A Review on Hysteresis in Magnetic Bearing-Rotor System

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***Abstract*—There are inevitably hysteresis phenomena in magnetic bearing-rotor systems. On the one hand, the nonlinear relationship of the restoring force and deformation of the engineering materials, especially composites and internal friction between the rotor components may result in hysteresis. On the other hand, there is inherently magnetic hysteresis in the actuator. The hysteresis is a typical one of nonlinear characteristics. The property may have a strong effect on the stability of systems, and make system exhibit complex, unpredictable nonlinear behaviors. It is crucial to analyze magnetic bearing-rotor system by taking account of hysteresis. In this paper, the literature about the magnetic bearing-rotor system with hysteresis are reviewed. The characteristics of nonlinear systems with hysteresis and the effects of hysteresis on systems' dynamics are summarized. The modeling approaches are researched and the advantages and disadvantages of each method are pointed out. This paper introduces various methods applied to nonlinear systems with hysteresis and illustrates corresponding results. The methods which could be appropriate for magnetic bearing-rotor systems subjected to hysteresis are discussed and the future research directions have been prospected.**

I. INTRODUCTION

As a typical mechatronic product, a magnetic bearings system can generate electromagnetic forces through the magnetic bearings to suspend the rotor with no mechanical contact. The magnetic bearings system comprises sensors, controllers, power amplifiers, magnetic bearings and the corresponding mechanical equipment, etc. In the magnetic bearings system, the displacement of the rotor is measured through the sensor and the measurement signal is transmitted to the controller. Then it turns into a control signal. The power amplifier receives the control signal and exports the current which plays a key role in magnetic bearings to make the magnetic bearings generate electromagnetic forces to suspend the rotor. There is no mechanical contact between the rotor and the stator in the magnetic bearings system. Compared with the conventional mechanical bearings, magnetic bearings have a number of advantages, such as no mechanical friction, no wear, lubrication free, etc. The magnetic bearing system can be operating at high speed for a long time with low maintenance cost. Especially, dynamic behaviors of the rotor could be controlled actively through magnetic bearings during operation. So, magnetic bearings are used widely in high-speed rotating machines, especially those operating in special environments.

In many previous researches, for simplicity, the modeling, performance analysis and controller design of the active magnetic bearing-rotor systems were usually based on linear models which could describe characteristics of the systems in the linear region [1]. And system identifications were always conducted based on those linear models [2,3]. Nonetheless, the system was inherently nonlinear so the linear model sometimes can't describe the characteristics of the system. For example, there are inevitably hysteresis phenomena in magnetic bearing-rotor systems. In magnetic bearings systems, the hysteresis is one of the inherent nonlinear characteristics. In electromagnetic actuators, the magnetic hysteresis which can be expressed by B-H curve is obviously existing. In fact, the voltage saturation phenomenon existing in the power amplifier could be regarded as the hysteresis characteristic, which would be explained later. Moreover, the nonlinear relationship between the restoring force and deformation of the engineering materials especially composites, and internal friction between the rotor components may result in hysteresis phenomena.

The hysteresis property may have a strong effect on the stability of systems and make system exhibit more complex, unpredictable dynamical behaviors. The analysis of magnetic bearing-rotor systems neglecting hysteresis can't reflect the nonlinear dynamics of the systems. It is crucial to analyze the magnetic bearing-rotor system by taking account of hysteresis property. The paper reviewed the literature about the nonlinear systems with the hysteresis and summarized the relevant research contents that have been done.

The content of this paper is as follows. In section 2, the hysteresis was presented and various modeling approaches were shown. Moreover, this section gave an outline of the influences of the hysteresis in some typical systems and the corresponding methods. In section 3, the hysteresis phenomena which could exist in the magnetic bearing-rotor system were summarized and the methods applied to the systems were outlined. The summary of this paper was made in section 4.

II. HYSTERESIS MODELS AND APPLICATIONS

The hysteresis phenomena widely exist in various fields, such as biology, magnetics, ferromagnetics, mechanics, and so on. The hysteresis refers to a memory effect, where there is a lag between the input and the output of the system and the output is not only dependent on the instantaneous state but also associated with the history of the state of the system, as shown in Fig. 1.

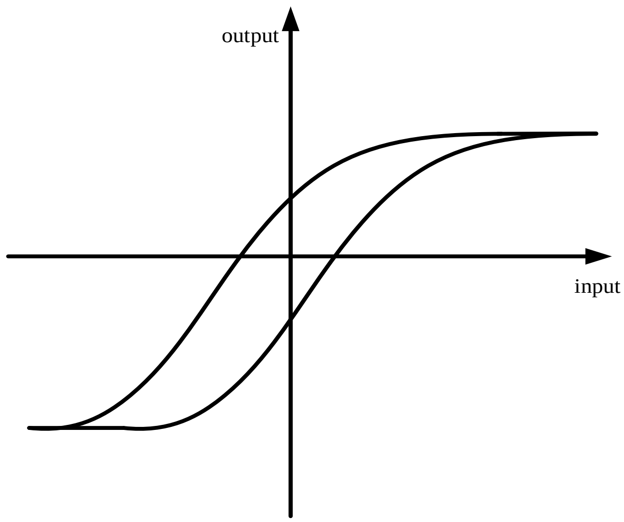

Figure 1. Hysteresis loop

To study hysteresis, modeling of the hysteresis systems is necessary. However, it is a challenge to obtain the physical model of hysteresis whose characteristics is so complex. And it is too complex to apply the physical model to the analysis of systems with hysteresis in various fields. Therefore, mathematical models fitting the hysteretic phenomena and semi-physical models are often used to approximate hysteresis to reflect some of its characteristics or phenomena. Some classic models of the hysteresis are following.

The bilinear hysteresis model was used in [4,5], which were formed with four piecewise affine hysteresis terms. In the bilinear model, the phenomena of hysteresis in different phases were fitted by piecewise lines simply. The model was simple. To a certain degree, the bilinear model could reflect the characteristics of the hysteresis. And it is easy to be applied to the analysis of the system subjected to hysteresis. Nevertheless, there were discontinuities in the model. And it was not in good agreement with the nonlinearity of the hysteresis. In some cases, the bilinear model could not meet the analytical precision.

Ref. [6] was organized to illustrate two types of the hysteresis models, namely the operator-based model including Preisach model, Krasnosel'skii-Pokrovskii (KP) model, Prandtl-Ishlinskii (PI), and Maxwell-Slip model, and the differential-based containing Bouc-Wen model and Duhem model. Various kinds of models were introduced to readers and the survey on the hysteresis in terms of modeling, control, and identification of the systems was made.

The most popular model of hysteresis in civil and mechanical engineering is Bouc-Wen model. The model is formulated in a first-order nonlinear differential equation, which relates the output restoring force to the input displacement in a hysteretic way. In this model, the parameters chosen properly could modulate the position and width of the hysteretic loop to make the model accord with the actual hysteresis. The parameters are usually found through identification techniques [7].

The plentiful developed models based on the classical Bouc-Wen model have arisen. For example, two dynamical models combining the position information and the accelerate information separately were developed based on Bouc-Wen model [8]. The model using position information is rate-independent and could reproduce some kinds of double hysteretic loops that original Bouc-Wen could not. The model employing acceleration information is insensitive to linear time-scale variations. These two models broadened the application of the Bouc-Wen model.

In addition, there are also other forms models enriching the hysteresis models. A phenomenological approach which described the hysteresis was proposed [9]. The force and kinematic parameters of the system were associated with a first-order ordinary differential equation. The right-hand side is chosen in the class of functions ensuring the asymptotic approach of the solution to the curves of the enveloping. This method is analytical, so it has a number of advantages in analyzing the nonlinear behaviors of hysteretic systems.

There is abundant literature reporting the hysteresis models. When the analysis of the system is conducted, it is essential to choose a proper hysteresis model. Since the hysteresis is a complicated nonlinear characteristic, the systems subjected to hysteresis could exhibit many unexpected phenomena and the stability may be affected. Many types of research on the behaviors of the systems subjected to hysteresis have been done. Various analysis methods were proposed and valuable results were acquired [4,5,10,11].

A bilinear hysteretic oscillator with clearance and sinusoidal excitation was investigated in [5]. This paper matched the hysteresis based on a bilinear model and showed how to calculate the equivalent stiffness and damping of the system. This paper utilized chaos phenomena to determine the switching time when discontinuities appeared and analyzed the system response in a piecewise linear way. The bifurcation diagram obtained numerically predicted the chaotic motion. Relatively, aiming at oscillations with bilinear hysteresis and sinusoidal excitation [4], a robust numerical technique was proposed to describe the steady and transient response, which could determine the switching time between the various linear regimes robustly and effectively, thus could patch the local solutions to form the global solution. However, in [4], the subharmonic motion was found but chaos not by using bifurcation and continuation techniques.

Based on the bilinear hysteresis, the nonlinear analysis of the systems may deviate from the original results. Ref. [10] introduced additional state variable (internal variable) to construct a hysteresis model based on the Masing-Bouc-Wen model and applied an effective approach based on the analysis of wandering trajectories to determine the conditions leading to chaotic responses of systems. In [11], the harmonic balance method was adopted to obtain frequency response curve of hysteretic two-degree-of-freedom systems based on Masing hysteresis model and the frequency response characteristic was analyzed. Ref. [11] pointed out that the combination of the harmonic and the path-following algorithm was suitable for the analysis of this kind of systems.

The differential equations of these different systems have the similar forms in common. The analyses on these systems are beneficial to characterize this kind of dynamic systems. The unexpected dynamical behaviors in the systems with hysteresis were verified and some analysis methods applied to the systems were pointed out.

In this section, some classical hysteresis models were introduced. For different analysis objects, the hysteresis model should be chosen felicitously. The investigations on the systems with hysteresis were recommended. The systems mentioned above have the similar motion equations with the rotor system. The methods and the corresponding results can provide some references for the analysis of the bearing-rotor system.

III. Hysteresis in Magnetic Bearing-Rotor System

There are many articles about the analysis of vibration and stability in the rotor. Ref. [12] investigated the dynamics of the energy storage flywheel rotor-active magnetic bearings system. But, the analysis in the paper was conducted based on the linear model that was simplified by ignoring the hysteresis and other nonlinearities existing in the system.

The magnetic bearing-rotor system is inherently nonlinear. There was a wide variety of literature on nonlinearities of active magnetic bearing-rotor system, in which the various analytical methods applied could be divided into two categories: the numerical methods and the approximate analytical methods. The bifurcation behaviors of the flexible rotor supported by active magnetic bearings were investigated numerically in [13,14]. Nonlinear oscillations of a rigid magnetic bearing-rotor system taking account of the delay of control force were analyzed using the approximate analytical method in [15]. The influences of the delay of control force and other system parameters on the amplitude of the rotor were clarified and the analysis results were verified by experiments. Ref. [16] applied the method of multiple scales to obtain an analytical approximate solution of an active magnetic bearing-rotor system which subjected to primary resonance excitations at the presence of 1:1 internal resonance. Based on the solution, a verity of nonlinear phenomena were studied and the analysis results of the approximate solution were validated by numerical simulations. In [17,18], the dynamic response of active magnetic bearing-rotor system with periodic time-varying stiffness was investigated by the asymptotic perturbation method and the method of multiple scales. And the nonlinear response of the system was illustrated.

It can be seen from these papers that the nonlinear factors in the system have unpredictable effects on the system response as well as the stability, and many effective methods dealing with the nonlinear dynamics of the system were proposed. However, little literature deals with the nonlinear systems with hysteresis. As a typical nonlinear characteristic of the bearing-rotor system, the hysteresis characteristics have a non-negligible influence on the rotor's dynamics. The analysis methods on nonlinear systems mentioned in above papers have referential value for the analysis of the magnetic bearing-rotor systems subjected to hysteresis.

In the practical system, the hysteresis may occur in several ways. The magnetic hysteresis of the ferromagnetic material exists intrinsically in the electromagnetic actuator. The power amplifier will exhibit hysteresis characteristic when the voltage saturation phenomenon occurs. In addition, material, internal friction, and internal damping of the rotor may result in hysteresis in the rotor.

A. Hysteresis in The Rotor

The hysteresis always occurs along with some irreversible thermodynamic changes, such as phase transition and internal friction. The material properties and internal friction in the rotor may lead to hysteresis.

The bifurcating self-excited vibrations of a rotating viscoelastic shaft was investigated based on the Hopf bifurcation theory in [19]. This paper concentrated on hysteresis due to the material of the rotor. The influence of hysteresis on the stability of the rotor was verified.

The internal friction in the rotor material is one of the major causes of the hysteresis which has a significant effect on the stability of the rotor. Ref. [20] discussed the stability capacity of the flexible rotor with hysteresis due to three forms of internal frictions separately. The results showed that stability limits may vary considerably with different hysteresis mechanisms. It can be seen from the conclusions of this paper that the proper hysteresis model was essential for the analysis result. But, ref. [20] did not define the type of internal friction specifically, but just discussed the influence of different mechanism hysteretic forces on the system stability abstractly. Ref. [21] concentrated on the internal friction existing actually in the rotor, which was defined to be hysteretic-type. The emphasis was the stability in the primary resonance region and the averaged equations in primary resonances were obtained for the stability analysis by using the averaging method. The method of singularity was utilized to investigate the bifurcation in the synchronous and non-synchronous whirl. The instability thorough Hopf bifurcation was found.

Sorge utilized the linear model and the nonlinear Coulombian force model to express the internal friction separately and illustrated the effect of the hysteresis on the stability separately in [22]. The paper indicated that the nonlinear Coulombian model was more applicable than the linear model which was actually consistent with the model in [21]. The averaging approaches of the Krylov-Bogoliubov were proposed to solve the nonlinear motion equations of the rotor with internal friction in [22–24]. The destabilizing effect of the internal friction was indicated and the beneficial influence of the support stiffness anisotropy was clearly identified.

It can be concluded that the stability of the rotor system is affected by the internal friction which has the hysteretic characteristic. The researches on the problem make a contribution to the rotor dynamics and the proposed methods help successor to study the problem further.

The internal damping of the rotor may also lead to the instability of the system. As we know, the material losses of many engineering materials are frequency-independent, which was not consistent with the viscous damping model used before. Hysteretic type damping can better reflect the internal damping of the rotor than the viscous damping. Therefore, when analyzing the influence of rotor internal damping on rotor stability and other properties, hysteretic internal damping has attracted the attention of the researchers.

Ref. [25] used the finite element method to discuss the stable regions of the rotating shaft with internal damping based on a Euler-Bernoulli model. The threshold speeds of the rotating shaft with different forms of the internal damping, namely the viscous damping and the hysteretic damping, were compared. This paper pointed out that the hysteretic damping model could reflect the characteristic of internal damping better. In this paper, hysteretic damping was modeled by including an equivalent viscous damping term. However, in [26], the internal hysteretic damping was expressed by the exponential model which was more general than the classical viscous damping model. The rotating viscoelastic shaft with hysteretic internal damping was investigated in [27]. In this paper, the boundary of the stability and the conditions for a double zero eigenvalue bifurcation and Hopf bifurcation was determined based on the linear model. Then based on the nonlinear model, the analytical solutions describing the behaviors of the shaft in the neighborhood of the bifurcations were deduced using the center manifold theory and the method of the normal form. The methods proposed in this paper enriched the theory inestigating the dynamic behavior of a rotor system with hysteresis characteristics, which were of great significance.

The internal damping of the rotor will cause the hysteresis and the hysteretic characteristic has a non-negligible effect. So many analyses on the rotor with internal damping were conducted. The proper selection of the model makes sense.

There are kinds of literature about the rotor subjected to hysteresis. The inducements of the hysteresis in the rotor were classified and the corresponding modeling methods were introduced. The analysis methods and corresponding results about the effect of the hysteresis on the dynamic behaviors and the stability of the rotor were outlined.

B. *Hysteresis in Magnetic Bearing*

However, the dynamics of the bearing-rotor system may change when the rotor is supported by magnetic bearings. Investigating the hysteresis in the magnetic bearings systems is indispensable. The magnetic hysteresis in the magnetic bearing system is inherent. And the system behaviors have some change with magnetic bearings.

The existence of the hysteresis in magnetic bearings was pointed out in [28]. The hysteretic characteristics should be taken into account in the dynamic analysis, design of the magnetic bearings, and controller design. In [29], the dynamics of the rotor supported by the magnetohydrodynamic bearings was illustrated separately in case of soft magnetic materials and rigid magnetic materials. In case of soft magnetic materials, the approximate analytical solution which described vibration of the rotor was acquired using the method of multiple scales. In case of rigid magnetic materials, the hysteresis which was modeled by Bouc-Wen model was taken into account and the condition when chaos occurred was determined by the approach based on the analysis of the wandering trajectories.

In practice, the devices in the magnetic bearings system always have the capacity limitation. For example, the operating voltage in the voltage-type power amplifier is limited in certain ranges. So the voltage saturation phenomena often occur. As a matter of fact, the input voltage and the output current which plays a key role in the magnetic bearing have a hysteretic type relationship, as shown in Fig. 2. So the voltage saturation can be regarded as the hysteresis property in the magnetic bearings system.

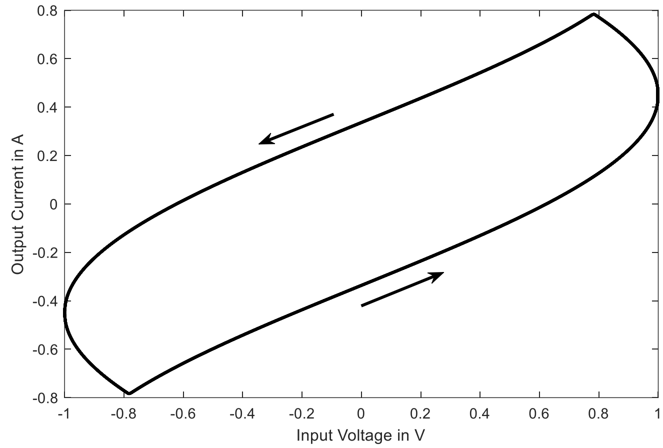

Figure 2. Hysteresis loop betweenthe voltage and the current

Ref. [30] utilized the analytical method to analyze the single-degree-of-freedom magnetic bearing-rotor system with saturation effects. The symmetric period-one solutions were obtained and the analysis of the stability was determined. Moreover, other kinds of solutions, such as asymmetric, subharmonic periodic and chaotic motions and multiple-crossing saturation region per cycle periodic solutions was also found by means of numerical simulation. The voltage saturation was introduced into the magnetic bearings system and the influence on the dynamics of the rotor was analyzed. However, the investigation based on the piecewise linear model could not reflect the comprehensive characteristics of the voltage saturation.

The voltage saturation was also mentioned in [31–33]. But these papers concentrated on the control design instead of the dynamic analysis. However, the analysis of the dynamic behaviors of the rotor could be of great benefit to the controller design. It is impending to investigate the voltage saturation in the magnetic bearings system further.

In this section, the potential hysteresis existing in different parts of the magnetic bearing-rotor was set forth through a review of the pertinent literature. The different hysteresis models were applied to the analyses on the rotor systems with hysteresis. Various methods including the numerical methods and the analytical methods, were utilized to investigate the dynamics of the rotor and corresponding results were shown. It can be seen that the hysteresis in the magnetic bearing-rotor system is common and has effects on the dynamics of the system. It is important to investigate the hysteresis in the system comprehensively using effective method and show the influence of the hysteresis on the dynamic behaviors of the system.

IV. CONSLUSIONS

In this paper, the literature about the magnetic bearing-rotor system with hysteresis was reviewed. The various modeling methods of the hysteresis were outlined and some hysteresis models applied to the specific systems was introduced. A variety of methods used to investigate dynamic behaviors of the system with hysteresis and corresponding results are shown. The potential hysteresis existing in different parts of the magnetic bearing-rotor was illustrated. And dynamic behaviors of the system with hysteresis and the analysis method were summarized.

As already reported, the hysteresis in the magnetic bearing-rotor system is always associated with the bifurcation, chaos, and the instability of the rotor. The methods and results mentioned in this paper are heuristic. This paper shows the importance of the investigations on the magnetic bearing-rotor system subjected to the hysteresis and provides the effective methods. The analyses of dynamic behaviors in the systems with the hysteresis would play a key role in the construction of the magnetic bearings and the controller design.

REFERENCES

[1] G. Schweitzer, and E. H. Malsen, Magnetic Bearings:Theory, design and application to rotating machinery, *Springer Verlag*, Berlin, 2009.

[2] Z. Sun, J.J. Zhao, Z.G. Shi, and S.Y. Yu, "Identification of flexible rotor suspended by magnetic bearings," *Proceedings of Int. Conf. on Nuclear Engineering*, 2013.

[3] Z. Sun, Y. He, J.J. Zhao, Z.G. Shi, L. Zhao, and S. Yu, "Identification of active magnetic bearing system with a flexible rotor," Mechanical Systems and Signal Processing, 49, pp. 302–316, 2014.

[4] T. Kalmr-Nagy, and A. Shekhawat, "Nonlinear dynamics of oscillators with bilinear hysteresis and sinusoidal excitation," *Physica D Nonlinear Phenomena*, vol. 238, no. 17, pp. 1768–1786, 2009.

[5] H. G. Li, J. W. Zhang, and B. C. Wen, "Chaotic behaviors of a bilinear hysteretic oscillator". *Mechanics Research Communications*, vol. 29, no. 5, pp. 283–289, 2002.

[6] V. Hassani, T. Tjahjowidodo, and T. N. Do, "A survey on hysteresis modeling, identification and control," *Mechanical Systems and Signal Processing*, vol. 49, no. 1-2, pp. 209–233, 2014.

[7] M. Ismail, F. Ikhouane, and J. Rodellar, "The hysteresis bouc-wen model, a survey," *Archives of Computational Methods in Engineering*, vol. 16, no. 2, pp. 161–188, 2009.

[8] F. Pozo, L. Acho, A. Rodrłguez, and G. Pujolz,"Nonlinear modeling of hysteretic systems with double hysteretic loops using position and acceleration information," *Nonlinear Dynamics*, vol. 57, no. 1-2, pp. 1–12, 2009

[9] A. N. Danilin, "Vibrations of mechanical systems with energy dissipation hysteresis," *Mechanics of Solids*, vol. 52, no. 3, pp. 254–265, 2017.

[10] J. Awrejcewicz, and L. Dzyubak, "Hysteresis modelling and chaos prediction in one- and two-dof hysteretic models," *Archive of Applied Mechanics*, vol.77, no.5, pp. 261–279, 2007.

[11] R. Masiani, D. Capecchi, and F. Vestroni, "Resonant and coupled response of hysteretic two-degree-offreedom systems using harmonic balance method," *International Journal of Non-Linear Mechanics*, vol. 37, no. 8, pp. 1421–1434, 2002.

[12] J. Liu, Z. Ren, S. Wu, and Y. Tang, "Theoretical vibration analysis on 600 Wh energy storage flywheel rotorłactive magnetic bearing system,". International Journal of Rotating Machinery,vol.8, 2013.

[13] M.-J. JANG, and C.-K. CHEN, "Bifurcation analysis in flexible rotor supported by active magnetic bearing," International Journal of Bifurcation and Chaos, vol.11, no.8, pp. 2163–2178, 2001.

[14] J. I. Inayat-Hussain, "Geometric coupling effects on the bifurcations of a flexible rotor response in active magnetic bearings," Chaos Solitons and Fractals, vol.41, no.5, pp. 2664–2671, 2009.

[15] T. Inoue, Y. Ishida, and S. Murakami,"Theoretical analysis and experiments of the nonlinear vibration in a vertical rigid rotor supported by the magnetic bearing system (case considering the delay of control force):(case considering the delay of control force)," Journal of System Design and Dynamics, vol.1981, no.1, pp. 295–306, 2007.

[16] N. A. Saeed, M. Eissa, and W. A. El-Ganini, "Nonlinear oscillations of rotor active magnetic bearings system," onlinear Dynamics, vol.74, no.1-2, pp. 1–20, 2013.

[17] M. Kamel, and H. S. Bauomy, "Nonlinear study of a rotorcamb system under simultaneous primary-internal resonance," Applied Mathematical Modelling, vol.34, no.10, pp. 2763–2777, 2010.

[18] W. Zhang, M. H. Yao, and X. P. Zhan, "Multi-pulse chaotic motions of a rotor-active magnetic bearing system with time-varying stiffness," Chaos Solitons and Fractals, vol.27, no.1, pp. 175–186, 2006.

[19] W. Kurnik, "Bifurcating self-excited vibrations of a horizontally rotating viscoelastic shaft". Ingenieur-Archiv, vol.57, no.6, pp. 467–476, 1987.

[20] H. F. Black, "The stabilizing capacity of bearings for flexible rotors with hysteresis," Journal of Engineering for Industry, vol. 98, no. 1, pp. 87-91 , 1976.

[21] Q. Ding, and Y. Chen, "Bifurcation of a shaft with hysteretic-type internal friction force of material," *Applied Mathematics and Mechanics(in English)*, vol.24, no. 6, pp. 638–645, 2003.

[22] F. Sorge, "Approach to rotor-shaft hysteretic whirl using krylov-bogoliubov techniques," *Journal of Vibration and Control*, vol. 21, no. 5, pp. 883–895 , 2015.

[23] F. Sorge, "Nonlinear analysis of cylindrical and conical hysteretic whirl motions in rotor-dynamics," *Journal of Sound and Vibration*, vol.333, no. 20, pp. 5042–5056 , 2014.

[24] F. Sorge, and M. Cammalleri, "On the beneficial effect of rotor suspension anisotropy on viscous-dry hysteretic instability," *Meccanica*, 47(7), pp. 1705–1722 , 2012.

[25] O. Montagnier, and C. Hochard, "Dynamic instability of supercritical driveshafts mounted on dissipative supportsłeffects of viscous and hysteretic internal damping," *Journal of Sound and Vibration*, vol. 305, no. 3, pp. 378–400, 2007.

[26] A. Skarolek, "Exponential damping as an approach to internal hysteretic damping of rotor systems: Fem model of timoshenko rotating beam with maxwell-weichert damping model," *Mechanisms and Machine Science*, vol. 8, pp. 253–261, 2012.

[27] S. A. A. Hosseini, "Dynamic stability and bifurcation of a nonlinear in-extensional rotating shaft with internal damping," *Nonlinear Dynamics*, vol. 74, no. 1-2, pp. 345–358, 2013.

[28] C. Cristache, I. Valienteblanco, E. Diezjimenez, M.A.A. lvarezvalenzuela, N. Pato, and J.L. Perezdiaz, "Mechanical characterization of journal superconducting magnetic bearings: stiffness, hysteresis and force relaxation," *Proceedings of European Conference on Applied Superconductivity*, 2014.

[29] J. AWREJCEWICZ, and L. P. DZYUBAK, "Chaos caused by hysteresis and saturation phenomenon in 2-dof vibrations of the rotor supported by the magnetohydrodynamic bearing," International Journal of Bifurcation and Chaos, vol. 21, no.10, pp. 2801–2823, 2011.