Identification of Stiffness and Damping of Auto-eliminating Clearance Auxiliary Bearing Devices based on Finite Element Method

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***Abstract*—In order to study the dynamic supportive stiffness and damping of auto-eliminating clearance auxiliary bearing devices (ACABD) after the rotor falls, this paper proposed an identification method of stiffness and damping based on finite element method. Fist, the model of the rotor supported by ACABD and the feasibility of the method was verified by simulation. Then, the test bench of magnetic bearing system contained the ACABD was designed. And the unbalanced responses were obtained by the experiment method of unbalanced excitation. In the end, the actual supportive stiffness and damping of ACABD were obtained by the parameter identification method based on finite element method. The results indicate that during the moderation process from 500Hz to 20Hz after the drop of the rotor, the values of supportive stiffness and damping of ACABD change continually, which generally decrease with the rotational speed.**

Key words: supporting characteristics; ACABD ; finite element method ; unbalanced response;

1. *Introduction*

Active magnetic bearings (AMBs) have the advantages of high speed, no friction, clean and can be controlled online. And they are more and more applied in rotation machinery, especially high speed rotation machinery [1]. AMBs still need auxiliary bearings for protection device when glitch occurs [2]. Traditional auxiliary bearings are common rolling bearings. When AMBs suffer overloads or failure, there may be reverse whirling generated by the friction between rotor at high speed and rolling bearings, which will lead to instability, even the damage of the raceway. Thus, the life of the auxiliary bearings will be shortened dramatically [3]. Therefore, it is necessary to study a new kind of auxiliary bearing to improve the stability of the auxiliary bearings. Aiming at the research of new type of auxiliary bearing, Chen proposed a zero gap auxiliary bearings and tested the reliability of the new auxiliary bearing [4]. Wu et al proposed a device that using an electromagnet as auxiliary bearing [5], this device could eliminate the clearance between the rotor and bearing by releasing the device under power-down conditions. In addition, there are foreign scholars that use aerodynamic foil bearing as auxiliary bearing and have deep researches [6, 7].

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Our group proposed a kind of device that can gradually eliminate the gap between the outer ring of the rolling bearing and the surface of bearing base, as known as ACABD (auto-eliminating clearance auxiliary bearing devices). The structure, operating mechanism and the statical and dynamci analysis of the device are introduced in Ref [8]. The dynamic models of the rotor dropping on the ACABD were established and the modes of lubrication and the influences of support shape on the performance and execution time of clearance elimination were discussed in Ref [9]. The research results above indicates that when the rotor falls with high speed, the ACABD can eliminate the gap between the outer ring of the rolling bearing and the base, reducing the vibration and impact and has protective effect on the rotor. In order to verify the working performance after the ACABD eliminated the gap and the state of the rotor from the high speed to static state, it is necessary to study the supportive parameters after the ACABD works.

In recent years, the research of parameter identification has made great progress. De Santiago [10, 11] tested the dynamic supportive parameters of radial journal bearings and rolling bearings at the left and right ends of the rigid rotor by the experimental method based on impact force and unbalanced excitation respectively. San Andrés [12] proposed the dynamic supportive parameters identification method of flexible rotor system of the rotor based on the finite element model of the rotor. Tiwar [13] used the least square method to process the displacement data of the experiments, obtaining the supportive parameter of the bearing in different speed. Perry [14] applied the genetic algorithm to the identification and damage of the structural parameter. The values and experimental results indicated that it can effectively identify the structural parameter and the damage by only using the incomplete acceleration measurement. Lyu M [15, 16] proposed a new method based on Hilbert transform to identify the tail responses and designed the algorithm to suspend the rotor again. Xu Y [17] took magnetic suspension bearing system as research object, identifying the supportive parameters of the magnetic suspension bearing based on the identification method of finite element method. And the validity was verified by simulation and experiments.

This paper studied the characteristics of dynamic supportive stiffness and damping of ACABD after the rotor falls with high speed, proposing the identification method of ACABD supportive parameters based on finite element method. The rotor finite element dynamic equations are established and the expressions of the support stiffness and damping are derived. The feasibility of parameter identification based on the finite element method is verified through simulation analysis. And the magnetic suspension bearing test bench that contains the ACABD was designed. And the dynamic supportive stiffness and damping of ACABD were obtained by the identification method based on finite element method with unbalanced responses obtained by unbalanced excitation experiment.

2. *Content Guidelines*

The ACABD is used as a protective device when the rotor is dropped. It is distributed on both ends of the rotor. The support characteristics of the protection bearings at both ends are basically the same. For the sake of research convenience, this article studies the situation in which one end of the rotor is protected by a ACABD support and the other end is supported by an active magnetic suspension, as shown in fig.1. In this paper, the supportive stiffness and damping of ACABD are identified and studied.

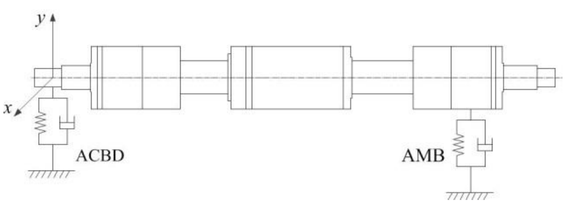

Fig. 1 the schematic diagram of the rotor supported by AMB and AB

According to finite element theory, the rotor can be divided into *N* units and *N*+ 1 node. Taking the two degree of freedom of translational motion (*x*, *y*) and the two degree of freedom of rotation (βx, βy) and ignoring the two axil degree of freedom, according to the theory of rotor dynamic, the dynamic equation of magnetic suspension bearing-rotor system can be expressed as:

$$M\ddot{q}+C\dot{q}-\Omega G\dot{q}+Kq=F_u \tag{2.1}$$

Where, *M*, *C*, *K*, *G* is the total mass matrix of the rotor, total damping matrix, total stiffness matrix and total gyroscopic matrix respectively. Ω is the rotation speed of the rotor. *q* is the displacement of every node. F_u is the generated unbalanced excitation. The displacement vector q and the unbalanced force can be represented as:

$$\begin{aligned} &q=[q_1 ... q_A ... q_B ... q_N]^{\mathrm{T}}; \\ &q_i=[x_i \quad y_i \quad \alpha_i \quad \beta_i]^{\mathrm{T}}{}_{i=1,2...N} \\ &F_u=\left[0\cdots f_u\cdots 0\right]^T \end{aligned} \tag{2.2}$$

In Eq. (2.2), q_A and q_B is the displacement vector of the ACABD and magnetic bearing at node. x_i and y_i are the translational displacement of the node. α_i and β_i are the rotational displacement of the node *i*. f_u is the unbalanced force generated by unbalanced mass of the node. The unbalanced force can be expressed as:

$$f_u=m_u r_u \omega^2 [e^{i(\omega t+\phi)} \quad -ie^{i(\omega t+\phi)} \quad 0 \quad 0]^{\mathrm{T}} \tag{2.3}$$

In Eq. (2.3), m_u is the imposed unbalanced force. r_u is the radius of imposed unbalanced mass point to the axle center. Because of that the excitation frequency of the unbalanced mass is same to that of the rotor speed, which is $\omega=\Omega$, Eq. (2.1) can be simplified to:

$$[(-M\Omega^2+K)+i\Omega(C-\Omega G)]q=F_u \tag{2.4}$$

The transfer function between the vector of rotor displacement and that of external excitation force can be represented as:

$$H=[(-M\Omega^2+K)+i\Omega(C-\Omega G)] \tag{2.5}$$

Under the circumstance of that the rotor is supported by the bearings, the total stiffness matrix and damping matrix is the sum of stiffness and damping of rotor and that of magnetic bearing and ACABD , that is:

$$K=K_R+\begin{bmatrix}0&0&0&0&0\\0&K_A&0&0&0\\0&0&0&0&0\\0&0&0&K_B&0\\0&0&0&0&0\end{bmatrix};C=C_R+\begin{bmatrix}0&0&0&0&0\\0&C_A&0&0&0\\0&0&0&0&0\\0&0&0&C_B&0\\0&0&0&0&0\end{bmatrix} \tag{2.6}$$

Where the supportive stiffness and damping of ACABD and magnetic bearing can be expressed as:

$$K_A=\begin{bmatrix}K_{xx1}&0\\0&K_{yy1}\end{bmatrix};C_A=\begin{bmatrix}C_{xx1}&0\\0&C_{yy1}\end{bmatrix} \tag{2.7}$$

$$K_B=\begin{bmatrix}K_{xx2}&0\\0&K_{yy2}\end{bmatrix};C_B=\begin{bmatrix}C_{xx2}&0\\0&C_{yy2}\end{bmatrix} \tag{2.8}$$

For convenience, Eq. (2.5) was decomposed into the transfer function of rotor H_R , the transfer function of ACABD and magnetic bearing H_B . At this time, the Eq. (2.4) can be represented as:

$$(H_R+H_B)q=F_u \tag{2.9}$$

Suppose Z_A and Z_B is the translational displacement of ACABD and magnetic bearing respectively, then

$$Z_A=[x_A \quad y_A]^T;\ \ Z_B=[x_B \quad y_B]^T \tag{2.10}$$

For convenience, Eq. (2.9) can be transformed into:

$$\bar{H}_R\begin{bmatrix}Z_A\\Z_B\\Z_O\end{bmatrix}+\bar{H}_B\begin{bmatrix}Z_A\\Z_B\\Z_O\end{bmatrix}=\begin{bmatrix}0\\0\\\bar{F}_u\end{bmatrix} \tag{2.11}$$

Where $\bar{H}_R$ and $\bar{H}_B$ is the transfer function. Z_O is the unbalanced response of other unknown node on the rotor. In order to precisely identify the supportive parameters of magnetic bearing, $\bar{H}_R$ and $\bar{H}_B$ will be divided into nine sub-matrixes of stiffness and damping, as shown in Eq. (2.12).

$$\bar{H}_R=\begin{bmatrix}H_{R11}&H_{R12}&H_{R13}\\H_{R21}&H_{R22}&H_{R23}\\H_{R31}&H_{R32}&H_{R33}\end{bmatrix};\ \ \bar{H}_B=\begin{bmatrix}H_a&0&0\\0&H_b&0\\0&0&0\end{bmatrix} \tag{2.12}$$

In Eq. (2.12), H_a and H_b is the supportive parameter of ACABD and magnetic bearing. According to Eq. (2.9), stiffness and damping matrix is consist of the coefficient of stiffness and damping, then

$$H_a = \begin{bmatrix} K_{xx1} + i\Omega C_{xx1} & 0 \\ 0 & K_{yy1} + i\Omega C_{yy1} \end{bmatrix} \qquad (2.13)$$
$$H_b = \begin{bmatrix} K_{xx2} + i\Omega C_{xx2} & 0 \\ 0 & K_{yy2} + i\Omega C_{yy2} \end{bmatrix}$$

Taking Eq. (2.12) into Eq. (2.11),

$$\begin{bmatrix} H_{R11} & H_{R12} & H_{R13} \\ H_{R21} & H_{R22} & H_{R23} \\ H_{R31} & H_{R32} & H_{R33} \end{bmatrix} \begin{Bmatrix} Z_A \\ Z_B \\ Z_o \end{Bmatrix} + \begin{bmatrix} H_a & 0 & 0 \\ 0 & H_b & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} Z_A \\ Z_B \\ Z_o \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \bar{F}_u \end{Bmatrix} \qquad (2.14)$$

According to Eq. (2.14):

$$Z_o = H_{R33}^{-1}\left[\bar{F}_u - H_{R31}Z_A - H_{R32}Z_B\right] \qquad (2.15)$$

$$H_{R11}Z_A + H_{R12}Z_B + H_{R13}Z_o = -H_aZ_A \qquad (2.16)$$
$$H_{R21}Z_A + H_{R22}Z_B + H_{R23}Z_o = -H_bZ_B$$

For simplification, the equivalent force at bearings f_A and f_B can be expressed as:

$$f_A = -(H_{R11}Z_A + H_{R12}Z_B + H_{R13}Z_o) \qquad (2.17)$$
$$f_B = -(H_{R21}Z_A + H_{R22}Z_B + H_{R23}Z_o)$$

The stiffness and damping matrix of the rotor can be represented by equivalent force as:

$$H_aZ_A = f_A \qquad (2.18)$$
$$H_bZ_B = f_B$$

Combining with Eq. (2.18), the stiffness and damping matrix of ACABD can be deduced:

$$H_a = \begin{bmatrix} K_{xx1} + i\Omega C_{xx1} & 0 \\ 0 & K_{yy1} + i\Omega C_{yy1} \end{bmatrix} = f_A \bullet Z_A^{-1} \qquad (2.19)$$

3. *Simulation study of parameters identification of ACABD*

In order to verify the vitality of the finite element parameters identification and the identification accuracy, the simulation study is needed for the identification method. First, the accurate model of the rotor was established. Then add the initial supportive stiffness and damping at ACABD and magnetic bearing in the simulation program. In order to obtain the unbalanced excitation force, an unbalanced mass of 1.5g was added into the rotor model. And the balanced responses of 10Hz~500Hz of the rotor were recorded. And the identified supportive parameters were obtained by the identification method of ACABD based on genetic algorithm. In the end, compare the identification results to the initial value. The specific process of simulation is shown in Fig. 2. Tab. 1 shows the initial value of the coefficient of stiffness and damping of ACABD.

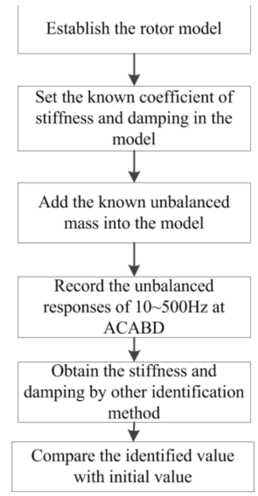

Fig. 2 Simulation analysis process of stiffness and damping identification

Tab. 1 Default value of Supportive stiffness and damping

Stiffness	Default value (N/m)	Damping	Default value (N s/m)
K_{xx1}	2.0×10^7	C_{xx1}	2000
K_{yy1}	3.0×10^7	C_{yy1}	3000

The unbalanced responses are recorded every 10Hz in simulation. Fig.3 shows the calculated theoretical unbalanced responses in x and y direction at ACABD under the excitation of external unbalanced mass. Fig. 4 and Fig. 5 show the supportive parameters of ACABD at 10Hz~500Hz identified by the identification method based on finite element method proposed above. The green points in the figure represent the stiffness and damping values identified in different speed. Red line represents the default value of stiffness and damping.

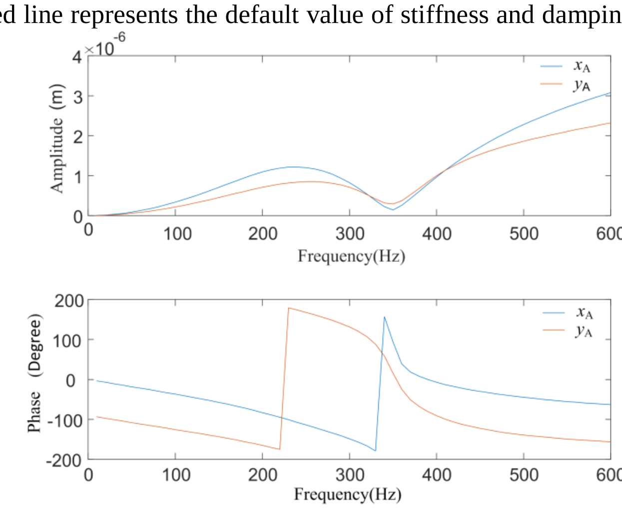

Fig. 3 Unbalanced responses at ACABD

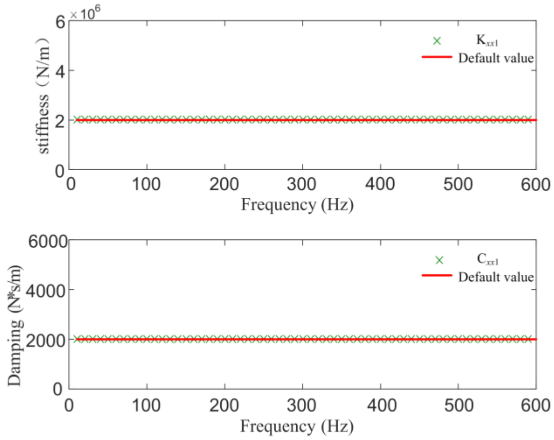

Fig. 4 The simulation values and default values of ACABD in *x* direction

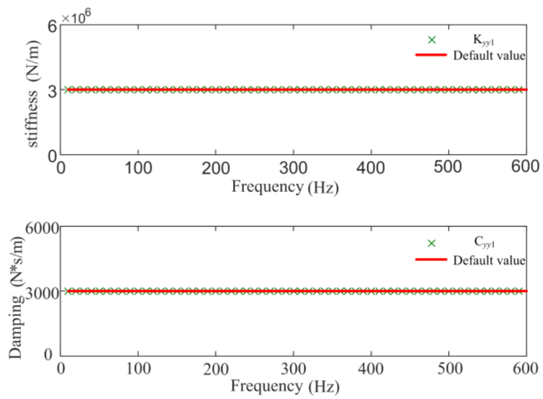

Fig. 5 The simulation values and default values of ACABD in y direction

It can be indicated form the figures that ignoring the effect of residual unbalance mass, the simulation values of ACABD is coincide with that of default values in the range of considered speed. It can be indicated form the results of simulation that the supportive parameters identification method based on finite element method is suit for the model in this paper.

4. Simulation study study on supportive characteristics experimental identification of ACABD

4.1 Unbalanced excitation experiment

In order to further verify the validity of the supportive parameters identification method based on finite element method, the experiment bench of five degree of freedom magnetic bearings system that contain ACABD, as shown in Fig. 6. In the bench, the NI signal acquisition card is responsible for collecting the displacement signals after the rotor drops. Crio-9215 inverter is responsible for driving the rotor in the magnetic suspension bearing system to rotate the rotor at a high speed. The photoelectric sensor is responsible for collecting the rotational speed changes after the fall of the rotor. The PC is responsible for observing the rotation of the falling of the rotor and saving the data. This paper adopts the testing method of supporting parameter based on unbalanced response method. The experimental test system can achieve the measurement and identification experiment by various kinds of sensors, acquisition cards and PC, the suspension of magnetic suspension bearing and high speed rotation experiment with DSP chip as the control chip, and the high-speed drop experiment of the rotor achieved by the single of power amplifier of the magnetic bearing controlled by the single chip.

The unbalanced mass is imposed on the end of the rotor and the rotor is suspended by the support of magnetic bearing. The rotor is drove by the inverter to the speed of 500Hz. Then drop one end of the rotor on the ACABD by the control of the rotor. Cut down the motor and collect every kind of single of the slowing down process with ACABD. Since the unbalanced excitation is very small with low speed, only the data in the range of 500Hz~20Hz were remained in the experiment.

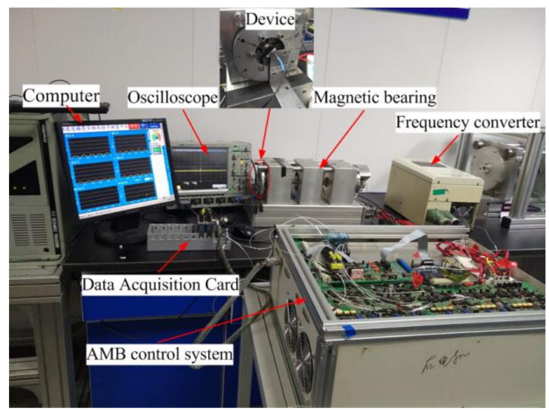

Fig. 6 The experimental test system of magnetic suspension bearing system

4.1 Data process of unbalanced excitation experiment

There are displacement sensors in the experimental bench, which can obtain the amplitude data of displacement vibration. But the phase information of the displacement data cannot be obtained directly. The phase data of the unbalanced response is the phase difference between displacement response and the unbalanced vibration force. Therefore, the data of response collected by the signal acquisition card needs to be processed. The processing procedure is shown in Fig. 7.

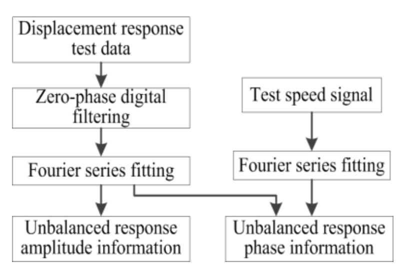

Fig. 7 The processing procedure of unbalanced displacement experiment data

In the unbalanced displacement experiment, there is a lot of high-frequency noise inevitably. In order to decrease the interference of high-frequency, the band-pass filter need to be carried out to the displacement responses data of the experiment. Conventional filtering method will lead to phase lag, phase information, however, is vital to stiffness and damping identification. In order to avoid the phase lag caused by filtering, this paper use zero-phase filter [18] to process the single in the time phase collected.

The filtered single can be fitted to the first order flourier series by least square method and can be represented as:

$$y = a_0 + a_1 \cos(\omega x) + b_1 \sin(\omega x) \qquad (3.1)$$

The amplitude of unbalanced displacement A and phase φ:

$$\begin{cases} A = \sqrt{a_1^2 + b_1^2} \\ \varphi = arc\tan(\dfrac{a_1}{b_1}) \end{cases} \qquad (3.2)$$

The phase of rotational speed φ_r can be obtained by the same process method of the single of rotational speed. At this time, the phase of unbalanced responses can be expressed as:

$$\varphi = \varphi_a - \varphi_r \qquad (3.3)$$

4.3 Experimental identification

The unbalanced responses of the rotor in the range of 500Hz~20Hz at ACABD can be obtained by data process, as shown in Fig. 8. It can be indicated that the unbalanced responses changes continually with the rotational speed. There is relatively large vibration on the rotor at about 120Hz, which is caused by the fact that the frequency is close to the first order rigid modal frequency.

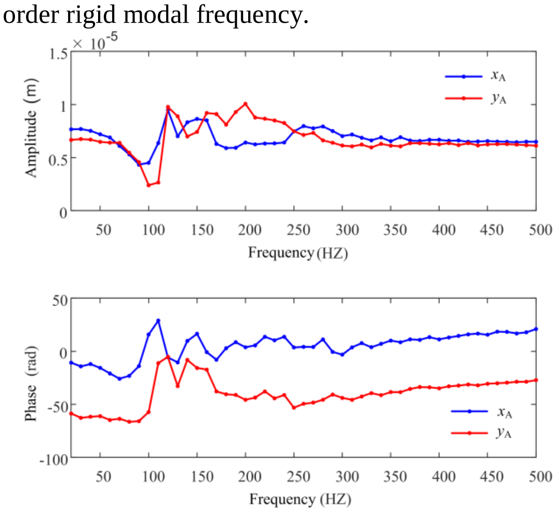

Fig. 8 Unbalanced responses at ACABD in the experiment

The amplitude of unbalanced responses were processed by the supportive parameters identification method of ACABD based on finite element method, obtaining the supportive stiffness and damping of ACABD, as shown in Fig. 9. It can be indicated from the figure that the stiffness and damping of ACABD changes with the speed of the rotor. For x and y direction of ACABD, because of the symmetry of the structure and the fact that the rotor is rotating at the center of ACABD, the supportive parameters in the two direction have the same trend. Since the rotational speed has not reached the first-order bending critical speed of the rotor, the general trend of supportive stiffness and damping is increased as the rotor speed increases.

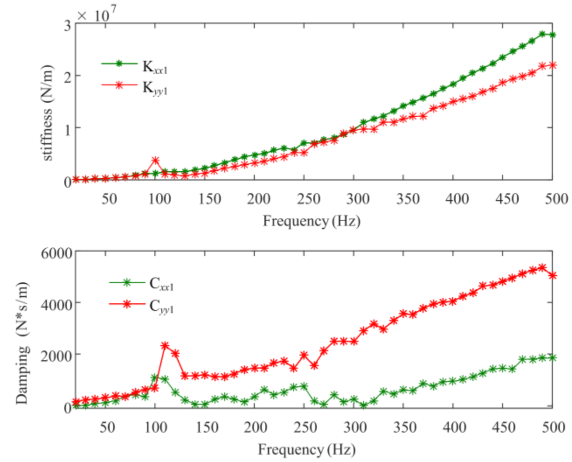

Fig. 9 Stiffness and damping identification results of ACABD in the experiment

5. *Conclusions*

1) This paper took the ACABD in the magnetic suspension bearing system as research object and proposed the supportive parameters identification method based on finite element method.
2) Established the identification model of ACABD and verified that the identification method is suit for the model.
3) Designed the experimental bench of magnetic suspension bearing system that contains ACABD and tested the identification method through the experiment, which obtained the supportive stiffness and damping of the ACABD.
4) The dynamic supportive parameters were obtained, which provides the theoretical basis to the structural design and optimization of ACABD.

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