required thick, which are spaced in place for the ring.

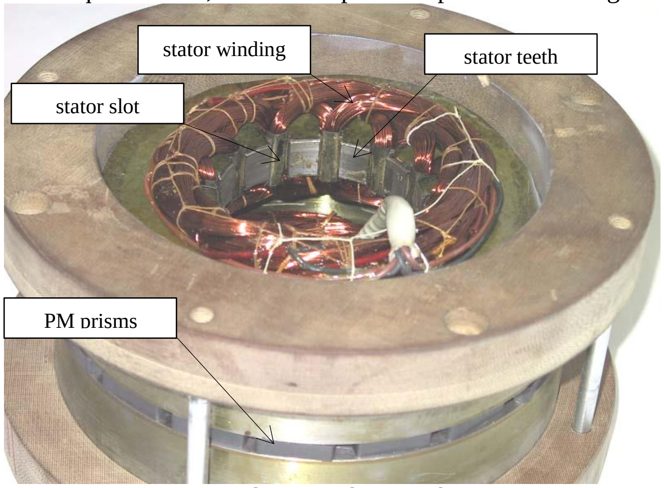

Figure 14 View on opened AMB with PM in the CTU in Prague – FEE laboratory

VI. Conclusion

Active magnetic bearings find use in rotary machines for some of their advantages. The heteropolar AMB are most frequently used in industrial applications and therefore the greatest attention is paid to them in the professional literature. On the contrary, about the homopolar AMBs with permanent magnets can find in the literature only a description of the principle of their work, but information about their design can not easily be found.

This paper deals with the analysis of the magnetic circuit of the homopolar AMB with permanent magnets and, on the basis of this analysis, describes the necessary equations for determining the main dimensions of its permanent magnets.

The described design method is one of the results of doctoral thesis T. Kupka's dissertation [6].

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