Design Method of Filter Cross Feedback Controller for Nutation Mode Suppression of AMBs-Rotors

Maolin Sun [a], Shiqiang Zheng, Jinlei Li, Yifan Liu

[a] Beihang University, Xueyuan Road 37, 100191 Beijing, China, sun_m_l@163.com

***Abstract*—This paper explores a design method of the filter cross-feedback for nutation mode suppression of AMBs-rotor systems. The derivation of a multiple-input-multiple-output model (MIMO model) about AMBs-rotor was presented. Then the nutation stability of the system is analyzed by the complex coefficient method. Because the filter cross- feedback is an additive compensation, we can get the desired compensation by analyzing the Nyquist diagram of the system. At last, the filter cross-feedback controller is designed according to the compensation value. Simulation and experimental results demonstrate that the filter cross-feedback designed by the proposed method works well for restoring the stability of the system.**

I. INTRODUCTION

Compared with the conventional mechanical bearings, special advantages of AMBs are high power density, operation with no mechanical wear, less maintenance and longer lifetime[1]- [3].

In order to make better use of these advantages, the AMBs-rotor often has a high operation speed. Due to the coil inductance of magnetic bearings, the power amplifier in the AMBs-control system can be approximated to a low pass filter, which will introduce a phase lag and degrade the stability of the nutation mode. Especially as the operational speed of rotor increases, the lag becomes more serious, which can easily lead to the instability of the high-speed rotor system and possibly induce destructive crashes. Therefore, an effective suppression of the nutation mode is the precondition for the stable operation of the AMBs system.

Many researchers have investigated the AMBs-rotor control issues introduced by nutation mode and proposes some solution methods such as multivariable H∞ control [4] and model-based decoupling control [5], etc. Akio Sanbayashi designed a gain scheduled (GS) control for an AMBs-rotor system. The rotational speed is treated as a time-varying parameter and Linear fractional transformation (LFT) is applied to design the GS controller [6]. The mixed μ synthesis technique is applied in [7] with respect to a particular test rig exposed to severe gyroscopic effects.

The aforementioned methods use modern control theory to cope with the gyroscopic coupling issues. However, the filter cross-feedback control is also an effective method to suppress gyroscopic effects [8]. It is widely used in engineering applications due to its simple structure. But, there is no simple and explicit method for designing the parameters of filter cross feedback. Generally the parameters of filter cross-feedback are designed from the simulation of root locus, which makes the filter cross-feedback control complex to be used in practice [9].

In this paper, an AMBs-rotor system is modelled using the complex coefficient method. Then, the parameters of filter cross-feedback are designed exactly from double-frequency Bode plot method, which ensure the stability of nutation mode. Finally, the experimental results on a MSCMG prototype validate the effectiveness of the proposed method.

II. MODELING

In this paper, the simulation and experimental investigations are based on a magnetically suspended control moment gyros (MSCMG). The whirl modes caused by the gyroscopic effect are only related to the tilting motion. Therefore, this paper mainly focuses on the tilting dynamics of the magnetically suspended rotor. According to Newton's law, the dynamical model of the AMB-rotor system can be presented as

$$\begin{cases} J_x \ddot{\alpha} + H \dot{\beta} = -F_{ay} a - F_{by} b \\ J_y \ddot{\beta} - H \dot{\alpha} = F_{ax} a + F_{bx} b \end{cases} \tag{1}$$

The whirl modes caused by the gyroscopic effect are only related to the tilting motion. Therefore, this paper mainly focuses on the tilting dynamics of the magnetically suspended rotor. According to the real rotor system, the tilting motion can be modeled as:

$$\begin{cases} J_x \ddot{\alpha} + H \dot{\beta} = -(-k_i k_s g_w g_c x_{say} + k_h x_{may}) a \\ \qquad\qquad - (-k_i k_s g_w g_c x_{sby} + k_h x_{mby}) b \\ J_y \ddot{\beta} - H \dot{\alpha} = (-k_i k_s g_w g_{ca} x_{sax} + k_h x_{max}) a \\ \qquad\qquad + (-k_i k_s g_w g_c x_{sbx} + k_h x_{mbx}) b \end{cases} \tag{2}$$

where k_i and k_h are the current stiffness and the position stiffness of magnetic bearing; g_w and g_c are the transform operators from input to output signal of the power amplifier and PID controllers respectively. That is

$L[g_w g_c x] = g_w(s)g_c(s)x(s)$. The transfer functions of the PID controller and the power amplifier are expressed by

$$g_{ca}(s) = K_p + \frac{T_i}{s} + \frac{T_d s}{T_f S + 1} \tag{3}$$

and

$$g_w(s) = \frac{b_w}{s + a_w} \tag{4}$$

where the parameters in (3) and (4) have been defined in Table II.

According to the geometric structure of the rotor, tilting angles about x-axis and y-axis are expressed by

$$\begin{cases} \alpha = \dfrac{-(x_{say} - x_{sby})}{2l_s} = \dfrac{-(x_{may} - x_{mby})}{2l_m} \\ \beta = \dfrac{x_{sax} - x_{sbx}}{2l_s} = \dfrac{x_{max} - x_{mbx}}{2l_m} \end{cases} \tag{5}$$

where l_s and l_m are the distance from O to sensor center and distance from O to bearing center. O is the geometric center of the rotor.

Substituting (5) into (2) yields

$$\begin{cases} J_x \ddot{\alpha} + H \dot{\beta} - 2l_m k_h \alpha = -l_s k_s k_i g_w g_c \alpha \\ J_y \ddot{\beta} - H \dot{\alpha} - 2l_m k_h \beta = -l_s k_s k_i g_w g_c \beta \end{cases} \tag{6}$$

The phase of α leads that of β by 90°. Accordingly, we can define a complex coefficient $\varphi=\alpha+j\beta$, where j is the imaginary unit and $j^2 = -1$. Then, using the complex coefficient method, the tilting equations in (1) can be converted a complex variable equation:

$$J_r \dot{\varphi} - jH \dot{\varphi} - 2l_m k_h \varphi = -l_s k_i k_s g_w g_c \varphi \tag{7}$$

Applying the Laplace transform to the differential equation (8) yields

$$(J_r s^2 - jHs - l_m k_h)\varphi(s) = -l_s k_i k_s g_w(s) g_c(s) \varphi(s) \tag{9}$$

Then, the equation described in (15) can be equivalent to a SISO feedback control system, as shown in Fig. 3. Its corresponding transfer function of the controlled plant and control scheme can be presented, respectively, as

$$g_{oe}(s) = \frac{1}{J_r s^2 - jHs - l_m k_h} \tag{10}$$

$$g_{ce}(s) = l_s k_i k_s g_w(s) g_c(s) \tag{11}$$

The open-loop transfer function of the SISO feedback control system is presented as

$$g_{OL}(s) = g_{oe}(s) g_{ce}(s) \tag{12}$$

That is, after the variable reconstruction, the original MIMO cross-coupled system with real coefficients is converted into a SISO control system with complex coefficients. The transformation system owes the advantages of simplicity and convenience of the single-valuable frequency-domain method.

III. COMPENSATION FOR NUTATION MODE BASED ON CROSS FEEDBACK

Because the imaginary unit j is in (16), the Bode plots of $g_{OL}(j\omega)$ are not symmetric for ω > 0 and ω< 0. The Bode plots of $g_{OL}(-j\omega)$ and $g_{OL}(+j\omega)$ for $\omega > 0$ are drawn as shown in Fig. 1 to analyze the stability of the overall system. Where $F_r = 250Hz$ and the system parameters have been defined in Table I.

Table I. PHYSICAL PARAMETERS OF THE AMB-ROTOR SYSTEM

Symbol	Value	Symbol	Value
m	16.7kg	l	0.111 m
J_r	0.08285 kgm^2	k_i	600 N/A
J_z	0.1302 kgm^2	k_h	2400000 N/m
l_m	0.0725 m		

Table II. CONTROL PARAMETERS OF THE AMB-ROTOR SYSTEM

Symbol	Value	Symbol	Value
Kp	4.5	k_s	13650000
Ti	0.000575	aw	585.543
Td	0.0051	bw	0.325
Tf	0.0002785		

The vertical dashed line in Fig. 1 represents a 180° Down-jump of phase frequency characteristic from the starting turning point to the termination turning point. The upward arrow indicates a half positive crossing, and the downward arrow indicates a half negative crossing [10]. the total number of crossing of double phase-frequency characteristic: $N = -1$.

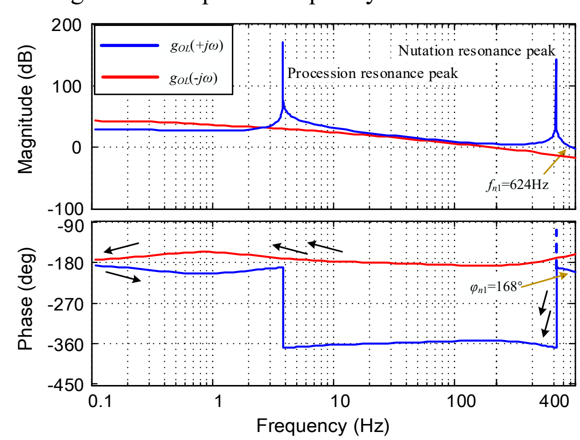

Figure 1. Block diagram of equivalent SISO control system.

The number of poles which have positive real part for $g_{OL}(s)$ is defined as Q. According to (12), we can get Q=0 . The number of poles which have positive real part for the close-loop SISO system is defined as Z. Then we know $Z = Q - N = 1 \neq 0$. So the system is unstable. The further analysis shows that this instability is caused by nutation mode. Fig. 1. shows the gain of $g_{OL}(+j\omega)$ at f_{n1} is 0 dB and the corresponding phase φ_{n1} is smaller than 180°, which makes once negative crossing. The added negative crossing is the reason of instability of natation mode. The idea of cross feedback compensation is to remove this added negative crossing. That is, keep the gain of $g_{OL}(+j\omega)$ at f_{n1} still 0 dB and make the corresponding phase change to $\varphi_{n2} = \gamma + 180^\circ$. Where γ is called phase margin and $\gamma > 0$. The Nyquist diagram of for positive frequency is shown in Fig. 2. O is the origin of the coordinate. n_1 represents the complex value of $g_{OL}(+j\omega)$, where $\omega = 2\pi f_{n1}$. That is, $on_1 = e^{j\varphi_{n1}}$. The transfer function of open-loop SISO system with filter cross- feedback is denoted as $g_{OLr}(s)$. The Nyquist diagram of $g_{OLr}(s)$ for positive frequency is also shown in Fig. 2. n_2 represents the complex value of $g_{OL}(+j\omega)$, where $\omega = 2\pi f_{n1}$. Because the filter cross-feedback is an additive compensation, we can get the desired compensation as follow:

$$\overline{n_2 n_1} = \overline{on_2} - \overline{on_1} = c_{nc} e^{j\varphi_{nc}} \tag{13}$$

The filter cross-feedback is an effective method to improve the nutation stability. Its theory is shown in Fig. 3. k_{ch} and ω_h are the cross coefficient and the cutoff frequency of the high pass filter, respectively. They both determine the effect of compensation, so our goal is to design the two parameters to make the AMBs-rotor system as stable as possible.

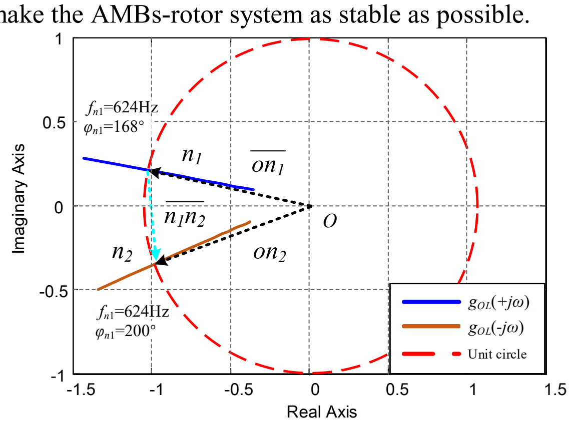

Figure 2. Nyquist diagram of the open-loop SISO system

According to Fig. 3, the model of the AMBs-rotor system with filter cross-feedback can be expressed as

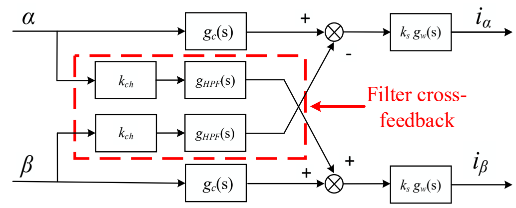

Figure 3. Block diagram of the filter cross-feedback method.

$$\begin{cases} J_x \ddot{\alpha} + H\dot{\beta} - 2l_m k_h \alpha = -2l_s k_i k_s g_w g_c \alpha + l_s k_s g_w g_{HPF} \beta \\ J_y \ddot{\beta} - H\dot{\alpha} - 2l_m k_h \beta = -2l_s k_i k_s g_w g_c \beta - l_s k_s g_w g_{HPF} \alpha \end{cases} \tag{14}$$

where g_{HPF} is the transform operator from input to output signal of the high pass filter.

Now, the transfer function of open-loop SISO system with filter cross-feedback is presented as

$$g_{OLr}(s) = g_{cer}(s) g_{oe}(s) \tag{15}$$

The new control scheme is deduced as

$$g_{cer}(s) = 2l_s k_i k_s g_w(s) g_c(s) + 2jl_s k_s g_w(s) g_{HPF}(s) \tag{16}$$

The additive compensation produced by filter cross-feedback can be expressed as

$$g_{nc}(s) = g_{cer}(s) g_{oe}(s) - g_{ce}(s) g_{oe}(s) \tag{17}$$

Substituting (10), (11) and (16) into (17) yields

$$g_{nc}(s) = 2jl_s k_s g_w(s) g_{HPF}(s) g_{oe}(s) \tag{18}$$

When the system is accurately compensated we have the relation as follows:

$$\begin{cases} g_{nc}(j\omega_{n1}) = \overline{n_2 n_1} \\ g_{OLr}(j2\pi f_{n1}) | = 1 \\ \angle g_{OLr}(j2\pi f_{n1}) = \gamma + 180^\circ \end{cases} \tag{19}$$

Considering both the stability and the respond speed of the system, the phase margin γ is designed to be 20°. Then we can get $\omega_h = 1712.3$ rad and $k_{ch} = 9.77$ by solving (19). The Double-frequency Bode plot of the open-loop SISO system with filter cross-feedback is shown in Fig. 4 For the positive frequency characteristic, the value of the crossover frequency f_{n1} on the right of the nutation resonant peak is still the same. But the corresponding phase becomes larger, which eliminates the negative crossing near f_{n1} . That is, the number of poles which have positive real part for the close-loop SISO system becomes zero and the AMBs-rotor system restores its stability.

The parameters of the filter cross-feedback ω_h and k_{ch} at different rotational speeds can be designed by the same method. Fig. 5 shows the values of ω_h and k_{ch} needed in the whole speed range.

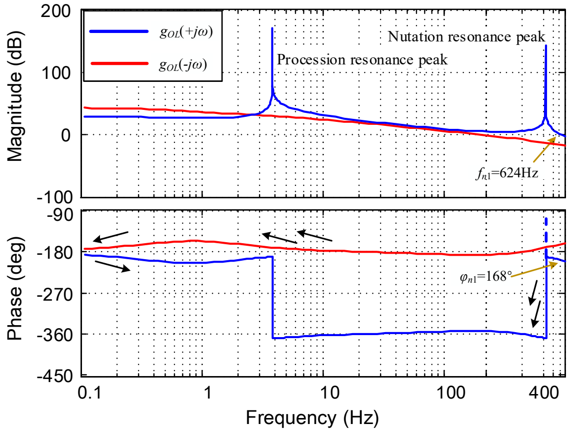

Figure 4. Double-frequency Bode plot of the AMB-rotor system with the filter cross-feedback.

The parameters of the filter cross-feedback ω_h and k_{ch} at different rotational speeds can be designed by the same method. Fig. 5 shows the values of ω_h and k_{ch} needed in the whole speed range.

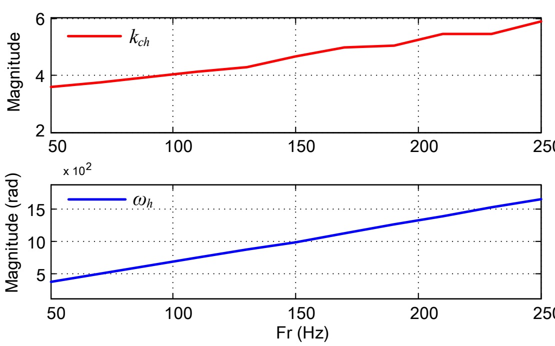

Figure 5. The values of ω_h and k_{ch} needed in the whole speed range.

IV. SIMULATION AND EXPERIMENTAL RESULTS

The experimental setup is shown in Fig. 6. The radial clearance of the auxiliary bearings is 300 *um*. In normal operation, the radial displacement signal contains only the vibration component caused by the mass imbalance of the rotor, which has the same frequency as the rotational speed. And the amplitude of the radial displacement signal generally does not exceed 15% of the bearing clearance. A high-speed Texas Instruments DSP (TM320C28335) with a sampling frequency kept at 6.67 kHz operates the digital control algorithm. The rotor displacements are detected by eddy current sensors.

A. Simulation Analysis

The simulation platform of the AMB-rotor system in MSCMG is established using the software MATLAB/ SIMULINK. In the simulation model, the operational speed is set 15000 r/min. First, we adopt a PID controller to track the reference signal. The parameters of the PID controller were selected to offer the best performance under a step input. The

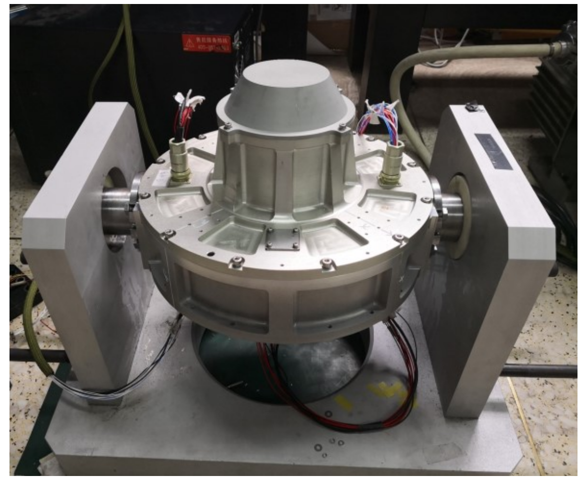

Figure 6. Experimental setups.

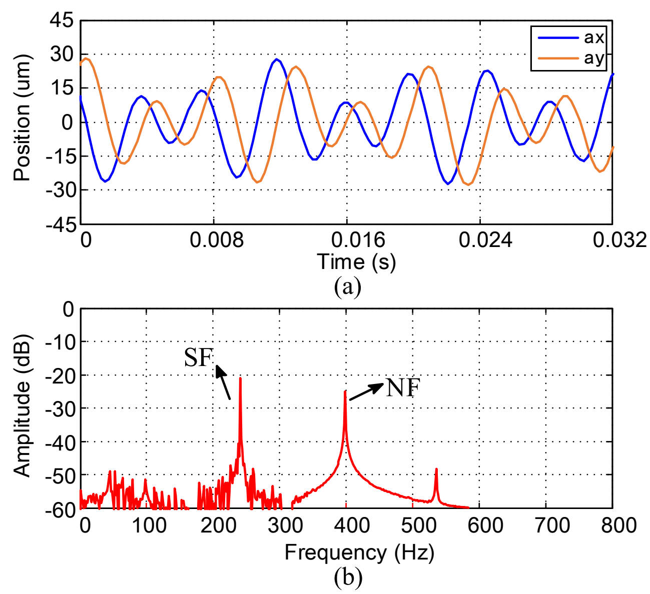

Figure 7. Simulition result of the AMBs-rotor system without the filter cross-feedback. The synchronous frequency is 250 Hz. (a) Radial displacement responses of channels *ax* and *ay*. (b) Frequency spectrum of displacement ax

radial displacements become significantly large, reaching 20% of the bearing clearance, as shown in Fig. 7 (a). The system tends to be unstable. Analyzing the frequency spectrum of displacement *ax*, we find that the amplitude of the nutation frequency (NF) is almost as large as the synchronous frequency (SF), as shown in Fig. 7(b). So the instability of the system is caused by the nutation mode. Adding the filter cross-feedback to the PID controller, parameters of which are designed by the proposed method, the radial displacements are reduced to less than 11% of the bearing clearance, as shown in Fig. 8. The system restores stability.

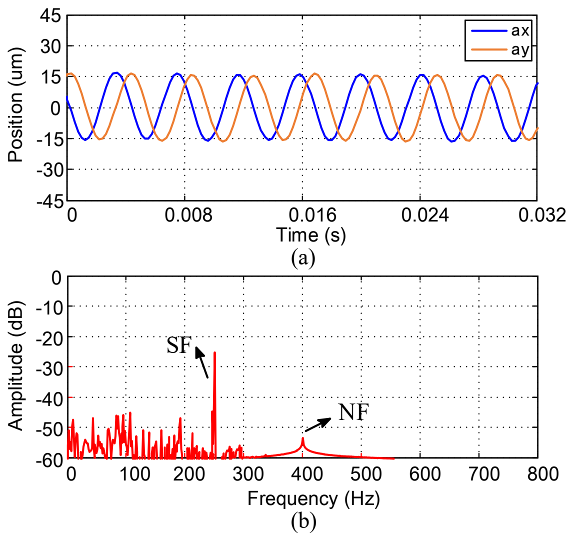

Figure 8. Simulition result of the AMBs-rotor system with the proposed filter cross-feedback. The synchronous frequency is 190 Hz. (a) Radial displacement responses of channels *ax* and *ay*. (b) Frequency spectrum of displacement *ax*.

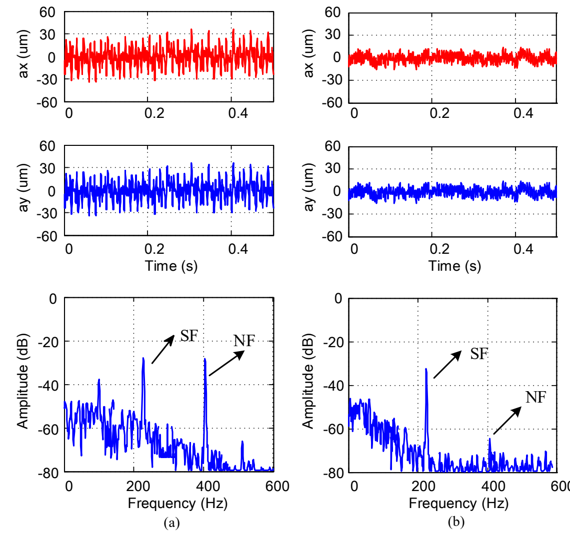

Figure 9. Experimental result: the synchronous frequency is 250 Hz. (a) The filter cross-feedback is not implemented in the controller. (b) The filter cross-feedback is implemented in the controller.

B. *Experimental Results*

To evaluate the performance of the AMB-rotor system under various operating conditions, the experiments are conducted in this section. The experimental results are recorded by Digital Oscilloscope Agilent 3024A. In the sampling record plotted by oscilloscope, the time domain signals of radial displacements (denoted by ax and ay) at End A are printed. The real-time power spectrum of the ax-axis displacement signal is also obtained by employing the math function of the oscilloscope using the fast Fourier transform.

Fig. 9 (a) shows the radial displacements reach 20% of the bearing clearance with a PID controller, when the rotational speed is 15000 r/min. The amplitude level of NF is up to -28.3 dB, lager than the amplitude of SF. This poses a potential risk to the stability of the AMBs-rotor system. Adding the filter cross-feedback designed by the proposed method to the PID controller, the experimental result is shown as Fig. 9 (b). The power spectrum level of NF dropped to -55.9 dB. These data mean the nutation mode is controlled within a very safe boundary. The filter cross-feedback, parameters of which are designed by the proposed method can effectively improve the stability of the AMBs-rotor system.

I. CONCLUSION

The stability of the nutation mode is an important concern for the applications of the AMBs-rotor system. This paper analyzes the compensation value of nutation mode of the system by the complex coefficient method. Then, the parameters of the filter cross-feedback is designed according to the compensation value. Simulation and experimental results demonstrate that parameters k_{ch} and ω_h designed by the proposed method can make the filter cross-feedback work well for restoring the stability of the AMBs-rotor system.

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