Stability Margin Evaluation of AMB Rotor Systems Using Singular Value Decomposition

Hyeong-Joon Ahn

[a] School of Mechanical engineering, Soongsil University, Seoul 06978, Korea ahj123@ssu.ac.kr

***Abstract*—Active magnetic bearings (AMBs) are a proven technology that has been used in numerous rotating machines and is primarily being applied to many new turbomachines. Therefore, proper evaluation of rotordynamic performances is very important for OEM and end-users of AMBs. API 617 requires unbalance response limit and separation margin while ISO 14839-3 checks stability margin using measured sensitivity. In addition, singular value decomposition is mathematical tool used extensively in control community and can be used to analyze problems of rotating machines such as unbalance response, model reduction and stability margin. This paper presents stability margin evaluation of AMB rotor systems using singular value decomposition (SVD). First, Two AMB rotor systems and their frequency measurement are introduced. Then, we review linear control theory related to stability and performance evaluation using SVD. Identifications of two AMB rotor systems are performed and exact stability margins of two AMB rotor systems are analyzed based on linear control theory and SVD. Finally, stability margin evaluation of linear control theory and ISO 14839-3 are compared.**

I. INTRODUCTION

AMBs offer an alternative way to support rotors in rotating machinery due to their clear advantages: non-contact, high speed, low maintenance and controllability of bearing dynamics [1]. Therefore, AMBs have been used in numerous rotating machines and are primarily being applied to many new turbomachines.

Rotordynamic performance such as unbalance response, stability and critical speed should clarified for AMB rotor systems like conventional rotor bearing system. Since AMBs are a typical mechatronic device, consisting of mechanical, electrical and information processing elements, proper evaluation of rotordynamic performances is not familiar with OEM and end-users of AMBs [2].

New version of API617 and ISO 14839-3 addresses rotordynamic requirements for AMB rotor system [3-5]. API617 requires amplification factor, separation margin like nd conventional turbomachines as well as much larger vibration limit than conventional bearing. On the other hand, ISO 14839-3 defines stability class based on measured closed-loop frequency responses.

SVD can be used to access rotordynamic problem such as unbalance response, model reduction and stability margin [6]. Originally, SVD is a mathematical tool used extensively in control community and hardly used so far for conventional rotor bearing system.

This paper presents stability margin evaluation of AMB rotor systems using singular value decomposition (SVD). First, ISO 14839-3 and some issues (force excitation and MIMO transfer function matrix) for evaluation of the stability margin of an AMB rotor system are introduced. The, Two AMB rotor systems and their frequency measurement are used to investigate the stability margin based on linear control theory and SVD. Finally, stability margin evaluation of linear control theory and ISO 14839-3 are compared.

II. METHOD

A. ISO 14839-3 stability margin

Closed-loop transfer function should be measured for evaluating the stability margin of an AMB rotor system, as shown in Figure 1 [5]. The closed loop of an AMB-rotor system is simplified using the notation of the transfer function G_r of the AMB control part and the transfer function G_p of the plant (rotor, sensor, and actuator). At a certain point of this closed loop, an excitation (E) such as harmonic or random signals are injected and the closed-loop response V_1 and V_2 are measured directly after or before the injection point. The ratio of these V_1 and V_2 signals provides an open-loop transfer function G_o and the closed loop frequency transfer function G_c and the sensitivity function G_S can be evaluated with Eq. (1) and (2).

$$G_c(s) = -V_2(s)/E(s) \quad (1)$$

$$G_S(s) = -V_2(s)/E(s) \quad (2)$$

The method requires an injection of an excitation signal into an axis of the control loop, then measurement of ratio between the excitation and the response, as shown in Eq. (2). This ratio as a function of frequency (up to 2kHz or three time of the rated speed) is the “sensitivity function”. The lower value of the sensitivity function implies the more robust system. The maximum values of the sensitivity function at four zones are summarized in Table 4.

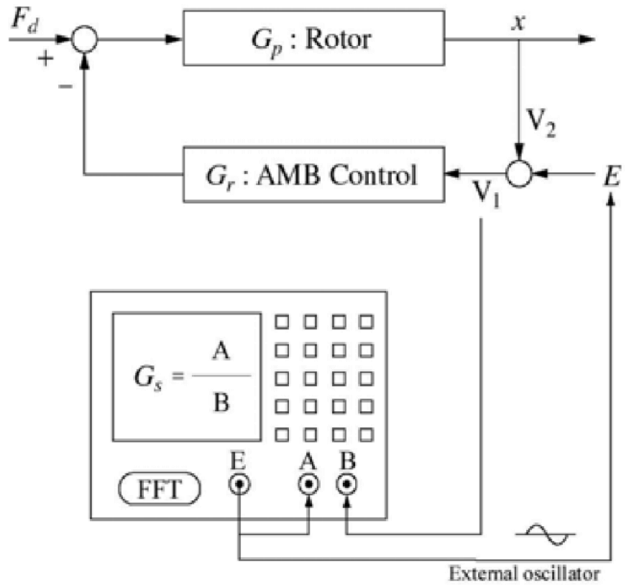


Figure 1. Measurement of transfer function of an AMB rotor system

Table 1. ISO 14839-3 Sensitivity function limits

Zones	Boundaries	Description
A	Max (G_S)<3	New machines
B	3≤Max(G_S)<4	Acceptable for long term operation
C	4≤Max(G_S)<5	Excessively high vibration
D	5≤Max(G_S)	Machine damage expected

B. Transfer function measurement using force excitation

Identification or diagnosis of a rotating machinery is performed using force or displacement excitation. All rotating machine has unavoidable unbalance and this unbalance excites the vibration of the rotating machine. On the other hand, misalignment between elements of a rotating machine is typical displacement excitation source that causes the rotating machine to vibrate.

Although ISO 14839-3 provides sensitivity function measurement using displacement perturbation, closed-loop transfer function can be measured either force or displacement perturbation, as shown in Figure 2. Force perturbation is more simple and intuitive than displacement perturbation since allowable or necessary force excitation is easily calculated based on operating condition. However, actuator input for

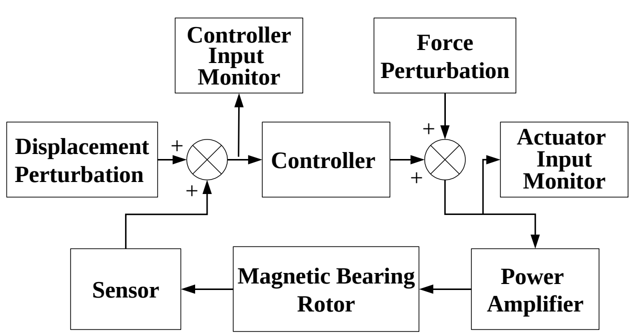


Figure 2.Measurement of transfer function of an AMB rotor system using displacement or force perturbation

magnetic actuator was not commonly available in conventional AMB rotor system. As digital controller and its monitoring system become common, force perturbation is not difficult to access. In this study, we measure closed-loop transfer function using force perturbation and evaluate the stability margin of the AMB system.

C. MIMO transfer funtion

Although robustness of MIMO control system is evaluated using maximum singular value, the stability margin of an AMB system is evaluated based on axis-by-axis or SISO measurement of transfer function. In general, peak of sensitivity function of a control system is recommended to be less than 2 [7]. However, ISO 14839-3 allows much higher peak of the SISO sensitivity function than 2. SISO measurement may not capture maximum singular value or potential risk of a MIMO control system so that peak values may be relieved. In this study, we evaluate the sensitivity function in either SISO or MIMO transfer function. In particular, MIMO closed-loop transfer matrix is measured and its singular value are used for stability evaluation.

III. RESULTS

A. AMB rigid rotor system

A rigid AMB rotor system is shown in Figure 3 and its specifications are summarized in Table 2. The AMB machine is designed for high speed grinding machine. The AMB rigid rotor system includes three radial sensor planes and two actuator plane.

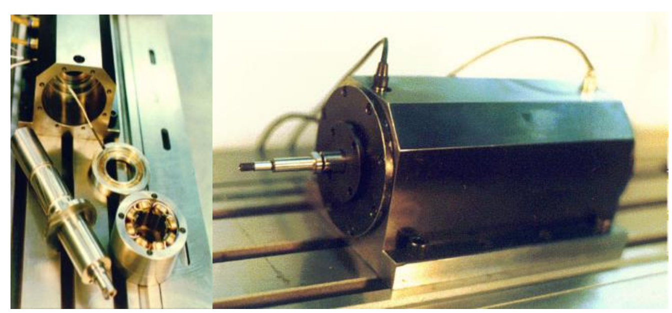
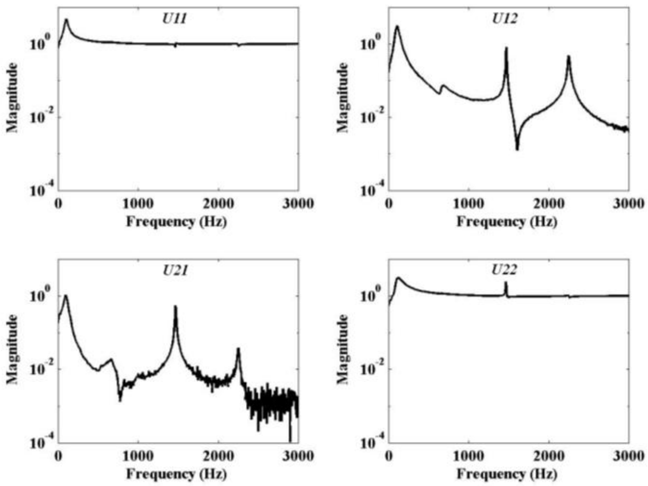


Figure 3. AMB rigid rotor system

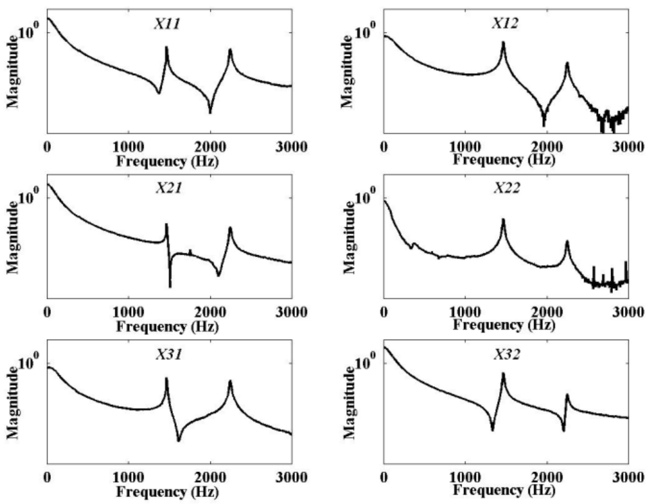
Table 2. Specifications of the AMB rigid rotor system

Specifications	Value
Max. rotating speed	60,000 rpm
Motor power	3kW
Max. radial force	200N
AMB diameter	39mm

Closed-loop MIMO transfer function matrices of the AMB rigid rotor system are measured using force perturbation, as shown in Figure 4. Single sinusoidal force perturbation is injected into every control input in turns from 2 to 1,999 Hz and both all control and actuator inputs are measured. U denotes the MIMO transfer function matrix from the force perturbation to the actuator input monitor, while X denotes the MIMO transfer function matrix from the force perturbation to the controller input monitor. We can get open-loop transfer function matrix $G(s)$ (3×2) by $Y(s)U(s)^{-1}$.



(a) *U* transfer function matrix



(b) *X* transfer function matix

Figure 4. MIMO transfer fuction matrices of the AMB rigid rotor system

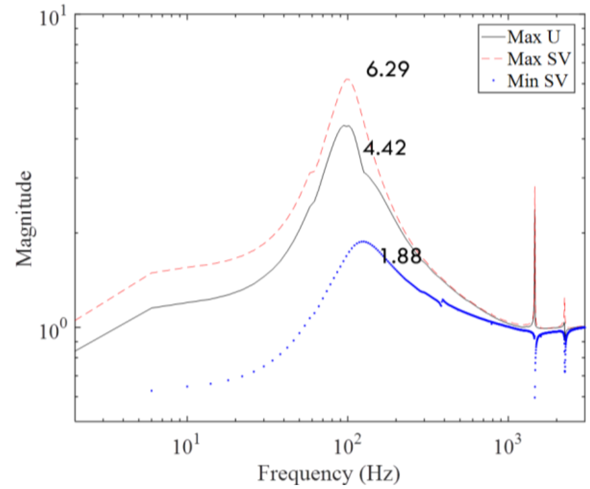


Figure 5. Evaluation of stability margin of the AMB rigid rotor system

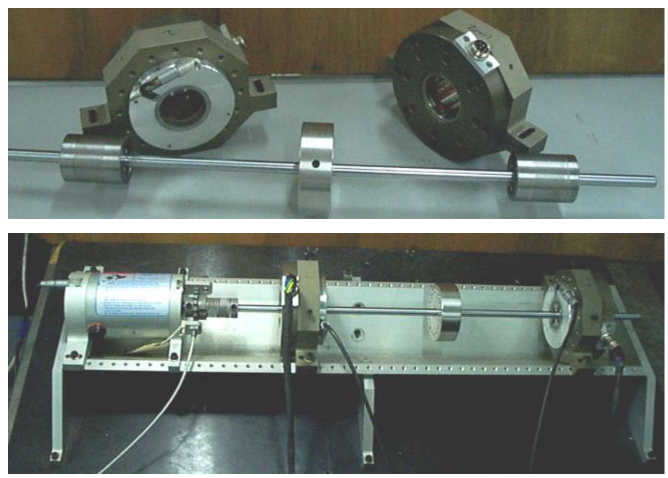


Figure 6. AMB flexible rotor system

Stability margins of the AMB rigid rotor system are analyzed and shown in Figure 5. "Max U" denotes stability margin evaluation of ISO 14839-3 by picking up the peak value of *U* transfer function matrix. The AMB rigid rotor system may have excessive vibration in bearing mode and is not acceptable for long term operation. In addition, peak of maximum singular values of *U* transfer function matrix is 6.29 while peak of the minimum singular values of *U* transfer function matrix is 1.88. Peak of the maximum singular value is too high while peak value of *U* transfer function matrix is not too high.

B. AMB flexible rotor system

The AMB flexible rotor system was modified from the rotor-kit of GE rotor kit, as shown in Figure 6. Each AMB unit has magnetic bearing, displacement sensor (cylindrical capacitive sensor) and back-up ball bearing. The rotor has 10 mm diameter and is 560 mm in length. The AMB had a maximum load capacity of 170 N. The unbalance disk had a weight of 0.8 kg.

Open-loop transfer function matrix of the AMB flexible rotor system are measured using force perturbation, as shown in Figure 7. Single sinusoidal force perturbation is injected into every control input in turns from 1 to 599 Hz and both all control and actuator inputs are measured. We can get open-loop transfer function matrix $G(s)$ (3×2) by $Y(s)U(s)^{-1}$.

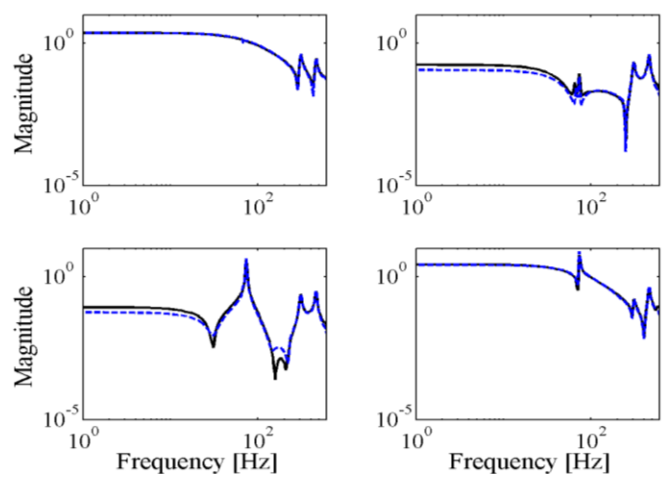


Figure 7. Open-loop transfer function matrix of the AMB flexible rotor system

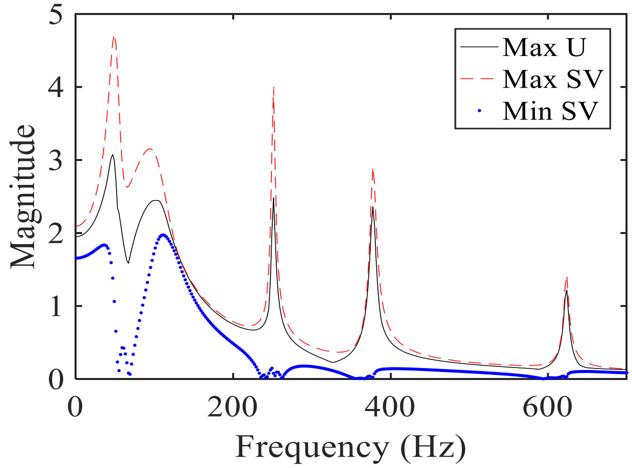


Figure 8. Evaluation of stability margin of the AMB flexible rotor system

Stability margins of the AMB flexible rotor system are analyzed and shown in Figure 8. Stability margin evaluation of ISO 14839-3 is around 3 and the AMB flexible rotor system can be operated for long term. However, peak of maximum singular values of *U* transfer function matrix is little less than 5 while peak of the minimum singular values of *U* transfer function matrix is less than 2.

IV. Conclusion

This paper presents stability margin evaluation of AMB rotor systems using singular value decomposition (SVD). First, ISO 14839-3 and some issues (force excitation and MIMO transfer function matrix) for evaluation of the stability margin of an AMB rotor system are introduced. The, Two AMB rotor systems and their frequency measurement are used to investigate the stability margin based on linear control theory and SVD. Finally, stability margin evaluation of linear control theory and ISO 14839-3 are compared. For accurate evaluation of the stability margin of an AMB rotor system, further study should be done considering MIMO transfer function matrix.

References

[1] Schweitzer, G. and Maslen, E. H. (editors), 2009, Magnetic Bearings: Theory, Design and Application to Rotating Machinery, Berlin, Germany; Springer-Verlag.

[2] Swanson, E.E., Maslen, E.H., Li, Guoxin, Li, Cloud, C.H., Rotordynamic design audits of AMB supported machinery, Proceedings of the 37th Turbomachinery Symposium, https://doi.org/10.21423/R19S7N

[3] Swanson, E., Masala, A., Hawkins, L., 2014, A new Active Magnetic Bearing Requirements for Compressors in API 617 Eighth Edition,@ Proceedings of the 43rd Turbomachinery Symposium, Tutorial 10.

[4] American Petroleum Institute, 2014, API Standard 617, Eighth Edition, Axial and Centrifugal Compressors and Expander-Compressors, Washington, D.C., USA, API Publishing Services.

[5] ISO 14839-3:2006, Mechanical vibration – Vibration of rotating machinery equipped with active magnetic bearings – part 3: Evaluation of stability margin

[6] Cloud, C. H., Failes, W. C., Li, G. , Maslen, E. H., Barrett, L. E., Practical applications of singular value decomposition in rotordynamics, AUSTRALIAN JOURNAL OF MECHANICAL ENGINEERING; 2, 1; 21-32,2005

[7] Skogestad, S. and Postlethwaite I., 2005, *Multivariable feedback control: analysis and design*, 2nd edition, Willey.