Random response analysis of magnetic bearing rotor system

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***Abstract*—In the practical application, the magnetic bearing-rotor system is subject to various external disturbances in practical application. Under certain control conditions, the random response characteristics of the magnetic bearing-rotor system is a particular concern. This paper analyzes the response characteristics of the magnetic bearing subjected to Gaussian random excitation in the horizontal direction. First, the magnetic bearing-rotor system model is deduced. Then, the random response of the rotor under Gaussian random excitation is derived. The probability of the collision of the rotor between the auxiliary bearing is calculated and the example is given. The paper conclusion provides a theoretical basis for the collision detection and prediction of the magnetic bearing-rotor system.**

I. PREFACE,

Magnetic Bearing uses the magnetic force to suspend the rotor in the air, so that there is no mechanical contact between the rotor and the stator. Compared with the traditional ball bearing, sliding bearing and oil film bearing, the magnetic bearing has no mechanical contact. The rotor can run to a high speed. It has the advantages of small mechanical wear, low energy consumption, low noise, long life, no lubrication, no oil pollution and so on. So it is especially suitable for high speed, vacuum and ultra clean environment. The theoretical research and Application Research of magnetic suspension bearings have been continued until now.

The mechanical engineers who study probability do so specifically so that they may better estimate the likelihood that some engineering system will provide satisfactory service. This is often stated in the complementary way as estimating the probability of unsatisfactory service or failure. Thus, the study of probability generally implies that the engineer accepts the idea that it is either impossible or infeasible to devise a system that is absolutely sure to perform satisfactorily.

Because most of the magnetic levitation bearings are used in high-speed rotating machinery, the working gap of the magnetic bearing is very small. The high speed rotor produces violent vibration during the fall process, which can cause damage to the rotor and auxiliary bearings, even result in the system failure.

Therefore, it is very important to predict the probability of the collision between the rotor and the auxiliary bearing in the practical application of the magnetic bearing. There are few literatures in this respect. This paper tries to make a preliminary study on this aspect and analyzes the probability of collision between the rotor and the auxiliary bearing under the random excitation conditions.

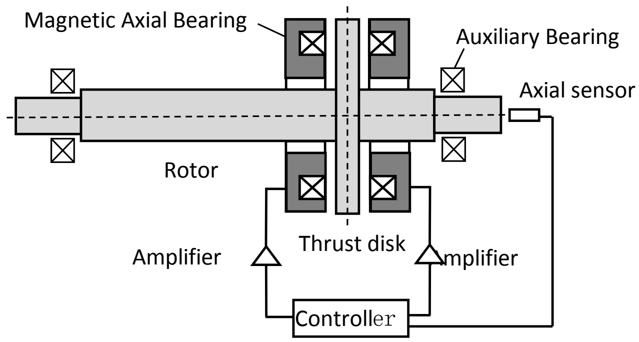

Figure 1. Schematic diagram of magnetic bearing rotor system.

II. AXIAL MAGNETIC BEARING MODEL.

As shown in figure 1

the axial magnetic bearing system consists of a rotor, a thrust disk, an axial magnetic bearing, an auxiliary bearing, a sensor, an amplifier, and a controller.

The electromagnetic force of a magnetic bearing is[1].

$$f_x = f_+ - f_- = \frac{\mu_0 n^2 A}{4}\left[\frac{(i_0+i_x)^2}{(x_0+x)^2} - \frac{(i_0-i_x)^2}{(x_0-x)^2}\right] \quad (1)$$

where: μ_0 is the vacuum permeability, n is the number of coil turns, A is the section area of the air gap.

Equation (1) is a non-linear equation. Because the working gap of the magnetic bearing is very small, it can be liberalized at the work center position, and Taylor is deployed at the center of the work and the high-order and small-quantity are omitted. The dynamic force is can be expressed as

$$f_x \approx k_x x + k_i i \quad (2)$$

Where: $k_x = -\frac{\mu_0 n^2 A i_0^2}{x_0^3}$ is the force-displacement stiffness coefficient, $k_i = \frac{\mu_0 n^2 A i_0}{x_0^2}$ is force-current stiffness.

When studying the magnetic bearing system, it is often compared with similar mechanical systems. Because mechanical system model has a very complete maturity theory. A typical single-degree-of-freedom mechanical system consists of three basic elements: the mass m, spring k, damping c. The mechanical system motion differential equation is[2]

$$m\ddot{x} + c\dot{x} + kx = f \quad (3)$$

For an axial magnetic bearing rotor system, its differential equation of motion is

$$m\ddot{x} + k_x x + k_i i_x = f \quad (4)$$

Therefore, the rotor system supported by the axial magnetic suspension bearing can be regarded as a single-degree-of-freedom system, if the two systems are equivalent, then there is.

$$c\dot{x} + kx = k_x x + k_i i_x \quad (5)$$

Therefore, the selection of control current should follow:

$$i_x = \frac{(k-k_x)x + c\dot{x}}{k_i} = Px + D\dot{x} \quad (6)$$

Above formula (6) shown that the control current i_x requires at least Two parts of proportional control and differential control.

Obviously, once the control parameters P and D are selected, then *k* and *c* in the equivalent typical single-degree-of-freedom system mechanics model can be determined.

III. RANDOM RESPONSE MODEL.

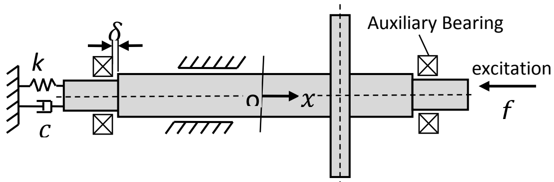

Figure 2. The single-degree-of-freedom mechanical system model

The magnetic bearings have been applied in many engineering fields. such as on road vehicles, on aircraft and on ships. In the applications environments there are vibrations, and these real vibrational environments are often random. These random excitations will have an unstable influence on the normal operation of magnetic bearing system. Gaussian excitation is a common steady-state distributed excitation, A Gaussian process is fully determined by knowledge of the mean value and of the standard deviation. Therefore, the response under Gaussian excitation is mainly studied below.

The response characteristics of the equivalent single-degree-of-freedom mechanical system model under random excitation disturbance conditions are studied below [3][4].

The equivalent single-degree-of-freedom mechanical system model (3) according to Fig. 2 can be written as:

$$\ddot{x} + 2\xi\omega_0\dot{x} + \omega_0^2 x = \ddot{a}(t) \quad (7)$$

Where $\xi = \frac{c}{2\sqrt{mk}}$ is the damping of the system, $\omega_0 = \sqrt{k/m}$ is the natural circle frequency, $\ddot{a}(t)$ is stationary random excitation

Let $\ddot{a}(t) = a(\omega)e^{i\omega t}$; $x(t) = X(\omega)e^{i\omega t}$

Then, the frequency response function(FRF) available in (7) is:

$$H(\omega) = \frac{1}{\omega_0^2 - \omega^2 + 2i\xi\omega_0\omega} \quad (8)$$

The square of the modulus of the FRF H(ω) is:

$$|H(\omega)|^2 = H(\omega)H^*(\omega) = \frac{1}{(\omega_0^2 - \omega^2)^2 + (2\xi\omega_0\omega)^2} \quad (9)$$

IV. .STATISTICAL SOLUTION OF RANDOM RESPONSES.

let $\ddot{a}(t)$ is a Gaussian process, its mean value is $E[\ddot{a}]$ and spectral density function is $S_a(\omega)$ are known, then the statistical solution for each response is

Mean value: $E[x] = H(\omega)E[\ddot{a}]$ (10)

Mean square error:

$$E[x^2] = \frac{1}{2\pi}\int_{-\infty}^{\infty}|H(\omega)|^2 \; S_a(\omega)d\omega \quad (11)$$

Variance: $\sigma^2 = E[x^2] - E^2[x]$ (12)

Since the excitation is a Gaussian process, the probability density function of the response is also subject to the Gaussian distribution:

$$P_x = \frac{1}{2\pi\sigma}\exp[-(x - E[x])^2/2\sigma^2] \quad (13)$$

Therefore, the probability that the random response is greater than the gap δ is

$$P(|x| > \delta) = 1 - \frac{1}{\sqrt{2\pi\sigma}}\int_{-\delta}^{\delta} exp[-(x - E[x])^2/2\sigma^2]dx \quad (13)$$

This formula can estimate the probability of collision between rotor and auxiliary bearing.

V. .EXAMPLE

This section gives an example to illustrate the application of the above theory. As shown in Fig.2, the system damping coefficient is $\xi = 0.027$, a natural frequency is $\omega_0 = 3500$, the clearance between the rotor and the auxiliary bearing is $\delta = 0.2\text{mm}$. the system is disturbed by horizontal acceleration excitation. Let this acceleration excitation is a stationary Gaussian excitation with a mean of zero and a unilateral power spectral density is $S_{\ddot{a}}(f) = 5\,g^2/H_z$,What is the probability that the system will not collide?

To solve the following,

the power spectral density is

$$S_0(\omega) = S_{\ddot{a}}(f)/4\pi = 0.3979\,g^2/H_z$$

Mean squared error is

$$E[x^2] = S_0\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{(\omega_0^2 - \omega^2)^2 + 4\xi^2\omega_0^2\omega^2}d\omega$$

$$= \frac{S_0}{4\xi\omega_0^3} = \frac{0.3979g^2}{4 * 0.027 * 3500^3} = 8.593 * 10^{-9}\text{m}$$

The standard deviation of the displacement is

$$\sigma = \sqrt{E[x^2]} = \sqrt{8.593 * 10^{-9}} = 9.3 * 10^{-5}\text{m}$$

Let:

$$z = \frac{\delta - 0}{\sigma} = \frac{0.0002 - 0}{0.000093} = 2.151$$

the probability that Probe{|x(t)| > 0.0002}

$$= P(|x| > 0.0002) = 1 - \frac{1}{\sqrt{2\pi}}\int_{-2.151}^{2.151} e^{-z^2/2}dz = 0.0315$$

In the example, the probability of collision between the rotor and the auxiliary bearing is not large, and it can be further based on 3σ or 6σ rules to determine the protection gap between the rotor and the auxiliary bearing, but to incorporate the design requirements of the levitation force to consider.

VI. .CONCLUSION

This paper analyzes the random response characteristics of the axial magnetic levitation system under random disturbance conditions and gives the control model of the basic magnetic levitation system. According to the equivalent principle, the corresponding one-degree-of-freedom mechanical system mechanical model is determined, the response characteristics under random acceleration excitation of model is analyzed. the probability formula is given for predicting the collision between the rotor and the auxiliary bearing. At the same time, a calculation example is given.

It can also be based on 3σ or 6σ rules combine the requirements of the levitation force to consider the design of the clearance between the rotor and the auxiliary bearing, the paper provide a new attempt for the design of the magnetic suspension bearing rotor system.

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