Optimal Vibration Control for a Centrifugal Compressor with Magnetic Bearings by a Phase-shift Notch Filter

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***Abstract*—Undesired vibration caused by mass imbalance is common for a centrifugal compressor supported by magnetic bearings. It poses a potential threat to system stability and operation performance. In this work, a phase-shift notch filter is adopted to suppress the synchronous vibration and stability analysis is given to ensure the system stability in the varying speed range. In order to achieve optimal dynamic performance of vibration control, the objective function is set up based on the time domain response of current, and the optimal notch filter is obtained based on the minimization of the objective function value. Finally, the effectiveness of the proposed optimal notch filter on the advancement of dynamic performance is validated through simulation results.**

I. INTRODUCTION

Centrifugal compressor is widely used in industrial processes for the pressurization and transportation of gases and fluids [1]. With a tendency of high energy efficiency and compactness, high-speed machine supported by magnetic bearings is a promising structure of the compressor system. Active magnetic bearings(AMBs) have the advantages of no contact, no need of lubrication, low maintenance cost and long service life, which is the better supporting device compared with mechanical bearings [2].

Vibration control is an important issue for high-speed rotating machinery since the amplitude of vibration force is proportional to the square times of the rotational speed. Mass imbalance, namely the misalignment of geometrical axis and inertia axis, is known as the dominant vibration source and will introduce synchronous disturbance in the displacement of rotor and coil current of magnetic bearings. Considerable efforts have been dedicated to attenuate the vibration by making the rotor rotate around its inertia axis. Betschon [3] presented a gain scheduled adaptive control method for current reduction by employing an adaption-gain matrix. The test results on a high-speed spindle showed that significant reductions in bearing currents were achieved over a wide range of operating speeds, which was also accompanied by a great attenuation in rotor vibration. Jiang [4] proposed an algorithm based on the real-time identification for the fourier coefficients of the rotor imbalance disturbance and automation balancing was realized by locating the pickup point behind the injection point. The experimental results showed that the orbit of control current visibly converged toward its center. A novel autobalancing method based on the synchronous rotating frame transformation was proposed in [5] for synchronous vibration suppression on a motor driveline system. It was also revealed that the peak-to-peak values of control currents can be used to evaluate the strength of vibration force. All in all, the above research works have demonstrated that synchronous current suppression can directly indicate the reduction of the synchronous vibration.

Notch filter is one of the effective methods for vibration control, which is widely used in industrial applications for its simplicity and practicality. Considering the degradation of system stability when inserting the notch filter into the basic control system, especially in the varying speed range, Herzog [6] proposed a generalized narrow-band notch filter, of which parameters strongly depended on the inverse sensitivity matrix evaluated at rotational speed. Cui [7] employed multiple phase-shift notch filters to suppress the harmonic current of the AMB rotor system and the phase angle was adjusted at different rotation speeds to ensure the closed-loop stability in the whole speed range. In the situation that a rotational sensor was not available due to the limited size of the machine and high operating temperature, Chen [8] studied a modified adaptive notch filter(ANF) to estimate the rotational frequency, which was also used to eliminate the synchronous current. The convergence of the proposed ANF algorithm was analyzed. However, the above literatures are mainly focused on the system stability without discussing the dynamic performance.

In order to achieve the better transient response of current suppression, Zheng [9] presented a parallel-mode scheme that the notch filter was parallel connected with the controller instead of the series connection. The results showed that the closed-loop system with a parallel-mode notch filter had a deeper notch depth and faster convergence. However, the advancement was merely obtained on the basis of the proportion-derivative (PD) controller. From the viewpoint of the root locus, Zhang [10] designed an analytical formula of optimal compensation phase angle so as to achieve superior dynamic performance. While the effectiveness of the optimal parameter in theory on the practical application relies on the precise system parameters.

For this reason, an objective function is built up in this paper to evaluate the dynamic performance based on the time domain response of control current. The optimal notch filter is designed through the minimization of the objective function value. The proposed method is a more direct way to implement optimal vibration control without considering the accurate modelling and limitation of some specific control algorithm.

Therefore, this paper will model the synchronous vibration arising from mass imbalance and present the control system with a phase-shift notch filter for eliminating the synchronous current. Then, the stability analysis of the closed-loop system will be given. Furthermore, to achieve better dynamic performance, the objective function will be set up for optimal notch filter design. Finally, simulation results will be presented to validate the effectiveness of the proposed method for a magnetic suspended centrifugal compressor.

II. Dynamic Model for an AMB Rotor System with Mass Imbalance

A general view of a magnetic suspended centrifugal compressor is shown in Fig. 1. The structure is composed of two permanent-magnet-based axial and radial active magnetic bearings (PARAMB) and a brushless DC motor[11].

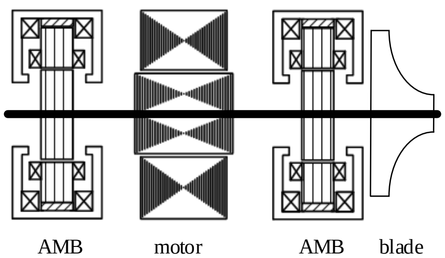

Fig. 1. Structure of the magnetic suspended centrifugal compressor

With the consideration of manufacturing error, mass imbalance is inevitable. The imbalanced rotor is illustrated in Fig. 2, in which $O(x,y)$ is the geometric center and $C(x_c,y_c)$ is the mass center, ε is the eccentricity and φ is the initial phase angle, Ω is the rotational speed.

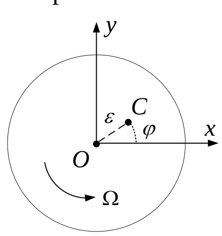

Fig. 2. Illustration of the imbalanced rotor

The relationship between the geometric center and mass center can be expressed as

$$\begin{cases} x_c = x + \varepsilon\cos(\Omega t + \varphi) \\ y_c = y + \varepsilon\sin(\Omega t + \varphi) \end{cases} \tag{1}$$

Since the axial and radial magnetic circuits of the PARAMB are decoupled, the electromagnetic forces can be independently generated near the equilibrium position. For simplicity, taking 2-DOF radial electromagnetic forces as examples, they can be modeled as

$$\begin{cases} m\ddot{x}_c = k_{ix}i_x + k_x x \\ m\ddot{y}_c = k_{iy}i_y + k_y y \end{cases} \tag{2}$$

where Fx and Fy are electromagnetic forces in the horizontal and vertical directions respectively, i_x and i_y are the corresponding coil currents, x and y are relative displacement signals of the rotor, k_{ix} and k_{iy} are the current stiffness parameters, while k_x and k_y are the displacement stiffness parameters.

Then, the dynamic model is obtained as follows

$$\begin{cases} m\ddot{x} = k_{ix}i_x + k_x x + m\Omega^2\varepsilon\cos(\Omega t + \varphi) \\ m\ddot{y} = k_{iy}i_y + k_y y + m\Omega^2\varepsilon\sin(\Omega t + \varphi) \end{cases} \tag{3}$$

It is confirmed that the amplitude of the vibration force caused by mass imbalance is proportional to the square times of the rotational speed and the frequency is the same as the rotational speed. Therefore, the synchronous vibration control is urgent especially for the high-speed operation situation.

III. Optimal Vibration Control with a Phase-shift Notch Filter

In this work, a phase-shift notch filter is adopted to suppress the synchronous current. In order to achieve the optimal dynamic performance, the objective function is set up for optimal notch filter design.

A. *Phase shift notch filter*

The schematic diagram of a phase-shift notch filter is displayed in Fig. 3. Compared with the conventional notch filter, an adjustable phase angle is introduced to ensure the system stability in the varying speed range[2].

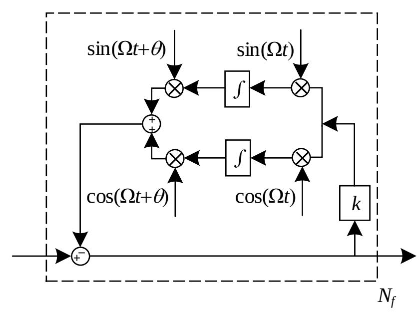

Fig. 3. Schematic diagram of a phase-shift notch filter

The transfer function of the phase-shift notch filter can be deduced as

$$N_f(s) = \frac{s^2 + \Omega^2}{s^2 + \Omega^2 + k(s\cos\theta - \Omega\sin\theta)} \tag{4}$$

where Ω is the notch frequency, θ is the adjustable phase angle and k is the gain that is related to the bandwidth of the notch filter.

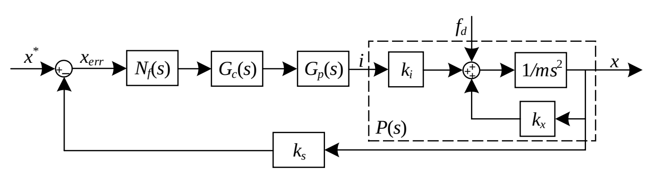

Fig. 4. Block diagram of AMB control system for synchronous vibration control

In theory, the notch filter can be inserted to any position in the control loop to realize the synchronous current suppression[4]. In practical digital control, for the convenience of data extraction and algorithm implementation, the phase-shift notch filter is connected to the suspension controller in series and the pickup point is located behind the injection point. The block diagram of AMB control system for synchronous vibration control is shown in Fig. 4, where x^* is reference position, x is the actual displacement signal, x_{err} is the error signal, i is the control current, f_d is the synchronous vibration force, $G_c(s)$ is the suspension controller, $G_p(s)$ is the power amplifier, $P(s)$ displayed in dash box is the AMB-rotor, k_s is the sensor.

B. Stability analysis

Stability analysis plays an important role in the design of the control system. As mentioned above, the phase angle can be adjusted to satisfy the overall system stability requirements.

First of all, the sensitivity function of the original AMB control system without a notch filter can be represented as

$$S_0(s)=\frac{1}{1+k_sG_c(s)G_p(s)P(s)} \tag{5}$$

It is known that the system stability depends on the location of poles of the closed-loop transfer function. Hence, the characteristic equation of the vibration control system shown in Fig. 4 is given as

$$1+k_sN_f(s)G_c(s)G_p(s)P(s)=0 \tag{6}$$

Since the original control system is stable, all the poles are placed at left half complex plane. The characteristic equation is further deduced as

$$s^2+\Omega^2+k(s\cos\theta-\Omega\sin\theta)S_0(s)=0 \tag{7}$$

As observed from (8), in the case of k=0, the roots can be $s=\pm j\Omega$, which means that there are two poles located on the imaginary axis. In practical application, k is a positive integer. Therefore, to ensure the system stability, the root locus near the imaginary axis ought to deviate to the left half plane so that all the poles have negative real parts. Accordingly, let s take a derivative with respect to k, and the condition given in (9) should be satisfied.

$$\arg\left[\left.\frac{\partial s}{\partial k}\right|_{k=0,s=j\Omega}\right]\in(90°,270°) \tag{8}$$

According to (8) and (9), the stability condition is derived as

$$-90°<\theta+\arg\left[S_0(j\Omega)\right]<90° \tag{9}$$

It can be concluded that the system stability can be maintained by adjusting the phase angle θ along with the sensitivity function in the varying speed range.

C. Optimal parameter tuning

The phase angle of the notch filter will not only influence the stability of the system, but also have an effect on the dynamic performance. Usually, the theoretical optimal parameters cannot achieve the superior performance in practical application. Hence, on the premise of system stability, the objective function is set up based on the integral of time-weighted absolute error(ITAE) criterion to optimize the dynamic performance directly.

The ITAE criterion can be expressed as

$$J_{ITAE}=\int_0^{\infty}|e(t)|\cdot tdt=\min \tag{10}$$

where $e(t)$ tends to be 0 and the parameter is optimized through the minimization of the integral.

In the AMB control system, the amplitude of the current will nearly decrease to 0 due to the significant attenuation of the synchronous disturbance. To make sure that the transient response of the current suppression is fast and smooth, $e(t)$ in (11) is replaced by current $i(t)$ and the upper limit of the integral is substituted by some specific time t_0 which is greater than settling time. Therefore, the objective function for designing the optimal phase angle is deduced as

$$J_{ITAE}=\int_0^{t_0}|i(t)|\cdot tdt\to\min \tag{11}$$

When the value of phase angle is different, the objective function ends up with different results. In order to obtain the optimal phase angle, the optimization process is presented.

Step 1: Obtain the rotational speed of the rotor;

Step 2: Determine the stable range of phase angle on the basis of stability analysis.

Step 3: The notch filters with different phase angles are applied into the AMB control system individually to suppress the synchronous current. The objective function value is acquired according to the transient response of the current.

Step 4: Find out the minimum of the objective function values and the corresponding phase angle is set to be the optimal parameter.

Step 5: At different rotational speed, repeat Step 1 to Step 4, the optimal phase angle which is suitable for corresponding synchronous current suppression can be obtained.

Although the traversal search is a tedious process, but the optimal phase angle at different rotational speed is identified in advance. So, it is still an acceptable method in engineering practice.

IV. Simulation Results

In order to verify the effectiveness of the proposed method for optimal vibration control, a simulation model of the AMB control system for the magnetic suspended centrifugal compressor is built in the MATLAB/Simulink. The related parameters of the system are listed in table 1.

TABLE I. PARAMETERS RELATED TO THE AMB CONTROL SYSTEM

	Parameter	Value
m	Rotor mass	3.6 kg
k_{ix}	Current stiffness	67 N/A
k_x	Displacement stiffness	2.38×10^5 N/m
k_P	Proportional coefficient	5
k_I	Integral coefficient	50
k_D	Differential coefficient	0.008

As is shown in Fig. 4, the notch filter is working together with the suspension controller to suppress the synchronous vibration. First of all, the stability of the closed-loop control system is validated. As the rotational speed of the rotor changes from 0 to 40,000r/min, the phase angle of the notch filter is adjusted according to the stability condition. Then the root locus of AMB control system is shown in Fig. 5. It can be seen that all the poles are located on the half plane in the whole speed range. It is demonstrated that the phase shift notch filter can maintain the system stability by adjusting the phase angle in varying speed range.

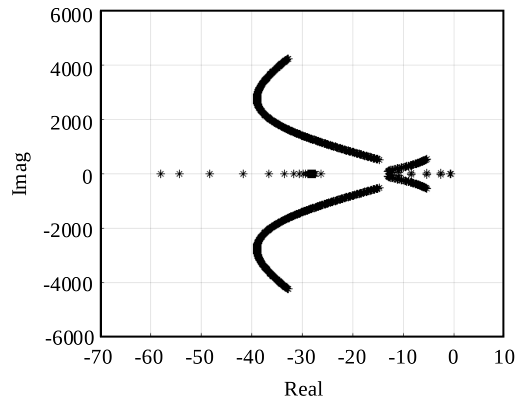

Fig. 5. Root locus of the AMB control system with a phase-shift notch filter

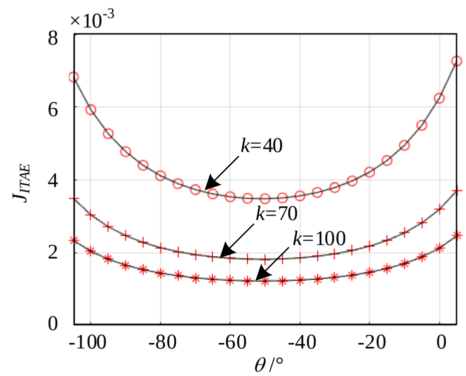

Fig. 6. Simulation results of phase angle optimization with different gain at the rotational speed of 30,000r/min

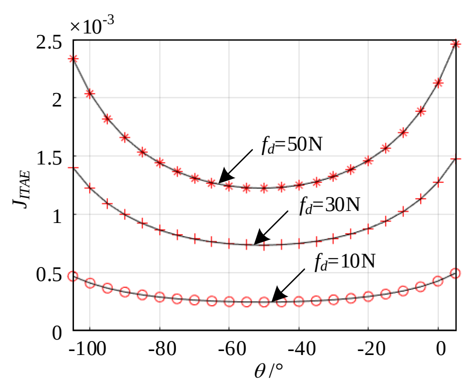

Fig. 7. Simulation results of phase angle optimization with different vibration force at the rotational speed of 30,000r/min

According to the optimization process mentioned above, the simulations are carried out to acquire the optimal phase angle of the notch filter at the rotational speed of 30,000r/min. The notch filters with different phase angles are inserted into the control system individually to suppress the synchronous current and the values of objective function are calculated and compared.

The first simulation presented in Fig. 6 is using notch filters with different phase angles to suppress the synchronous vibration force, of which the amplitude is 50N. The simulation results have demonstrated that there exists an optimal phase angle when the objective function is minimized, at the same time the gain is a constant. Furthermore, as the gain increases, the objective function drops to a smaller value, which means the dynamic performance is improved. Most importantly, no matter what the value of the gain is, the optimal phase angle θ_{op} still remains -50°. Another simulation has been implemented to investigate the influences of different amplitudes of vibration force on the phase angle optimization meanwhile the gain stays unchanged, which is shown in Fig. 7. It is also found that whatever the amplitude of the vibration force is, the optimal phase angle θ_{op} keeps the same.

The current suppression is usually used to evaluate the attenuation of vibration force. In order to present a direct view of the superiority of optimal notch filter, comparison results of current suppression are shown in Fig. 8. It can be concluded that the notch filter with optimal phase angle provides faster convergence. In detail, the peak to peak values of control currents are both reduced from 0.6A to almost 0A stably in these two circumstances. However, the convergence time of optimal notch filter is 0.15s, which is nearly half of the convergence time in the upper situation.

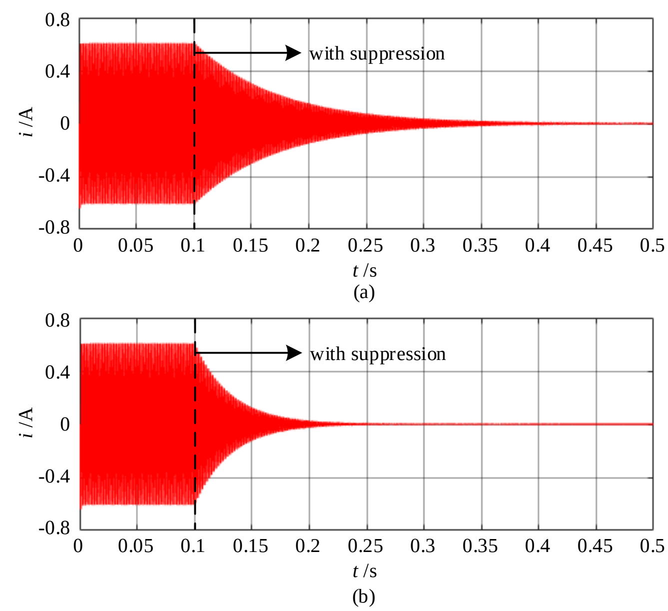

Fig. 8. Simulation results of current suppression with different phase angles of notch filter (a) $\theta=\theta_{op}$-60° (b) $\theta=\theta_{op}$

Finally, the simulations are implemented to validate the effectiveness of the proposed optimal notch filter for vibration control. At the rotational speed of 30,000r/min, the optimal notch filter is inserted into the AMB control system at 0.2s to attenuate the vibration force. The responses of current, displacement and vibration force for radial directions are shown in Fig. 9. It can be seen that the peak to peak value of the current is greatly reduced to 0A and the displacement signal is slightly decreased from 3.1um to 2.9um. The amplitude of vibration force is significantly attenuated to 1N, which is only 2.5% of that without suppression. It is also revealed that the current suppression plays a key role in vibration control.

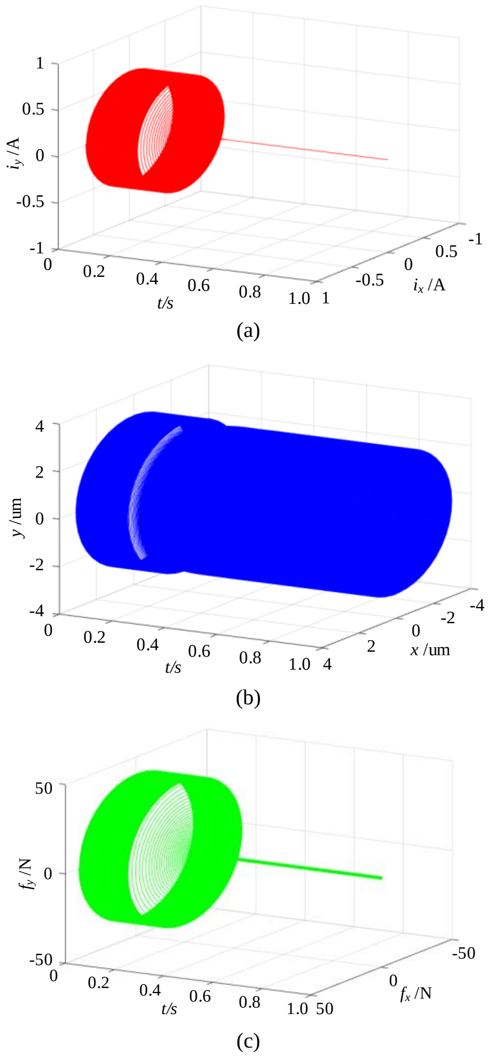

Fig. 9. Simulation results of the AMB control system using the optimal notch filter at the rotational speed of 30,000r/min (a) current (b) displacement (c) vibration force

V. CONCLUSIONS

Mass imbalance is an inevitable problem of rotating machinery. The resulting synchronous vibration force is attenuated stably in full speed range by employing a phase-shift notch filter in this work. Furthermore, an objective function based on the transient response of current is proposed to optimize the dynamic performance directly with no need to acquire the accurate system parameters. At a fixed rotational speed, the optimal phase angle of notch filter is obtained through the minimization of the objective function value, which remains invariant when the gain of the notch filter and the amplitude of vibration force changes. Simulation results have demonstrated that the proposed optimal notch filter provides faster convergence of current suppression and the synchronous vibration force is significantly reduced at the rotational speed of 30,000r/min. The future study will focus on the experimental validation and applying the proposed optimal notch filter in engineering practice considering the rotational speed fluctuation.

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