Model Predictive Control for Active Magnetic Bearings

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***Abstract*—Active magnetic bearings (AMBs) have lots of advantages than conventional mechanical bearings so that they are being applied to various industrial rotating machineries. However, AMB system is a complex and nonlinear open-loop unstable mechatronic system. For most AMBs, the bias current of actuator is needed which means the input constraint. In this paper, a model predictive control (MPC) technique is developed for eight-pole AMB system with input constraints to improve the ability of resisting external disturbances, e.g. sensor noise and external force. And stability analysis is given and shown that this method enlarging the region of attraction. Finally, the performance of the proposed control method is verified via the simulations.**

***Key Words*—Model Predictive Control (MPC), Active Magnetic Bearings, Input constraint.**

A. *Introduction*

The manufacturing industries develop rapidly in recent years, which result in higher requirements with machines. Conventional rotating machineries can no more completely satisfy demands such as higher precision and higher rotating speed. Active magnetic bearings (AMBs) are such kinds of good substitutes, which suspend the rotor in the air without contact. Compared with conventional mechanical bearings, AMBs enjoy advantages of no fray, needless of lubrication, high rotating speed, controllable stiffness and damping etc.

However, AMB system is a complex nonlinear mechatronic system involves electronics, mechanical engineering and control engineering etc [1]. Moreover, AMB system is an open-loop unstable system. All of those make it challenging yet indispensable task to stabilize the AMB system and resist external disturbances. Efforts have been devoted by numerous companies and scholars. Queiroz and Dawson designed a nonlinear controller with a back stepping approach and achieved global exponential rotor position tracking [2]. To deal with the issue of coil resistance variation, Lindlau and Knospe used the feedback linearization method, in which, a μ-controller was designed for the feedback linearized system to minimize a beam compliance performance specification [3]. Lei and Palazzolo provided the approach of controller design for flexible rotor system with AMBs in detail [4]. Schuhmann et al. conducted a quadratic Gaussian controller consisted of an extended Kalman filter and an optimal state feedback regulator for real-time AMB controlling, which shows better performance compared with conventional PID approaches [5].

For the phenomenon of magnetic saturation and the current working point, there are usually existing input constraint. Yet a few researches take input constrain into considerations. In this paper, a model predictive control method is apply to the eight-pole AMB system with input constraints. We are aiming to develop control methods for AMB system with input constrains and improve its ability of resisting external disturbances, e.g. sensor noise and external force.

B. *Model of AMB system*

In this paper, an eight-pole radial active magnetic bearing system is main research object, which is composed of rotor, displacement sensors, amplifiers and controller etc.

As it shows in Fig. 1, the angle between sensor direction and coil direction is $\theta = \pi/4$. x_0 is the distance of air gap from auxiliary bearing center to the coils.

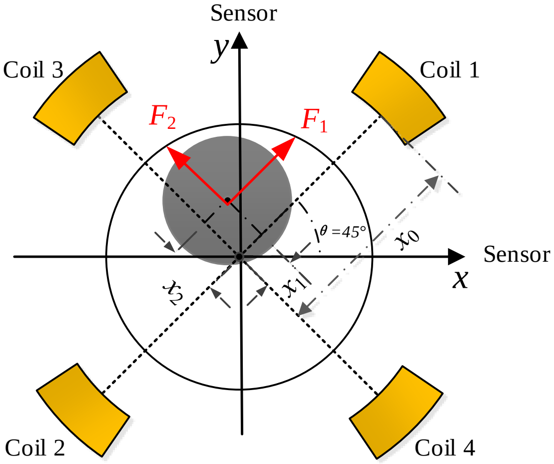

Figure 1. The radial magnetic bearings and rotor

The two orthogonal forces F_1 and F_2 are:

$$\begin{cases} F_1 = k_0[\frac{(I_b+I_{c1})^2}{(x_0-x_1)^2} - \frac{(I_b-I_{c1})^2}{(x_0+x_1)^2}] \\ F_2 = k_0[\frac{(I_b+I_{c2})^2}{(x_0-x_2)^2} - \frac{(I_b-I_{c2})^2}{(x_0+x_2)^2}] \end{cases} \tag{1}$$

where I_{c1} , I_{c2} are control currents and I_b is bias current. $I_1 = I_b + I_{c1}$, $I_2 = I_b - I_{c1}$, $I_3 = I_b + I_{c2}$, $I_4 = I_b - I_{c2}$ are the currents of four coils respectively, $k_0 = 1/4 \times \mu N^2 s \cos(\theta/2)$, μ is the permeability in the air, N is the number of coil turns and *s* is the polar area. x_1, x_2 are the displacements of the rotor.

$$x_1 = \sqrt{2}/2\, x_s + \sqrt{2}/2\, y_s, y_1 = \sqrt{2}/2\, y_s - \sqrt{2}/2\, y_s \tag{2}$$

where x_s and y_s are the movement of rotor in sensor directions.

The linearization [6] of equation (1) is

$$F_1 = K_i I_{c1} - K_x x_s,\quad F_2 = K_i I_{c2} - K_x y_s \tag{3}$$

where $K_i = 4kI_b / x_0^2$, $K_x = 4kI_b^2 / x_0^3$ are current stiffness and displacement stiffness respectively. The dynamic of the rotor is derived as,

$$\begin{cases} m_0\ddot{x}_1 = F_1 = K_i I_{c1} - K_x x_s \\ m_0\ddot{x}_2 = F_2 = K_i I_{c2} - K_x y_s \end{cases} \tag{4}$$

As is showed in equation (4), if the same control variables are given to two couple of coils in F_1, F_2 directions, the rotor would display the same state in both directions. Thus the study is simplified to one direction.

Let $x = [x_1, x_2]^T$, $y = x_1$, $u = I_{c1}$, then the state-space representation of (4) is

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{5}$$

Where $A = \begin{bmatrix} 0 & 1 \\ -K_x / m_0 & 0 \end{bmatrix}$, $B = [0 \quad K_i / m_0]^T$, $C = [1 \quad 0]$.

C. *Mode predictive controller design*

Mode predictive control is a control method depends on the current state. In the processing of each sampling instant, the current control action is achieved by the optimization of the system in a finite horizon online while the current state is taken as initial state [6].

The main advantage of MPC is its ability of dealing with input constraints [7]. MPC algorithm is thus employed in this paper, the design procedures is presented as follow.

The discrete version of equation (3) is

$$\begin{aligned} x(k+1) &= A_d x(k) + B_d u(k) \\ y(k+1) &= C_d x(k+1) \end{aligned} \tag{6}$$

with $x \in \mathbb{R}^m, y \in \mathbb{R}^n, u \in \mathbb{R}^q$ and $m = 2, n = 1, q = 1$. An invariant set for $x(k)$ is introduced as follow,

$$S_x := \left\{ x(k) \mid x(k)^T P_x x(k) \le 1 \right\} \tag{7}$$

The first step is to find the optimal state-feedback matrix K for system (6), in which, $\Phi_K = A - BK$ should be Hurwitz. Here, the Linear Quadratic Regulator (LQR) algorithm is used, and the cost function is

$$J = \sum_{k=0}^{\infty} (x^T(k) Q x(k) + u^T(k) R u(k)) \tag{8}$$

where $Q \in \mathbb{R}^m, \mathrm{R} \in \mathbb{R}^q$ are positive definite matrixes.

Then, the predictive controller is designed as

$$\hat{u}(k \mid k) = L[\hat{x}(k \mid k)^T, \hat{\xi}(k \mid k)^T]^T \tag{9}$$

where $L = [K, G]$, $\hat{x}(k \mid k) = \hat{x}(k)$, $\hat{u}(k \mid k) = u(k)$ and $G = [I_{q\times q}, 0_{q\times(h-1)\times q}]$, and h is the predictive horizon ($h \ge 1$), the prediction vector $\hat{\xi}(k \mid k) = \xi(k)$ where

$$\xi(k) := [\varepsilon(k)^T, \ldots \varepsilon(k+h-1 \mid k)^T]^T ,$$

and $\varepsilon(k+i \mid k)^T \in \mathbb{R}^{q\times 1}$ is a perturbation vector, $\xi(k)$ is defined as

$$\begin{aligned} \hat{\xi}(k+i \mid k) &= V \hat{\xi}(k+i-1 \mid k) \\ V &= \begin{bmatrix} 0 & I_{(h-1)\times m} \\ 0 & 0^T \end{bmatrix}_{hq\times hq} \end{aligned} \tag{10}$$

with $i = 1,2,\ldots,h$. Obviously, $\hat{\xi}(k+i \mid k)$ will decrease to zero under less than h steps with operator V . As a result, only the state-feedback control part will still work and $u = Kx$. While the input of AMB system (5) is limited by constrains, constrained MPC is used to ensure the stability and feasibility of AMB system. Consider $\hat{\tilde{x}}(k+i \mid k) = \left[\hat{x}(k+i \mid k) \quad \hat{\xi}(k+i \mid k) \right]^T$, which means

$$\hat{x}(k+i \mid k) = H\hat{\tilde{x}}(k+i \mid k) \tag{11}$$

where $H = [I_m, \mathbf{0}_{m\times hq}]$. And with equation (6) and (9), we can get

$$\begin{aligned} \hat{\tilde{x}}(k+i \mid k) &= M\hat{\tilde{x}}(k+i-1 \mid k) \\ \hat{y}(k+i \mid k) &= C_d \hat{x}(k+i-1 \mid k) \end{aligned} \tag{12}$$

with $M = \begin{bmatrix} \Phi_k & B_d G \\ 0 & V \end{bmatrix}$, $i = 1, \ldots h$, An invariant set for $\hat{\tilde{x}}(k)$ is introduced here,

$$S_{\tilde{x}} := \{ \tilde{x}(k) \mid \tilde{x}(k)^T P_{\tilde{x}} \tilde{x}(k) \le 1 \} \tag{13}$$

with positive-definite matrix $P_{\tilde{x}}$.

$\hat{\xi}(k+i \mid k)$ is achieved by quadratic optimization algorithm [7],

$$\begin{aligned} &\min_{\xi(k)} J(k) = \xi(k)^T \xi(k) \\ &s.t.\, \hat{\tilde{x}}(k+i \mid k)^T P_{\tilde{x}} \hat{\tilde{x}}(k+i \mid k) \le 1, i = 0, \ldots, h-1 \\ &\quad | \hat{u}_j(k+i \mid k) | \le u_b \end{aligned} \tag{14}$$

where $i = 1,2,\ldots,h-1$, $j = 1,2,\ldots,q$, u_b is the hard input.

$$\left| \hat{u}_j(k+i \mid k) \right| = \left| L_j \hat{\tilde{x}}(k+i \mid k) \right| \le u_b \tag{15}$$

in which

$$\begin{aligned} \left| L_j \tilde{x} \right| &= \left| L_j P_{\tilde{x}}^{-1/2} P_{\tilde{x}}^{-1/2}{}_{\tilde{x}} \right| \\ &\le \left\| L_j P_{\tilde{x}}^{-1/2} \right\|_2 \bullet \left\| P_{\tilde{x}}^{-1/2}{}_{\tilde{x}} \right\|_2 \\ &\le \left\| L_j P_{\tilde{x}}^{-1/2} \right\|_2 \end{aligned} \tag{16}$$

equation (15)will be satisfied when

$$\begin{vmatrix} -u^2{}_b & L_j \\ L_j{}^T & -P_{\tilde{x}} \end{vmatrix} \le 0 \tag{17}$$

Some assumptions are put forward as follow to ensure the stability of the system (6), (9).

Assumption 1: Equation (17) holds and equation (14) is solvable. The system (6) is stabilizable and K can be find be satisfy that $\Phi = A_d - B_d K$ is Hurwitz.

Assumption 2: There exist constants θ and symmetric matrix $P_{\tilde{x}}$ satisfy

$$\begin{aligned} &M^T P_{\tilde{x}} M - \theta P_{\tilde{x}} \le 0 \\ &P_{\tilde{x}} > 0 \\ &0 < \theta \le 1 \end{aligned} \tag{18}$$

Based on Assumption 2, if $\hat{\tilde{x}}(k+i \mid k)^T P \hat{\tilde{x}}(k+i \mid k) \le 1$, then

$$
\begin{aligned}
&\hat{\hat{x}}(k+i+1|k)^T P_{\tilde{x}}(k+i+1|k) \\
&= [\mathrm{M}\hat{\hat{x}}(k+i|k)]^T P_{\tilde{x}}[\mathrm{M}\hat{\hat{x}}(k+i|k)] \\
&= \hat{\hat{x}}(k+i|k)^T \mathrm{M}^T P_{\tilde{x}} M\hat{\hat{x}}(k+i|k) \\
&\le \theta\hat{\hat{x}}(k+i|k)^T P_{\tilde{x}}\hat{\hat{x}}(k+i|k) \le \theta \\
&\le 1
\end{aligned}
$$

That's to say $S_{\tilde{x}}$ is an invariant set.

Rewrite $P_{\tilde{x}}$ in the form as

$$
P_{\tilde{x}} = \begin{bmatrix} (P_1)_{m\times m} & P_2 \\ P_2^T & (P_3)_{hp\times hp} \end{bmatrix}
$$

substitute it into equation (13)

$$
x(k)^T P_1 x(k) \le 1 - 2\xi(k)^T P_2^T x(k) - \xi(k)^T P \xi(k) \qquad (19)
$$

$x(k)$ achieves the maximum value when

$$
\xi(k) = \xi_m(k) = -P_3^{-1} P_2 x(k)
$$

substitute $\xi_m(k)$ into equation (19), a new set S_m for initial $x(k)$ is achieved

$$
S_m := \{x(k) \mid x(k)^T P_m x(k) \le 1\} \qquad (20)
$$

where $P_m = P_1 - P_2 P_3^{-1} P_2^T$ is positive-definite and $P_m \le P_1$. When $\xi(k) = 0$, with equation (7) and (19), it's clearly that $P_m = P_1$. Thus $P_m \le P_x$ and $S_x \subseteq S_m$, the allowable initial state is expanded. We can achieve the maximum S_m by maximize $\det(P_m^{-1})$. Since (11) and (13), we have

$$
P_m^{-1} = H P_{\tilde{x}}^{-1} H^T \qquad (21)
$$

which means we can maximize $\det(H P_{\tilde{x}}^{-1} H^T)$ to obtain the maximum invariant set S_m.

D. Simulations

In this section, our objective is to verify the effectiveness of the proposed controller through simulations. The parameters within simulation are presented in Table I. As for the model predictive controller, the predictive step is set as $h = 15$.

An PID controller is introduced here to make a comparison between traditional controller with Ours. In our simulations, two set of PID parameters are applied which conducted a rapid result with overshoot and slow result of levitation. And the parameters P,D are the same as the LQR parameter $k1$,$k2$.

In Fig. 2, the result shows rising time is sharply reduced form 0.06s to 0.03s. And the simulation of Fig. 3 implies the overshoot decrease with an approached rising time. And in simulation a measure random disturb which operation time T = 0.004s, and the max noise $\Delta x = 20\mu m$ is conducted at t = 0.3s. The MPC controller has better performance than PID.

TABLE I: Parameters of the AMB

Data	Value	Units
Rotor mass m_0	8	kg
Air gap x_0	0.50	mm
Bias current I_b	1	A
Current gain K_i	511.35	N/A
Position gain K_x	4.0926E6	N/m

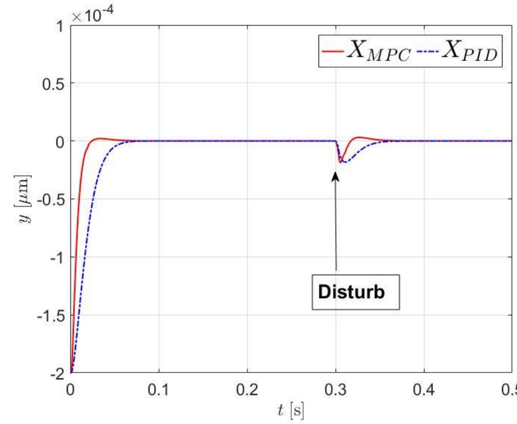

Fig. 2: Low speed levitation of PID and MPC, P = 2000, I = 100, D = 10

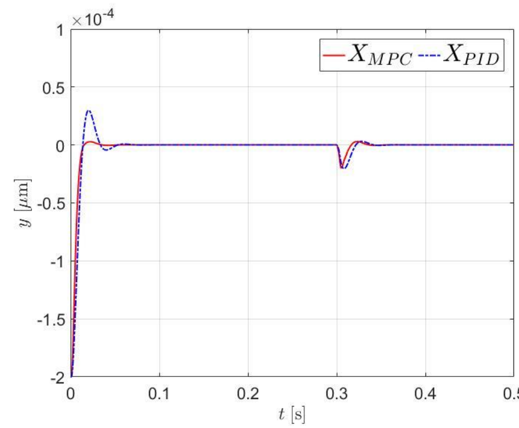

Fig. 3: Rapid speed levitation of PID and MPC, P = 3000, I = 100, D = 10

E. Conclsusion

In this paper, we study a model predictive controller for AMB system with input constrain. By introducing the system model, we designed the MPC controller for the levitation of AMB. And then we conducted some simulations of the comparison between conversional PID controller and the designed controller. The results indicate the designed MPC controller has a better levitating response performance of AMB and a stronger ability to resist disturbs. In the future, we are aiming to apply our methods to the real AMB system applications.

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