Research on Fault Tolerant Operation of Multi-Winding Control Loop Failure in Active Magnetic Bearing System

Zhao Wenjie [a], Jiang Kejian [a]

[a] College of Informatics and Electronics, Zhejiang Sci-Tech University, No. 2 street , Hangzhou, China, 1106502203@qq.com

*Abstract***— Active magnetic bearing system is a complex control system of electromechanical integration. Normal operation of active magnetic bearing system usually needs to drive all winding loops to work simultaneously. However, once one of the winding loops fails, the rotor suspension will run out of control and cause an accident. In order to improve the reliability of the whole operation of multi-winding loop, a method which implements the fault tolerant for the winding control loop is proposed. First, according to the distribution strategy of the same magnetic force produced by the minimum power consumption, the magnetic force of the fault winding will be redistributed to other normal windings to maintain the basic suspension state of the rotor, so as to win the time of the rotor speed reduction and eliminate or reduce the loss caused by uncontrolled suspension at high speed. All the failure modes that can achieve fault tolerant control are categorized. Each fault type only needs to solve one current distribution matrix. Then the fault tolerant control of different fault modes in a multi-winding loop can be achieved through coordinate transformation and winding remapping matrix. Finally, the experiments demonstrates the effectiveness of this fault tolerant method.**

1. INTRODUCTION

Active magnetic bearing (AMB) is a kind of mechatronic bearing which can stably suspend the rotor in a desired position by controlled magnetic force. It eliminates the limitation of traditional bearing and has such advantages as no friction, no lubrication and no heat. Nowadays, the application of magnetic bearings is used more and more widely. When the magnetic force of one or several magnetic poles fails, the rotor suspension will be out of control and cause an accident. Therefore, the fault tolerance of the magnetic bearing system has received many special attentions.

In the 1990s, American researchers Maslen and Meeker proposed controller based fault tolerant coil control [1]. The generalized bias current linearization theory can convert the magnetic force of any structure radial magnetic bearing into a linear function of the current. The theory has become the theoretical basis for the study of fault tolerant control problems. On the basis of Maslen's bias current linearization theory, Schroder used Lagrange method to solve the current distribution matrix[2], achieving a stable suspension of magnetic bearings under faulty conditions. Ming extended the bias current linearization theory originally used for radial magnetic bearings to magnetic bearings in composite structures [3]. Na studied eight-pole radial magnetic bearings with different polar distribution, finding that fault tolerant control can still be achieved when five winding control loops fail at the same time[4,5], but it turned out that the more the faulty windings, the more the magnetic bearing's stiffness and load capacity decreases. Won adopted a Lagrange multiplier method to calculate the current distribution matrix[6,7], achieving a fault tolerant control of magnetic bearings for a permanent magnet partial four-pole structure.

Domestic scholars also have some research on this. Wu Buzhou took eight-pole coupled radial magnetic bearings as the research object[8,9], calculating the current distribution matrix and simulating it by matlab. Han Fujun studied a permanent magnet partial magnetic bearing with homopolar structure[10], obtaining the current distribution matrix according to principle of force distribution. Cui Donghui, from Nanjing University of Aeronautics and Astronautics, simulating the radial magnetic bearing which owns six-pole strong coupling structure and proposing to classify the current distribution matrix based on coordinate transformation to reduce the number of current distribution matrix [11,12]. Geng Qingling studied the radial magnetic bearing which owns twelve-pole weak coupling structure[13], and proposed the corresponding current reconstruction method. Hu Chuntao used matlab to simulate the dynamic characteristics of rotor under different magnetic pole failures caused by actuator failure, and the results provided the basis for implementing compensation strategy to the magnetic bearing reliability[14]. X. Cheng proposed a fault tolerant control strategy for active radial magnetic bearings with tightly coupled redundant support structures, and analyzes the influence of partial flow coefficients on magnetic forces[15].

In order to improve the reliability of the whole operation of multi-winding of magnetic bearings, this paper takes the eight-pole radial magnetic bearing as the research object, and proposes the fault tolerance control algorithm of the magnetic bearing winding. The algorithm solves the current distribution matrix with minimal energy consumption, so that when one or more winding loops fail, the magnetic force of the fault winding can be distributed to other normal windings to maintain the basic suspension state of the rotor. The proposed fault tolerant algorithm provides the magnetic force required for the stable suspension of the magnetic bearing, and at the same time ensures that the energy consumed by the current in each winding loop is at a minimum. In addition, all failure modes that can implement fault tolerant control are categorized. The basic current distribution matrix corresponding to each basic fault type is obtained. The current distribution matrix of the same type of fault can be obtained by multiplying the basic current distribution matrix by the winding remapping matrix and the coordinate transformation matrix, thereby reducing the number of times of the current distribution matrix and simplifying the fault tolerant control process.

2. **CONTENT**

2.1 *General Expression of Magnetic Force*

For the stator structure of the eight-pole radial magnetic bearing, the windings of two adjacent stator poles are usually connected in series to form a C-shaped magnetic pole, as shown in Fig. 1 .

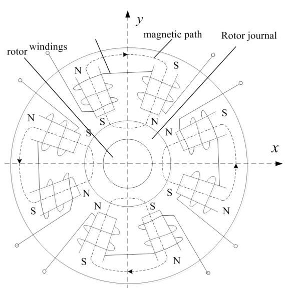


Fig.1 C-shaped stator structure of eight-pole radial magnetic bearing

According to the Maxwell's equation, the magnetic force produced by the winding control loop of the j-th (j=1,2,3,4,5,6,7,8) stator pole is obtained:

$$F_j = \frac{AN^2 \mu_0 I_j^2}{2X_j^2} \tag{1}$$

where, μ_0 is the permeability of the vacuum, A is the effective cross-sectional area of the magnetic circuit, N is the number of winding turns for each stator pole, I_j is the current of the j-th winding circuit, the column vector of each winding loop current is

$$I = [I_1 \ \ I_2 \ \ I_3 \ \ I_4 \ \ I_5 \ \ I_6 \ \ I_7 \ \ I_8]^T$$

X_j is the length of the air gap between the j-th stator and the rotor, the diagonal matrix of each air gap length is

$$X = \begin{bmatrix} X_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & X_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & X_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & X_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & X_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & X_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & X_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & X_8 \end{bmatrix}$$

Generally, the windings on the two stator poles of each C-type pole are connected in series, which is $I_1 = I_2$, $I_3 = I_4$, $I_5 = I_6$, $I_7 = I_8$, forming a series of four windings in series, thereby generating four magnetic forces in the direction of the axis of the rotor.

However, according to current literatures[4], to achieve fault tolerant operation of multi-winding loops of magnetic bearings, each C-type magnetic pole must be split into two independently magnetic poles, forming a total of eight independent winding control loops, as shown in Fig. 2.

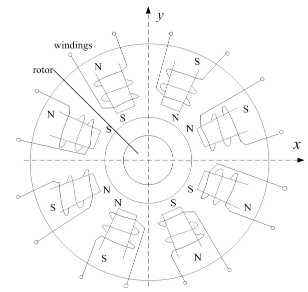


Fig.2 Independent stator structure of eight-pole radial magnetic bearing

Each winding control loop of the eight-pole radial magnetic bearing generates magnetic force in different directions, so the magnetic resultant force in the x-axis and y-axis directions are:

$$\begin{cases} F_x = \dfrac{AN^2 \mu_0}{2} I^T \left(X^{-1}\right)^T D_x X^{-1} I \\ F_y = \dfrac{AN^2 \mu_0}{2} I^T \left(X^{-1}\right)^T D_y X^{-1} I \end{cases} \tag{2}$$

$$\begin{cases} D_x = diag(\cos\theta_1 \ \ \cos\theta_2 \ \ \cos\theta_3 \ \ \cos\theta_4 \ \ \cos\theta_5 \ \ \cos\theta_6 \ \ \cos\theta_7 \ \ \cos\theta_8) \\ D_y = diag(\sin\theta_1 \ \ \sin\theta_2 \ \ \sin\theta_3 \ \ \sin\theta_4 \ \ \sin\theta_5 \ \ \sin\theta_6 \ \ \sin\theta_7 \ \ \sin\theta_8) \end{cases}$$

where θ_j is the angle between the magnetic force generated by the j-th winding loop and the x-axis. When the magnetic bearing is fixed, D_x and D_y are constant matrices.

2.2 *Reconstruction of Current Distribution Matrix*

Normal operation of magnetic bearing system usually needs to drive all winding loops to work simultaneously. Once one of the winding loops fails, the rotor suspension will run out of control and cause an accident. In this study, a method which implements the fault tolerant for the winding control loop is proposed. By the reconstruction of current distribution, the magnetic force of the fault magnetic pole will be redistributed to other normal magnetic poles.

The magnetic bearing controller generates a magnetic force drive signal according to the current rotor levitation position, and includes ternary information such as a bias current, an x-axis direction magnetic force drive and y-axis direction magnetic force drive. The current distribution matrix converts this ternary information into actual drive current for the eight magnetic poles of a magnetic bearing:

$$I = WI_c \tag{3}$$

where $I_c = [i_b \ \ i_x \ \ i_y]^T$ is a current control vector representing the controller output signal, i_b is the bias current and i_x, i_y are the control currents in the x-axis and y-axis directions, respectively. W is the current distribution matrix. From the above equation, the current distribution matrix is a matrix of eight rows and three columns. Substituting Eq.(3) into Eq.(2) leads to

$$\begin{cases} F_x = \dfrac{AN^2\mu_0}{2} I_c^{\ T} W^T \left(X^{-1}\right)^T D_x X^{-1} W I_c \\ F_y = \dfrac{AN^2\mu_0}{2} I_c^{\ T} W^T \left(X^{-1}\right)^T D_y X^{-1} W I_c \end{cases} \tag{4}$$

When the winding control loop is faulty, the magnetic resultant force of the rotor in the *x*-axis and *y*-axis directions after reconstruction by the current distribution matrix is as shown above. The magnetic resultant force is a function of the air gap length *X* and the control current vector I_c. Given a current, the magnetic force at a certain air gap length can be determined. But in turn, due to the quadratic relationship, the magnetic force required for a given magnetic bearing cann't determine the unique set of control currents.

According to the generalized bias current linearization method [1], if the magnetic resultant force of the rotor in the *x*-axis and *y*-axis directions can be converted into a linear function of the control current in the *x*-axis and *y*-axis directions, as shown in the following formula:

$$\begin{cases} F_x = c_0 \cdot i_x \\ F_y = c_0 \cdot i_y \end{cases} \tag{5}$$

where c_0 is constant. At this time, the magnetic resultant force received by the rotor in the x-axis and y-axis directions is only related to the control current in this direction. As long as the magnetic force required for the stable suspension of the magnetic bearing is determined, the current in each winding circuit can be determined.

In order to convert the magnetic resultant force in the *x*-axis and *y*-axis directions into a linear function of the control current, the current distribution matrix must satisfy the following conditions:

$$\begin{cases} W^T \left(X^{-1}\right)^T D_x X^{-1} W = M_x \\ W^T \left(X^{-1}\right)^T D_y X^{-1} W = M_y \\ M_x = \begin{bmatrix} 0 & 0.5 & 0 \\ 0.5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ M_y = \begin{bmatrix} 0 & 0 & 0.5 \\ 0 & 0 & 0 \\ 0.5 & 0 & 0 \end{bmatrix} \end{cases} \tag{6}$$

Substituting Eq (6) into Eq (4), Eq (4) can be written as follows:

$$\begin{cases} F_x = \dfrac{AN^2\mu_0}{2} \begin{bmatrix} i_b & i_x & i_y \end{bmatrix} \cdot \begin{bmatrix} 0 & 0.5 & 0 \\ 0.5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} i_b \\ i_x \\ i_y \end{bmatrix} \\ F_y = \dfrac{AN^2\mu_0}{2} \begin{bmatrix} i_b & i_x & i_y \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0.5 \\ 0 & 0 & 0 \\ 0.5 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} i_b \\ i_x \\ i_y \end{bmatrix} \end{cases} \tag{7}$$

Simplified Eq.(7) to get the linear relationship between the magnetic resultant force and the control current in the *x*-axis and *y*-axis directions.

$$\begin{cases} F_x = c_0 \cdot i_x \\ F_y = c_0 \cdot i_y \end{cases} \qquad c_0 = \frac{AN^2\mu_0}{2} i_b \tag{8}$$

where i_b is the bias current and remains unchanged. when the magnetic bearing is fixed, c_0 is constant.

From the above analysis, when the current distribution matrix satisfies Eq.(6), the magnetic resultant force is converted into a linear function of the control current in the *x*-axis and *y*-axis directions. At this time, as long as the magnetic force required for the suspension is determined, the current control vector representing the controller output signal is

$$I_c = \begin{bmatrix} i_b \\ \dfrac{F_x}{c_0} \\ \dfrac{F_y}{c_0} \end{bmatrix} \tag{9}$$

From equation (3), the current vector in each winding loop can be determined as

$$I = W I_c = W \cdot \begin{bmatrix} i_b \\ \dfrac{F_x}{c_0} \\ \dfrac{F_y}{c_0} \end{bmatrix} \tag{10}$$

2.3 *Solution of Current Distribution Matrix with Minimum Power Consumption*

It is proved that the current distribution matrix satisfying formula (6) exists, and it can only be found by numerical calculation. To provide the magnetic force required for the suspension of an magnetic bearing, the current of each winding loop is not unique and there are numerous combinations.

In order to calculate the optimal current distribution matrix, this paper proposes to solve the current distribution matrix with minimal energy consumption. This method ensures the magnetic force required for the stable suspension of the magnetic bearing, and at the same time, the energy consumed by the current in each winding loop is the least. Therefore, the sum of the squares of the currents in each winding circuit is used as the objective function of the fault tolerant optimal control of the winding loop, which can be described as:

$$J(W) = I^T I \tag{11}$$

where, *J(W)* is the instantaneous value of the objective function. $I=[I_1\ I_2\ I_3\ I_4\ I_5\ I_6\ I_7\ I_8]^T$ is the column vector of the current in each winding loop after the current distribution matrix：

$$I = [I_1\ I_2\ I_3, I_4\ I_5\ I_6\ I_7\ I_8]^T = W \cdot I_c \tag{12}$$

where $$I_c = \begin{bmatrix} i_b \\ i_x \\ i_y \end{bmatrix}$$

In this case, the above equation is substituted into Eq. (11) to obtain the objective function as

$$J(W) = I^T I = I_c{}^T W^T W I_c \tag{13}$$

In order to achieve the magnetic force required to provide stable suspension of the magnetic bearing with minimal energy consumption, the core issue is to solve the current distribution matrix to satisfy the

$$\min J(W) \tag{14}$$

From the above analysis, we can see that the current distribution matrix satisfies Eq.(6) is a necessary condition for the linearization of magnetic force and control current, then the constraint condition for solving the minimum value of the target function is

$$\begin{cases} W^T \left(X^{-1}\right)^T D_x X^{-1} W - M_x = 0 \\ W^T \left(X^{-1}\right)^T D_y X^{-1} W - M_y = 0 \end{cases} \tag{15}$$

The current distribution matrix is a matrix of eight rows and three columns, where W_b, W_x, W_y are column vectors.

$$\begin{cases} W = \left[W_b\ W_x\ W_y\right] \\ W_b = \left[W_{b1}\ W_{b2}\ W_{b3}\ W_{b4}\ W_{b5}\ W_{b6}\ W_{b7}\ W_{b8}\right]^T \\ W_x = \left[W_{x1}\ W_{x2}\ W_{x3}\ W_{x4}\ W_{x5}\ W_{x6}\ W_{x7}\ W_{x8}\right]^T \\ W_y = \left[W_{y1}\ W_{y2}\ W_{y3}\ W_{y4}\ W_{y5}\ W_{y6}\ W_{y7}\ W_{y8}\right]^T \end{cases} \tag{16}$$

The current distribution matrix must satisfy equation (15) to achieve linear control, substituting Eq. (16) into Eq. (15). Since M_x and M_y are symmetric matrices, the constraint condition for solving the minimum value of the objective function are transformed into 12 different scalar equation equations:

$$\begin{cases} h_1(W) = W_b^T \left(X^{-1}\right)^T D_x X^{-1} W_b = 0 \\ h_2(W) = W_b^T \left(X^{-1}\right)^T D_x X^{-1} W_x - 0.5 = 0 \\ h_3(W) = W_b^T \left(X^{-1}\right)^T D_x X^{-1} W_y = 0 \\ h_4(W) = W_x^T \left(X^{-1}\right)^T D_x X^{-1} W_x = 0 \\ h_5(W) = W_x^T \left(X^{-1}\right)^T D_x X^{-1} W_y = 0 \\ h_6(W) = W_y^T \left(X^{-1}\right)^T D_x X^{-1} W_y = 0 \\ h_7(W) = W_b^T \left(X^{-1}\right)^T D_y X^{-1} W_b = 0 \\ h_8(W) = W_b^T \left(X^{-1}\right)^T D_y X^{-1} W_x = 0 \\ h_9(W) = W_b^T \left(X^{-1}\right)^T D_y X^{-1} W_y - 0.5 = 0 \\ h_{10}(W) = W_x^T \left(X^{-1}\right)^T D_y X^{-1} W_x = 0 \\ h_{11}(W) = W_x^T \left(X^{-1}\right)^T D_y X^{-1} W_y = 0 \\ h_{12}(W) = W_y^T \left(X^{-1}\right)^T D_y X^{-1} W_y = 0 \end{cases} \tag{17}$$

Therefore, the optimal control of winding fault tolerance based on current distribution matrix is transformed into the minimum problem of objective function under 12 constraints.

In numerical analysis, Lagrange multiplier method is usually used to solve the numerical optimal solution. The Lagrangian multiplier method is a method for finding the extremum of a multivariate function whose variables are constrained by one or more conditions. Through the introduction of Lagrange multipliers, an optimization problem with n variables and k constraints is transformed into an extremum problem for a system of n + k variables, whose variables are not subject to any constraints. Then partial differentiation is performed on n + k variables to find the optimal solution that can satisfy the constraint conditions.

According to Lagrange multiplier method, define the Lagrange function as

$$L(W) = I^T I + \sum_{j=1}^{12} \lambda_j h_j(W) \tag{18}$$

where λ_j is the Lagrange coefficient. According to the Lagrange multiplier method, solving the optimal solution of the objective function under the constraint condition, we need to find the partial derivative of the variables W and λ_j for $L(W)$. First, partial derivative of $L(W)$ for W is：

$$\begin{cases} f_i = \dfrac{\partial L(W)}{\partial W_{bi}} = 0,\ i = 1,2,3....8 \\ f_{8+i} = \dfrac{\partial L(W)}{\partial W_{xi}} = 0,\ i = 1,2,3....8 \\ f_{16+i} = \dfrac{\partial L(W)}{\partial W_{yi}} = 0,\ i = 1,2,3....8 \end{cases} \tag{19}$$

Then, partial derivative of $L(W)$ for λ_j (j=1,2,3...12) is：

$$f_{j+24} = \frac{\partial L(W)}{\partial \lambda_j} = h_j(W) = 0, j = 1,2,3....12 \tag{20}$$

From Eqs. (19) and (20), we get the nonlinear algebraic equations:

$$F(W,\lambda) = \begin{bmatrix} f_1(W,\lambda) \\ f_2(W,\lambda) \\ \vdots \\ f_{36}(W,\lambda) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{21}$$

From the above analysis, we can see that the Lagrange multiplier method transforms the optimal control problem of the current distribution matrix into the minimum extremum of the nonlinear equations. The solution of the nonlinear algebraic equations is the current distribution matrix under the minimum power consumption.

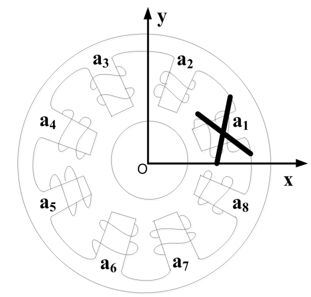
2.3 *Fault Tolerant Control Based on the Same Fault Type*

The general fault tolerance is that each fault needs a corresponding current distribution matrix. However, there are many kinds of fault combinations in the winding control loop. Since the stator of the magnetic bearing is a ring, all the failure forms can be classified by coordinate rotation and symmetry[7].

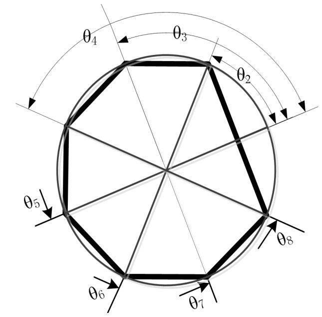
In order to simplify the fault tolerant control algorithm and improve the fault tolerant efficiency, this paper proposes to classify the same faulty electrode distribution obtained by rotation as the same fault type. Each fault type only needs to solve one current distribution matrix. The current distribution matrix of the same type of fault can be obtained by multiplying the basic current distribution matrix by the winding remapping matrix and the coordinate transformation matrix to form a unified fault tolerance algorithm for current distribution matrix reconstruction.

When there is a fault in one loop control loop, there are eight types of faults in the magnetic bearing of the independent winding structure. Through the rotation, these eight kinds of faults have exactly the same fault electrode distribution and belong to the same fault type. Only one current distribution matrix needs to be solved for the same fault type.

If the first winding control loop fails, as shown in Figure 3(a). If it is used as a basic fault type, the basic fault mode diagram is drawn as shown in Figure 3(b).



3(a) The fault model diagram



3(b) The basic failure mode diagram

Fig. 3 The fault model diagram and the basic failure mode diagram of the first winding loop fault

The current distribution matrix W_1 corresponding to the basic failure mode diagram is obtained, which distributes the control current vector I_c to the eight windings control loop. the

$$I = [I_1\ I_2\ I_3, I_4\ I_5\ I_6\ I_7\ I_8]^T = W_1 I_c = W_1 \begin{bmatrix} i_b \\ i_x \\ i_y \end{bmatrix} \tag{22}$$

where, I is the column vector of each winding control loop current. When the j-th winding loop fails, based on the general fault tolerant control algorithm of current distribution matrix reconfiguration, this situation is regarded as a different fault mode, and the corresponding current distribution matrix needs to be found again.

However, the angle θ_j of the reverse rotation of the basic failure mode diagram must coincide with the failure mode diagram of the j-th winding loop, so the j-th winding loop failure and the basic failure mode are the same type of failure, and the current distribution matrix can be calculated from the basic current distribution matrix. Corresponding Eq.(22), there is

$$I'_j = [I'_1, I'_2, I'_3, I'_4, I'_5, I'_6, I'_7, I'_8]^T = W_1 \begin{bmatrix} i_b \\ i'_x \\ i'_y \end{bmatrix} \tag{23}$$

where, i_x' and i_y' are control currents when the j-th winding loop fails. According to the coordinate transformation formula, there are

$$\begin{bmatrix} i_b \\ i_x' \\ i_y' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} i_b \\ i_x \\ i_y \end{bmatrix} \quad (24)$$

where
$$G_j = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \quad (25)$$

where G_j is the coordinate transformation matrix for the fault of the j-th winding circuit. According to the correspondence between the windings, the winding remapping matrix at this time is:

$$T_j = \left(E_{87}E_{76}E_{65}E_{54}E_{43}E_{32}E_{21}\right)^{j-1} T_1 \quad (26)$$

$$T_1 = \begin{bmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{bmatrix}$$

where, T_1 is the unit diagonal matrix of eight rows and eight columns, and the left multiply E_{ij} represents the exchange position of the i-th row and the j-th row of the unit matrix.

Since the fault of the j-th winding control loop and the fault of the first winding loop belong to the same fault mode, according to the fault tolerance control principle diagram of the same fault mode, the current distribution matrix after the j-th winding control loop fault is obtained :

$$W_j = T_j W_1 G_j \quad (27)$$

After fault tolerance control, the column vector of each winding current after the j-th winding control loop fault is:

$$I = W_j \cdot I_c \quad (28)$$

Substituting Eqs. (25), (26), and (27) into Eq. (28) yields:

$$I = T_j W_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_j & \sin\theta_j \\ 0 & -\sin\theta_j & \cos\theta_j \end{bmatrix} \cdot \begin{bmatrix} i_b \\ i_x \\ i_y \end{bmatrix} \quad (29)$$

From the above analysis, any one winding control circuit failure is the same type of fault for the entire system.Taking the fault mode diagram of the first winding loop fault as the basic failure mode diagram and calculating the current distribution matrix. According to the fault tolerant control method based on coordinate transformation and remapping of the windings, the current distribution scheme for any one winding loop failure can be found. The principle block diagram is shown in Fig. 4.

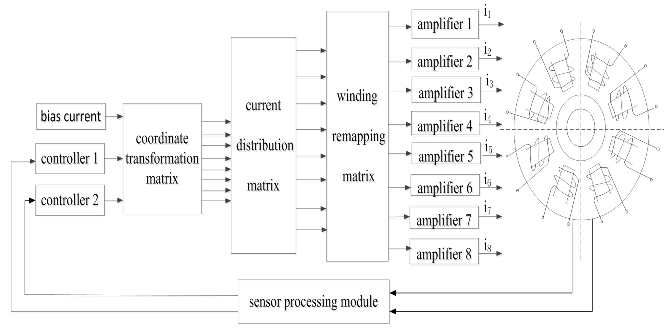


Figure 4. Functional block diagram of fault tolerant control of winding loop based on coordinate transformation and winding remapping matrix

Similarly, fault tolerant control methods for faults in multiple winding control loops are similar. The fault tolerance control criterion is: first, the basic current distribution matrix corresponding to the basic fault mode is solved. Secondly, after the basic failure mode map rotates a certain angle, it overlaps with the different failure combination pattern diagrams of the same failure type, and finds the corresponding coordinate transformation matrix and the winding remapping matrix. Finally, the current distribution matrix under different fault modes is obtained, and different fault tolerance control under the same fault type is finally achieved.

The fault tolerant control algorithm can reduce the number of calculations of the current distribution matrix and simplify the fault tolerant control process. In addition, the algorithm can maintain the basic levitation state of the rotor when one or more winding loops are faulty, provide time for the safe speed reduction of the rotor, eliminate or reduce the loss caused by uncontrolled suspension at high speed and improve the safety and reliability of the magnetic bearing system.

3. ANALYSIS OF EXAMPLES

3.1 *Numerical Analysis*

For a radial magnetic bearing with eight-pole independent winding structure, the current in each winding loop is I_1~I_8, and the sum of squared current is

$$\sum_{i=1}^{8} I_i^2 = I_1^2 + I_2^2 + I_3^2 + I_4^2 + I_5^2 + I_6^2 + I_7^2 + I_8^2 \quad (30)$$

The radial magnetic bearing of the C-type stator structure adopts differential control and usually connects the windings of two adjacent stator poles in series, that is

$$I_1 = I_2, I_3 = I_4, I_5 = I_6, I_7 = I_8$$

The sum of squared current is

$$\sum_{i=1}^{8} I_i^2 = 2I_1^2 + 2I_3^2 + 2I_5^2 + 2I_7^2 \quad (31)$$

When the rotor is in the equilibrium position in the x-axis direction and only the bias current provides the magnetic force, set the bias current to i_b=0.75A. The power amplifier used in this experiment is a unidirectional current type. The output current value of the rotor in the y-axis direction is 0-10A. When using differential control, set the current in the y-axis direction to 2.5A, 5A, 7.5A, and 10A respectively. In order to provide the same magnetic force as differential control, the current distribution matrix is solved by using the minimum power consumption, and the corresponding current distribution matrix of the eight-pole independent winding structure is obtained as：

$$W_1=\begin{bmatrix} 1.8855 & 1.4395 & 0.3292\\ -3.1200 & 0.0255 & -0.6400\\ 2.0029 & -0.1075 & 0.8030\\ -2.4350 & 1.0895 & -0.2568\\ 2.2233 & -0.6969 & -0.4463\\ 1.3612 & -0.5932 & -0.1758\\ -3.6750 & -0.3734 & 1.0557\\ 0.7026 & 0.3957 & 0.1152 \end{bmatrix} \quad W_2=\begin{bmatrix} 2.0525 & 1.5860 & 0.3715\\ -1.8983 & -0.0564 & -1.0015\\ 1.5957 & -0.3384 & 0.9866\\ -1.8746 & 1.1434 & -0.4569\\ 1.9038 & 0.9999 & -0.7690\\ 1.0800 & -0.9358 & -0.1438\\ -2.5624 & -0.6086 & 1.4056\\ 0.3942 & 0.2413 & 0.0567 \end{bmatrix}$$

$$W_3=\begin{bmatrix} 3.5806 & 0.9882 & -0.0127\\ -1.9712 & 0.7965 & -1.1756\\ -2.7910 & 0.3761 & -1.8638\\ -0.9456 & 0.1381 & 0.0468\\ 3.8269 & -0.9152 & -0.2729\\ -0.9377 & 0.1066 & 0.8213\\ -2.9667 & -0.5277 & 0.9329\\ -1.5050 & 0.0153 & -1.1093 \end{bmatrix} \quad W_4=\begin{bmatrix} 2.3443 & 1.5862 & -0.3177\\ -2.9382 & 0.6354 & -1.5598\\ -2.4082 & 0.7219 & -0.9941\\ -1.0214 & 0.5411 & 0.2978\\ 1.9211 & -1.3753 & 0.2995\\ 2.8851 & -0.2250 & -1.1695\\ -1.9454 & -0.5479 & 0.7628\\ -0.8963 & -0.3282 & 0.0556 \end{bmatrix}$$

From Eqs. (30) and (31), the energy consumption of the differential control and the energy consumption of the independent winding structure controlled by the current distribution matrix are shown in Table 1.

TABLE 1. ENERGY CONSUMPTION COMPARISON RESULTS

Control method / Y-axis current	Differential control	Independent control through current distribution matrix
2.5A	14.75	10.36
5A	52.25	44.89
7.5A	114.75	80.15
10A	202.25	152.86

Based on the data in Table 1, draw Fig.5. It can be clearly seen from Fig.5 that within the range of the y-axis output current, a current distribution matrix can always be found to provide the same magnetic force as the differential control.

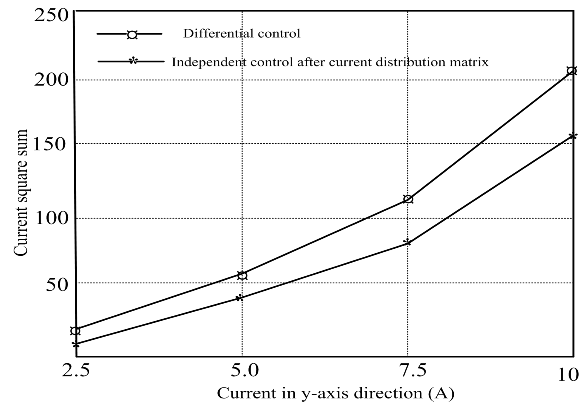


Figure 5. Comparison of energy consumption

As can be seen from Fig. 5, when the same magnetic force is provided, compared with the energy consumed by the differential control, the current distribution matrix obtained under the objective function of minimum power consumption enables the energy consumption of the magnetic bearing of the independent winding structure to be lower.

3.2 *Fault Tolerant Instance Analysis*

When one or several winding control loops of the magnetic bearing fail, the stable control of the rotor cannot be achieved by using the differential control. At this time, the current distribution matrix corresponding to each fault type can implement fault tolerant control to ensure that the rotor is suspended and the system works normally.

According to the method of minimum power consumption, the current distribution matrix when the second winding loop fails is W_2.

$$W_2=\begin{bmatrix} 0.2945 & -0.4880 & 0.0278\\ 0.0000 & 0.0000 & 0.0000\\ 1.6920 & 0.5620 & 1.1420\\ -0.8390 & 0.7620 & 0.8810\\ 0.1430 & -0.3020 & 0.7160\\ 1.9475 & -0.2787 & -0.3537\\ -1.0235 & 0.2415 & 1.3282\\ 1.4313 & 0.6566 & 0.9221 \end{bmatrix} \tag{32}$$

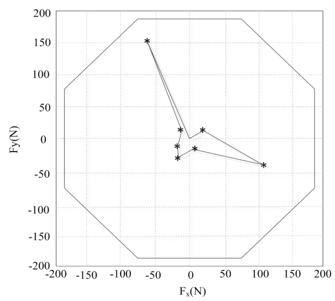
The W_{25} matrix for the 2-5th poles failed case is shown in Eq.(33).

$$W_{25}=\begin{bmatrix} 0.5017 & -0.3077 & 0.1035\\ 0.0000 & 0.0000 & 0.0000\\ 1.2887 & 0.2035 & 1.0961\\ -1.0863 & 0.1856 & 1.0852\\ 0.0000 & 0.0000 & 0.0000\\ 1.3175 & -0.7622 & -1.0553\\ -1.2598 & -0.0783 & 1.0193\\ 1.7539 & 1.3214 & -0.1128 \end{bmatrix} \tag{33}$$

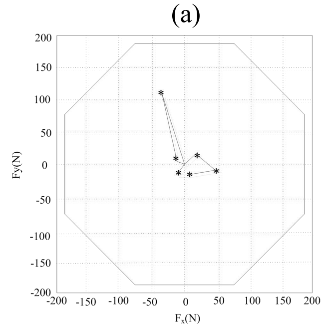
The W_{257} matrix for the 2-5-7th poles failed case calculated by the method of minimum power consumption is shown in Eq.(34).

$$W_{257} = \begin{bmatrix} -1.5017 & -0.1783 & -0.6299 \\ 0.0000 & 0.0000 & 0.0000 \\ 1.6900 & 0.4094 & 0.8391 \\ -1.0663 & 0.0376 & 1.3691 \\ 0.0000 & 0.0000 & 0.0000 \\ 1.8897 & -0.0822 & -1.0263 \\ 0.0000 & 0.0000 & 0.0000 \\ 2.971 & 1.1429 & -0.3010 \end{bmatrix} \quad (34)$$

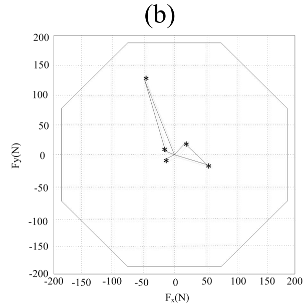
After the current distribution matrix reconstruction, the magnetic force distributions for the three different fault types are as shown in the fig. 6.



(a)



(b)



(c)

Figure 6. Force distribution of fault tolerant magnetic bearing for the 2-th winding loop failed(a), the 2-5th winding loops failed(b), the 2-5-7th winding loops failed(c)

From Fig.6(a), it can be seen that when the second winding circuit fails, the magnetic force in the positive direction of the *y*-axis is mainly provided by the first and third winding control loops, which need to overcome the gravity and magnetic force in the negative direction of the *y*-axis, to achieve the stable suspension of the rotor. When the 2nd and 5-th winding circuits fail, the magnetic distribution is shown in Figure 6(b). The magnetic force in the negative direction of the *y*-axis is provided by the 6-7-8 winding loops, and the magnetic force provided by the 3rd winding loop is slightly decreased compared to the Fig. 6(a).

When the 2-5-7th winding loops fail at the same time, the magnetic distribution is shown in Figure 6(c). The magnetic force in the negative direction of the *y*-axis is mainly provided by the 8-th winding loops, and the magnetic force generated by the 3rd winding loop is slightly higher than that in graph 6(b).

As can be seen from the figure, the three different winding failure modes are subjected to fault tolerant control, the resultant electromagnetic forces in the *x*-axis and *y*-axis directions are the same, that is, all provide the magnetic force required for the stable suspension of magnetic bearings. Therefore, when one or several winding circuits fail, the rotor can still be stably suspended.

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