The Motion of the Rotor Acrossing Critical Speed Levitated by AMBs with Anisotropic Stiffness

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***Abstract*— The stiffness and damping ratios in *x* and *y* radial degrees of freedom (DOF) of active magnetic bearing (AMB) may be different due to the machining and assembly errors. Therefore, the critical speeds in the two radial DOFs will not be equal. To study the rotor motion when its speed is between the two unequal radial critical speeds, a model of a rigid Jeffcott rotor levitated by AMBs with anisotropic bearing stiffness and damping ratio was built in this paper. The differential equation of motion of the model was derived from Lagrange equation firstly. Then the motion of the rotor geometric center was given. Through plotting the orbit of the rotor geometric center, it is found that the backward whirling movement of the rotor can be observed when the product of the damping ratios in *x* and *y* DOFs is lower than a critical value.**

A. Introduction

A typical active magnetic bearing (AMB) system consists of sensor, controller, amplifier, electromagnet, retainer bearing and the levitated object[1, 2]. Commonly, the bearing stiffness of AMB is determined mainly by the structure parameters, control method and control parameters[3]. Moreover, the machining and assembly errors also affect the bearing stiffness[4], which will make the stiffness unequal in the *x* and *y* radial degrees of freedom (DOF). And the damping ratio is also the same. The unequal stiffness in *x* and *y* DOFs will make the critical speeds unequal[5], which will affect the whirling motion of the rotor[6, 7], especially when its speed is between the two unequal critical speeds.

B. Anisotropic Stiffness of Radial AMB Affected by Machining and Assemble Errors

(1) Ideal Electromagnetic Force Model

Without the machining and assembly errors, the ideal electromagnetic force of the differential AMB system on single DOF can be calculated as equation (1)[1].

$$F = \frac{\mu_0 N^2 A}{4}\left[\frac{(i_0+i_x)^2}{(s_0-x)^2} - \frac{(i_0-i_x)^2}{(s_0+x)^2}\right] \tag{1}$$

Generally, equation (1) will be linearized to equation (2) at the balanced position (x=0， i_x=0) to simplify the controller design[1].

$$F = k_s x + k_i i_x \tag{2}$$

Where $k_s = k\frac{i_0^{\ 2}}{s_0^{\ 3}}$ is the force-displacement factor, $k_i = k\frac{i_0}{s_0^{\ 2}}$ is the force-current factor, and $k = \mu_0 N^2 A$.

(2) Real Electromagnetic Force Model

Due to the machining and assembly errors, the AMB axis and the retainer bearing axis are not always on the same line, shown as Fig.1. The parallel misalignment between the two axes is *e*. According to the equation (1), the real electromagnetic force can be transformed to equation (3). From equation (3), the real electromagnetic force F_e will be affected by the misalignment *e* and doesn't equal to the ideal force *F*.

$$F_e = \frac{k}{4}\left[\frac{(i_0+i_x)^2}{(s_0-e-x)^2} - \frac{(i_0-i_x)^2}{(s_0+e+x)^2}\right] \tag{3}$$

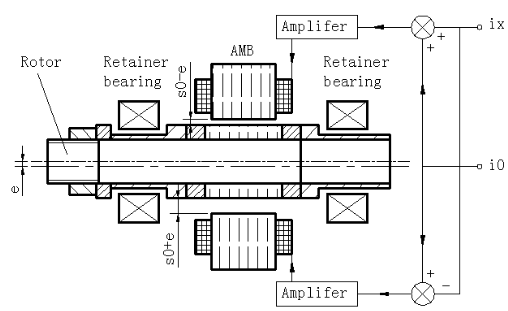


Figure 1. The AMB-rotor system with parallel misalignment

Equation (3) can be linearized to equation (4).

$$F_e = k_{se} x + k_{ie} i_x + F_0 \tag{4}$$

Where, $k_{se} = \frac{1}{1-e_r^{\ 2}} k_x$, $k_{ie} = \frac{1}{1-e_r^{\ 2}} k_i$, $F_0 = \frac{e_r k}{1-e_r^{\ 2}} \frac{i_0^{\ 2}}{s_0^{\ 2}}$, $e_r = e / s_0$.

The stiffness and damping of AMB system with parallel misalignment can be derived from equation (4), shown as equation (5).

$$\begin{cases} k_e = \dfrac{\partial F_e}{\partial x} = k_{se} + k_{ie}\dfrac{\partial i_x}{\partial x} \\ c_e = \dfrac{\partial F_e}{\partial \dot{x}} = k_{ie}\dfrac{\partial i_x}{\partial \dot{x}} \end{cases} \tag{5}$$

There are two radial DOFs x and y in one radial AMB system. It is difficultly to make the misalignments equal in the two radial DOFs actually. That is $e_x \neq e_y$, and $k_{sex} \neq k_{sey}$, $k_{iex} \neq k_{iey}$, where, e_x and e_y are the misalignment in x DOF and in y DOF respectively. It is assumed that the same control method and parameters are employed in the two radial DOFs. That is

$$\frac{\partial i_x}{\partial x} = \frac{\partial i_y}{\partial y}, \frac{\partial i_x}{\partial \dot{x}} = \frac{\partial i_y}{\partial \dot{y}} .$$

According to equation (5), the stiffness will be unequal in the two radial DOFs. And the damping is also the same. That is $k_{ex} \neq k_{ey}$ and $c_{ex} \neq c_{ey}$. Therefore, it is anisotropic of the stiffness and damping of AMB system with machining and assembly errors.

C. Dynamics and Motion Analysis of AMB- Jeffcott Rotor System with Anisotropic Stiffness

A rigid Jeffcott rotor levitated by two radial AMBs can be drawn as the Fig.2. The disc is in the middle of the shaft, the mass of the disc is m, and the shaft is rigid and without mass. The unequal stiffness of the AMB in x and y DOFs respectively is k_x and k_y, and the damping is c_x and c_y. The rotor speed is ω.

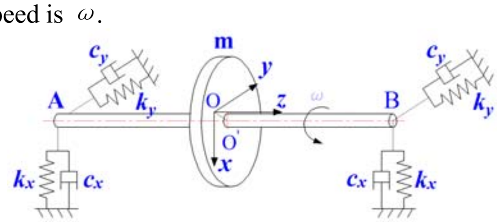


Figure 2. A Jeffcott rotor - AMB system

There is a mass eccentricity on the disc. The mass center C and the geometric center O' of the disc are not coincident, shown as the Fig.3. The eccentricity distance of the disc is e_m.

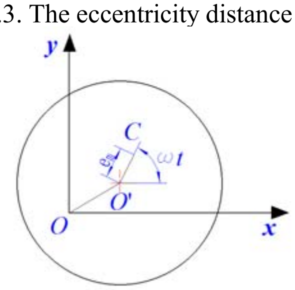


Figure 3. The disc

To simplify the analysis, it can be assumed that $x_A = x_{O'} = x_B, y_A = y_{O'} = y_B$. The differential equation of the system derived from Lagrange equation is shown as equation (6). The gravity of the disc is not considered. Then the equation (7) can be obtained. The x and y are the coordinates of the geometric center O'.

$$\begin{cases} m\ddot{x} + 2c_x\dot{x} + 2k_x x = me_m\omega^2\cos\omega t \\ m\ddot{y} + 2c_y\dot{y} + 2k_y y = me_m\omega^2\sin\omega t \end{cases} \tag{6}$$

$$\begin{cases} \ddot{x} + 2\xi_x\omega_{cx}\dot{x} + \omega_{cx}^2 x = e_m\omega^2\cos\omega t \\ \ddot{y} + 2\xi_y\omega_{cy}\dot{y} + \omega_{cy}^2 y = e_m\omega^2\sin\omega t \end{cases} \tag{7}$$

Where, $\omega_{cx} = \sqrt{\frac{2k_x}{m}}, \omega_{cy} = \sqrt{\frac{2k_y}{m}}, \xi_x = \frac{c_x}{\sqrt{2mk_x}}, \xi_y = \frac{c_y}{\sqrt{2mk_y}}$.

The damping ratio is often less than 1 in most actual systems. Therefore, the motion of the geometric center of the disc can be solved as equation (8).

$$\begin{cases} x = \dfrac{e_m\beta_x^2}{\sqrt{(1-\beta_x^2)^2 + (2\xi_x\beta_x)^2}}\cos(\omega t - \varphi_x) \\ y = \dfrac{e_m\beta_y^2}{\sqrt{(1-\beta_y^2)^2 + (2\xi_y\beta_y)^2}}\sin(\omega t - \varphi_y) \end{cases} \tag{8}$$

Where, $tg\varphi_x = \frac{2\xi_x\beta_x}{1-\beta_x^2}, tg\varphi_y = \frac{2\xi_y\beta_y}{1-\beta_y^2}$, $\beta_x = \frac{\omega}{\omega_{cx}}, \beta_y = \frac{\omega}{\omega_{cy}}$.

D. Analysis of the Orbit of Rotor Geometric Center

According to the equation (8), the orbits of the geometric center of the disc were plotted under different speeds and damping ratios to study the whirling motion of the rotor levitated by AMBs with anisotropic stiffness. It is assumed that the critical speeds of the AMB - Jeffcott rotor system in x and y DOFs respectively are 40Hz and 50Hz. That is $\omega_{cx} = 40\text{Hz}, \omega_{cy} = 50\text{Hz}$.

When $\xi_x = \xi_y$, the rotor orbits with different damping ratios were drawn in Fig.4. The differences of the two phase angles (φ_x - φ_y) under different speeds were shown in table 1.

When $\xi_x \neq \xi_y$, the rotor orbits with different damping ratios were drawn in Fig.5. The differences of the two phase angles (φ_x - φ_y) under different speeds were shown in table 2.

The disc rotates along the anti-clockwise direction around its own geometric center O'. The disk geometric center O' passes along the points with mark red+, blue*, and green● in chronological order in Fig.4 and Fig.5.

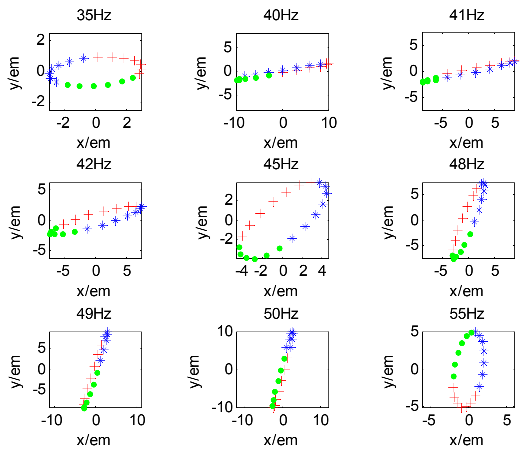
From the Fig.4, Fig.5, table 1 and table 2, it can be found that when the rotor speed is between the two critical speeds ω_{cx} and ω_{cy}, the backward whirling will appear once the difference of the two phase angels (φ_x-φ_y) is greater than 90 degrees, and the product of the damping ratios ($\xi_x \times \xi_y$) is less than 0.01246.

According to the above results, the critical condition of the backward whirling can be concluded as the equation (9). From this equation, the critical product of damping ratios $(\xi_x \times \xi_y)_c$ can be obtained and it changes with the ratio of the two critical speeds (ω_{cy}/ω_{cx}), shown as the equation (10) and Fig.6.

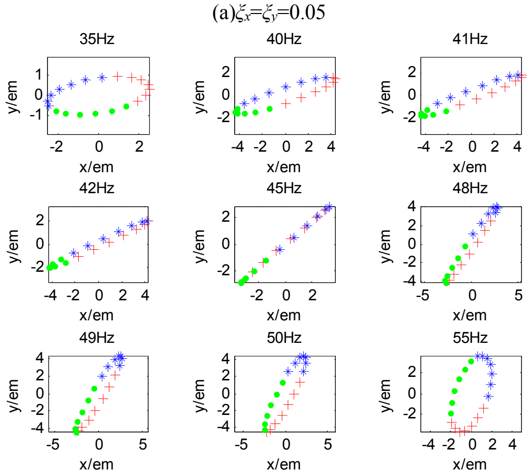
$$\begin{cases} tg\varphi_x tg\varphi_y = -1 \\ \omega = \dfrac{\omega_{cx} + \omega_{cy}}{2} \end{cases} \tag{9}$$

$$(\xi_x\xi_y)_c = -\frac{1}{16}\frac{[4-(1+p)^2][4-(1+\frac{1}{p})^2]}{(1+p)(1+\frac{1}{p})} \tag{10}$$

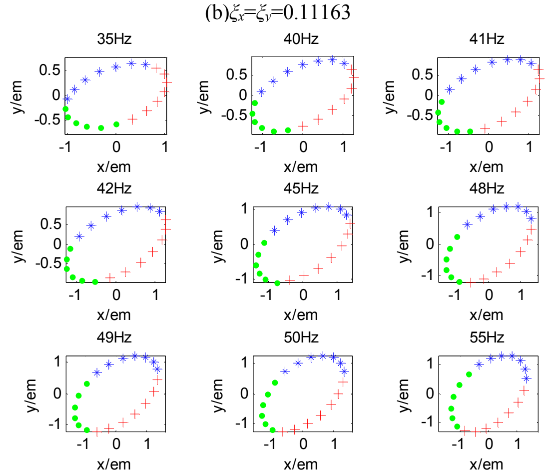
Where $p = \omega_{cy} / \omega_{cx}$.



(a)ξ_x=ξ_y=0.05



(b)ξ_x=ξ_y=0.11163

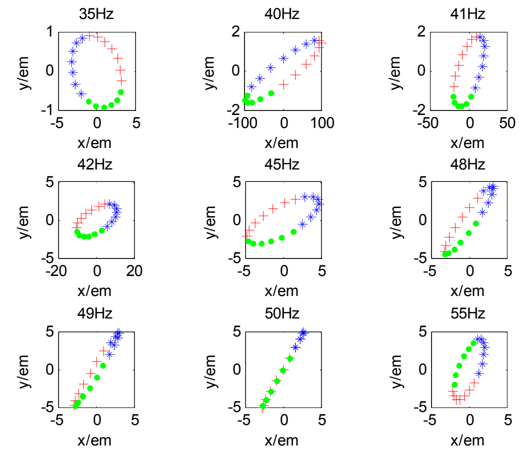


(c)ξ_x=ξ_y=0.4

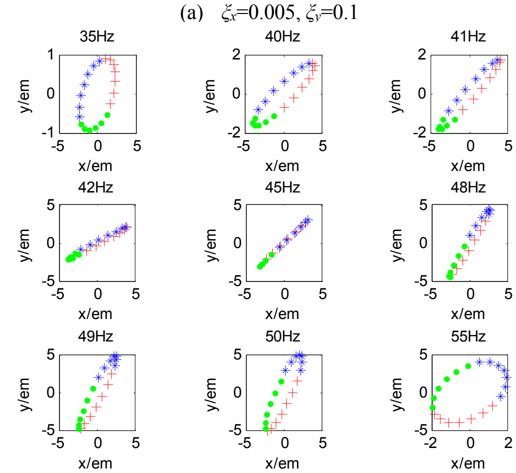
Figure 4. The orbits of the rotor with equal damping ratios

Table 1. The difference of the two phase angles (φ_x - φ_y) with equal damping ratios unit: degree

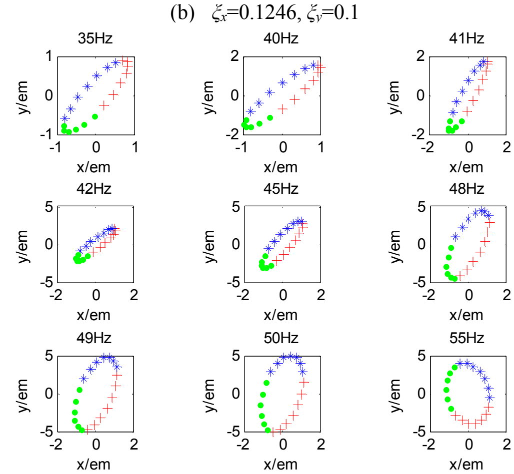
ω/Hz	35	40	41	42	45	48	49	50	55
ξ_x=ξ_y=0.05	13	78	102	118	131	114	98	78	19
ξ_x=ξ_y=0.11163	23	64	73	81	90	79	72	64	30
ξ_x=ξ_y=0.4	24	29	30	31	31	30	30	29	26



(a) ξ_x=0.005, ξ_y=0.1



(b) ξ_x=0.1246, ξ_y=0.1



(c) ξ_x=0.5, ξ_y=0.1

Figure 5. The orbits of the rotor with unequal damping ratios

Table 2. The difference of the two phase angles (φ_x - φ_y) with unequal damping ratios unit: degree

ω/Hz	35	40	41	42	45	48	49	50	55
ξ_x=0.005, ξ_y=0.1	-13	66	142	144	134	111	100	89	45
ξ_x=0.1246, ξ_y=0.1	28	66	75	82	90	78	70	61	25
ξ_x=0.5, ξ_y=0.1	60	66	66	66	60	42	34	24	-11

If $\xi_x\xi_y < (\xi_x\xi_y)_c$ and the rotor speed is between the two critical speeds, the backward whirling will be observed. However, if $\xi_x\xi_y > (\xi_x\xi_y)_c$, the backward whirling will not be observed, even if the speed is between the two critical speeds.

When p=50/40=1.25, the critical value $(\xi_x \xi_y)_c = 0.01246$ can be calculated from the equation (10), which equals the value derived from the above rotor orbits.

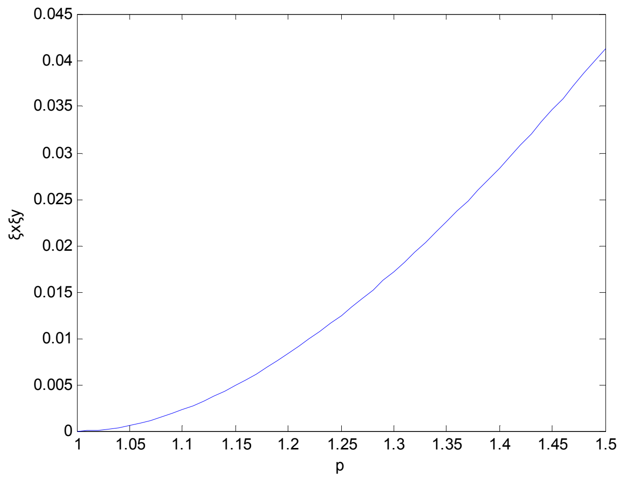


Figure 6. The relationship between the critical product of damping ratios and the ratio of the critical speeds

E. Conclusion

(1) The stiffness and damping ratio in x and y radial DOFs of an AMB may be unequal due to the machining and assembly errors. This will make the critical speeds unequal in the two DOFs of a maglev rotor system.

(2) If the stiffness is anisotropic in a maglev rotor system, the damping ratio product ($\xi_x \times \xi_y$) will determine whether the backward whirling motion will be observed when the rotor speed is between the two critical speeds. Moreover, the critical value $(\xi_x\xi_y)_c$ of the product of the two damping ratios, which varies with the ratio of the two critical speeds (ω_{cy}/ω_{cx}), is derived in this paper. If $(\xi_x\xi_y)<(\xi_x\xi_y)_c$ and the rotor speed is between the two critical speeds, the backward whirling motion will be observed. However, if $(\xi_x\xi_y)>(\xi_x\xi_y)_c$, the backward whirling motion will not be observed.

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