Study on the Nonlinear Suspension Characteristic of Magnetic Suspended Vibration Isolator

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***Abstract*—** The definition of stiffness and damping for magnetic suspended vibration isolator was proposed in this paper. The coupling relationship between system controller parameters, mechanical structure parameters and dynamic stiffness is revealed by studying the nonlinear stiffness and damping of magnetic suspension vibration isolator. The frequency domain characteristics of nonlinear stiffness of magnetic suspended vibration isolator are studied combined PID control law. The results show that the system has high support stiffness in low frequency and high frequency bands. The amplitude-frequency characteristics of lower stiffness in middle frequency band are bathtub-shaped. The dynamic stiffness of the system can flexibly designed by changing the controller parameters in the stable region of the system.

Keywords: magnetic suspended vibration isolator, nonlinear stiffness, damping, PID control

A. Introduction

Isolation technology is one of the key technologies to deduce submarine noise and improve submarine stealth capability. There are two kinds way of isolation, which are active isolation and passive isolation. Passive vibration isolation components were added during vibration transmission to reduce the vibration transmitted to base. The structure of passive isolation system was simple and easy to realize, however, the ability of low frequency interference and vibration isolation is poor, even the interference signal near the resonance frequency of the system was magnified. Active vibration isolation adjusts the bearing characteristic parameters of the system dynamically according to the established control law to solve the problem of noise reduction in the vicinity of low frequency and resonance frequency[1-3].

The magnetic suspended vibration isolator is a form of active vibration isolator, which uses controllable electromagnetic force to suspend the supported object. The supporting parameters of magnetic suspended isolator were adjustable during the course of operation, which depends not only on the structural parameters of the magnetic vibration isolator, also depends on its control system. Compared to other active vibration isolator, magnetic suspended isolator had many advantages, such as wide frequency response range, long life, fast response and no contact, which was an ideal active vibration isolator[4-6].

Adjustable supporting stiffness damping control is an important feature of magnetic suspended vibration isolator, and is closely related to the performance of vibration isolation system. However, the conventional magnetic suspended bearing model is based on displacement control model, and the base motion was largely ignored on the influence of the magnetic suspension bearing characteristics. As for the active vibration isolation system included magnetic suspended isolator, adjustable supporting stiffness and damping was its important features. And the magnetic vibration isolator installed base raft in the motion state. Stiffness and damping of magnetic suspended isolator based on stiffness control was deduced in this paper. The operating condition of magnetic suspended system was analyzed and mathematic model was constructed. The mathematic model of base motion was incorporated into the theoretical model and simulation model of the magnetic suspended isolator. The stiffness and damping of the whole vibration-isolating system are varied by the way of adjusting the stiffness and damping of the magnetic suspension isolator. The law of the variable stiffness and damping of magnetic suspension isolator, the mass and rotation inertia of the raft and base dimensions' influences on the transferring power flow of the vibration isolation system are obtained. The basis for the optimizing design of the magnetic vibration isolation system and magnetic suspension control system is provided.

B. The Stiffness and Dampingof Magnetic Suspended Vibration Isolator

The structure chart of magnetic suspended vibration isolator was shown in Figure1.The system consists of magnetic suspension bracket 1, E magnet 2, displacement sensor 3, armature4, thrust plate component 5 and coil 6 so on. The magnetic suspension bracket is fixed on the base, armature and thrust plate component fixed on the vibration isolation object, the gap between the magnet and the armature was measured by displacement sensor 3.

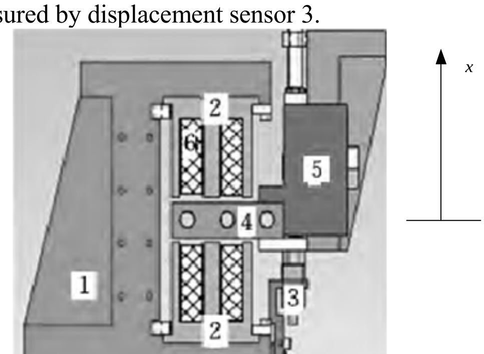


Figure1. The structure chart of magnetic suspended vibration isolator

Different from passive vibration isolator, the magnetic suspended vibration isolator uses controllable electromagnetic force to suspend the supported object. The supporting parameters of magnetic suspended isolator were adjustable during the course of operation, which depends not only on the structural parameters of the magnetic vibration isolator, also depends on its control system. So how to define the stiffness damping is the basis of the magnetic suspended vibration isolator. There is no clear statement of the stiffness of the magnetic suspended isolator from existing literatures, the stiffness of magnetic suspended vibration isolator was studied reference to the stiffness of magnetic suspend bearing[7-8].

There are two ways to define magnetic bearing stiffness. One kind definition is according to perspective of the change of the electromagnetic force to study the magnetic bearing characteristics. Its direction for a certain unit displacement stiffness along the direction of displacement of the electromagnetic force needed for the increment. The stiffness of the magnetic suspended isolator is defined as:

$$K_x = \frac{dF(i,x)}{dx} \tag{1}$$

The specific parameters of magnetic suspended vibration isolator were shown as table1.

Table1. Parameters of magnetic suspended vibration isolator

Coil cavity area (mm^2)	The magnetic poles area A (mm^2)	The coil number of turns$N$	The maximum current i_{max} (A)	The original gap X_0 (mm)
2700	9000	470	10	5

The electromagnetic force of the magnetic levitation vibrator is a function of the air gap and the coil current was shown as[1]:

$$F(i,x) = \frac{\mu_0 AN^2}{4}\left[\left(\frac{i_0-i}{x_0-x}\right)^2 - \left(\frac{i_0+i}{x_0+x}\right)^2\right] = k\left[\left(\frac{i_0-i}{x_0-x}\right)^2 - \left(\frac{i_0+i}{x_0+x}\right)^2\right] \tag{2}$$

Where μ_0 is the air permeability(H/m), N is the coil number of turns, A is the magnetic poles area, x_0 is the original gap, x is the air-gap variation, i_0 is the bias current and i is the coil current.

The relationship curve between electromagnetic force and armature displacement was drawn by matlab when the control current i = 1A, 2A, 3A, 4A, 5A, 6A respectively, which is shown as Figure2.

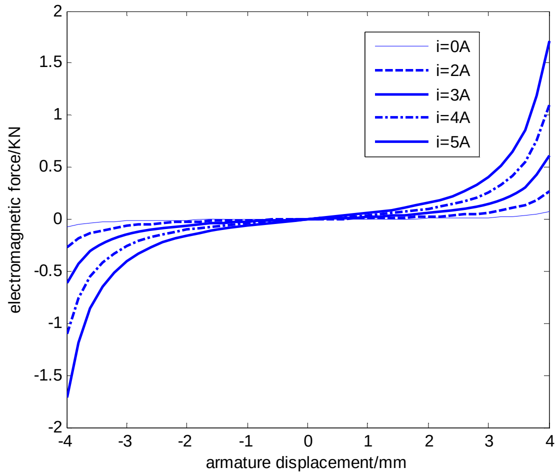


Figure2. The relationship curve between electromagnetic force and armature displacement.

However, the electromagnetic force equation (2) is given based on the three hypotheses: ① Uniform magnetic field distribution hypothesis; ② Core unsaturated hypothesis; ③ No magnetic flux leakage and magnetic hysteresis hypothesis. In addition, the effect of magnetic field coupling on magnetic force was ignored in equation (2). Therefore, there was error calculating the stiffness of magnetic suspended isolator by using equation (2).

According to the definition of stiffness equation (1), the equation (2) is introduced to the stiffness of the magnetic suspended vibration isolation shown as equation (3):

$$K_x = \frac{dF(i,x)}{dx} = \frac{\partial F(i,x)}{\partial i}\frac{di}{dx} + \frac{\partial F(i,x)}{\partial x} \tag{3}$$

The relation between the support stiffness of the magnetic suspended isolator and the control current of the isolator, the electromagnetic force of the vibration isolator and the displacement of the armature could be deduced from equation (3).The damping of the magnetic suspended isolator is defined as:

$$C_x = \frac{dF(i,x)}{d\dot{x}} = \frac{\partial F(i,x)}{\partial i}\frac{di}{d\dot{x}} + \frac{\partial F(i,x)}{\partial \dot{x}} \tag{4}$$

Electromagnetic actuator is the core of the magnetic suspension vibrator because of the adjustable stiffness and damping. The function relation between the support stiffness and the input current in different air gap is of great significance to realize the vibration active control of magnetic levitation vibration isolator.

$F(i,x)$ is the x-direction resultant force of magnetic suspended vibration isolator, which is the difference between the electromagnetic force generated by the upper electromagnet and the lower electromagnet on the armature. $F(i,x)$ is a nonlinear quadratic function about armature displacement x and control current i. It can be seen that magnetic suspended vibration isolator is a typical nonlinear system. For the convenience of research, it can be obtained by expanding the tate series near the equilibrium position ($x = 0$, $i = 0$) as equation(5):

$$F(i,x) = F(i_0,x_0) + \frac{\partial F(i_0,x_0)}{\partial i}(i-i_0) + \frac{\partial F(i_0,x_0)}{\partial x}(x-x_0) + \frac{1}{2!}\left[\frac{\partial^2 F(i_0,x_0)}{\partial i^2}(i-i_0)^2 + \frac{\partial^2 F(i_0,x_0)}{\partial i \partial x}(i-i_0)(x-x_0) + \frac{\partial^2 F(i_0,x_0)}{\partial x^2}(x-x_0)^2\right] + \cdots \tag{5}$$

When control current i and gap x changes in a small neighborhood near the equilibrium position, the error caused by its linear term could be ignored:

$$F(i,x) = F(i_0,x_0) + \frac{\partial F(i_0,x_0)}{\partial i}(i-i_0) + \frac{\partial F(i_0,x_0)}{\partial x}(x-x_0) \tag{6}$$

Obtaining partial derivative of equation (2):

$$\frac{\partial F(i,x)}{\partial x} = -2k\frac{i^2}{x^3}, \frac{\partial F(i,x)}{\partial i} = 2k\frac{i}{x^2} \tag{7}$$

Equation (7) was brought into equation (3) and equation (4) respectively:

$$K_x = -2k_i \bullet \frac{di}{dx} + 2k_x$$
$$C_x = -2k_i \bullet \frac{di}{d\dot{x}} \tag{8}$$

Where, $k_x = \dfrac{\mu_0 An^2 i_0^2}{2x_0^3}$, $k_i = \dfrac{\mu_0 An^2 i_0}{2x_0^2}$.

K_x in equation (8) is the stiffness of the magnetic suspended vibration isolator along *x*-direction and C_x is the damping of magnetic suspended isolator along *x* –direction. The stiffness and damping of magnetic suspended vibration isolator not only depend on the equilibrium position of structural parameters, but also depends on the control law of the coil current of the magnetic suspended vibration isolator.

In order to realize active vibration isolation of magnetic suspended vibration isolator, a control system must be introduced. The relationship among the stiffness of the magnetic suspended vibration isolator, the control parameters of the equilibrium position and the structural parameters of the isolator is the key of this paper, Therefore, the simplest PD control strategy was adopted:

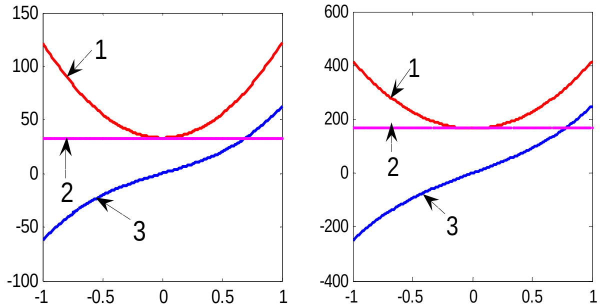
$$i = K_p x + K_d \dot{x} \tag{9}$$

K_p and K_d are the proportional coefficient and the differential coefficient of the PD controller respectively.

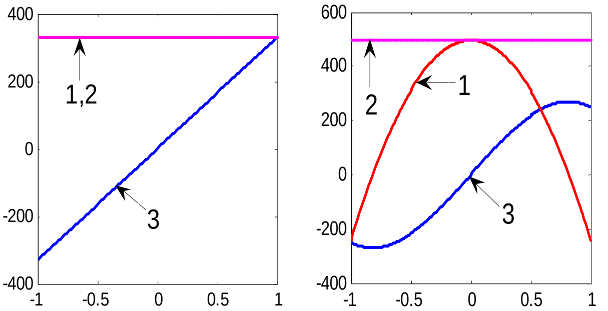
The focus of this paper is to study the stiffness and balance position control parameters of the magnetic suspension vibration isolator as well as the vibration isolator. Bring equation (9) into equation (8):

$$\begin{aligned} K_x &= -2k_i \cdot K_p + 2k_x \\ C_x &= -2k_i \cdot K_d \end{aligned} \tag{10}$$

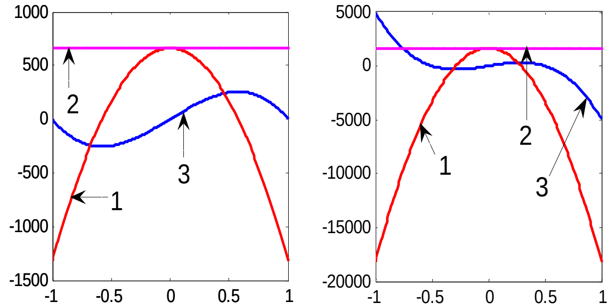
Equation (10) is the stiffness and damping of the magnetic suspended vibration isolator when PD control law was adopted. The parameters of the magnetic suspended vibration isolator are brought into equation (10), the relationship between linear stiffness, nonlinear stiffness and electromagnetic force was shown in figure 3.



(*a*) K_p=1.1 (*b*) K_p=1.5



(*c*) K_p=2.0 (*d*) K_p=2.5



(*e*) K_p=3.0 (*f*) K_p=6.0

Figure3. Variation law of electromagnetic force and stiffness with different K_p.

In Figure3, the abscissa is normalization displacement of armature, the ordinate is the stiffness value of the magnetic suspended vibration isolator under different displacements. It can be seen that stiffness of magnetic suspended isolator varied significantly with K_p. When K_p=1.1 and K_p=1.5, the electromagnetic force reach its maximum when | x|=1, in the same position, nonlinear stiffness reach its maximum. In Figure3(*a*), the minimum of nonlinear stiffness is 33.1 when x=0. In Figure3(*b*), the minimum of nonlinear stiffness is 4.03 when x=0. The relationship between electromagnetic force and displacement is linear when K_p=2, that is to say, the whole system is linear when $k_p=2i_0/x_0$, in this condition, stiffness is 331.1. In Figure3(*d*), When K_p=2.5, the electromagnetic force reach its maximum when | x|=0.816, the stiffness is 0 . And the stiffness becomes negative when the displacement of armature continues to increase. When K_p=3, the electromagnetic force reach its maximum when | x|=0.577, the minimum of nonlinear stiffness is 662. 1. When K_p=6, the electromagnetic force reach its maximum when | x|=0.289, the minimum of nonlinear stiffness is 1655.3.

The selection of controller parameters is greatly influenced by non-linear factors when the mechanical structure parameters were certain. The greater the proportional coefficient, the greater the stiffness of magnetic suspended vibration isolator at the equilibrium position , so do the greater the maximum support force that can be provided, however, the stable displacement interval of the system is smaller.

C. *Frequency domain characteristics of nonlinear stiffness of magnetic suspended vibratio isolator*

The stiffness characteristics of magnetic suspended vibration isolator are not only related to structural parameters, But also closely related to the frequency characteristics of the controller. The relationship between controller parameters and nonlinear stiffness was analyzed, which is helpful to study the stiffness characteristics of magnetic suspended vibration isolator. The control system block diagram of the magnetic suspended vibration isolator is shown in Figure 4[9-11].

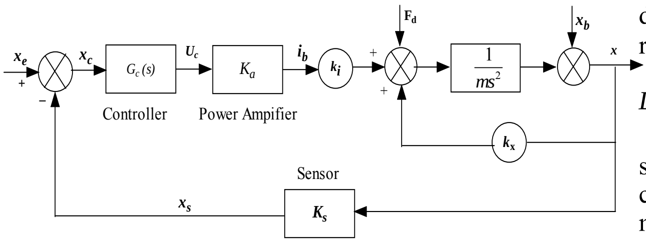


Figure4. The control system block diagram of the magnetic suspended vibration isolator

K_s and K_a are the gain of sensor and power amplifier respectively. The dynamic stiffness of magnetic suspended vibration isolator was derived from the control system block diagram. Taking the disturbing force F_d (s) as the input and armature displacement x(s) as the output , the transfer function could be deduced as :

$$\frac{x(s)}{F_d(s)}=\frac{1}{ms^2+k_iK_aK_sG_c(s)-k_x} \tag{11}$$

The stiffness is the external force required by the unit displacement of the armature, which is the reciprocal of equation (11):

$$k_d(s)=ms^2+k_iK_aK_sG_c(s)-k_x \tag{12}$$

The support stiffness is not a fixed value, which is the function of frequency ω. PID controller with low-pass filter was adopted:

$$G_c(s)=K_p(1+\frac{1}{T_is}+\frac{T_ds}{1+\tau s}) \tag{13}$$

Make $s=j\omega$, approximate expressions of dynamic stiffness in different frequency bands could be obtained as equation(14).

$$|K_d(j\omega)|\approx\begin{cases}K_sK_ak_iK_p/T_i\omega & \omega<1/T_i\\ K_sK_ak_iK_p & 1/T_i\le\omega\le 1/T_d\\ K_sK_ak_iK_pT_d\omega & 1/T_d\le\omega\le\omega_c\\ m\omega^2-k_x & \omega>\omega_c\end{cases} \tag{14}$$

Where, $\omega_c=K_sK_ak_iK_pT_d/m$.

The stiffness amplitude-frequency characteristic and asymptote of the system under PID controller was shown in Figure 5.

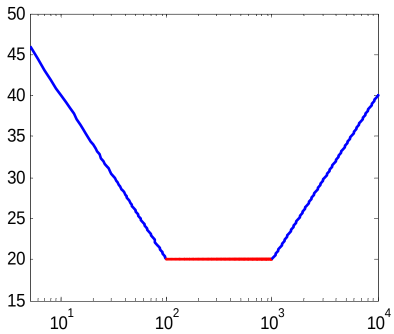


Figure5. Asymptote of stiffness amplitude-frequency characteristic of magnetic suspended vibration isolator

It could be seen that the system has high support stiffness in low frequency and high frequency bands, which looks like a bathtub. The amplitude-frequency characteristics of lower stiffness in middle frequency band are bathtub-shaped. The dynamic stiffness of the system can flexibly designed by changing the controller parameters in the stable region of the system.

D. Discussion and Conclusion

The definition of stiffness and damping for magnetic suspended vibration isolator was proposed in this paper. The coupling relationship between system controller parameters, mechanical structure parameters and dynamic stiffness is revealed by studying the nonlinear stiffness and damping of magnetic suspension vibration isolator. The frequency domain characteristics of nonlinear stiffness of maglev vibration isolator are studied combined PID control law. The results show that the system has high support stiffness in low frequency and high frequency bands. The amplitude-frequency characteristics of lower stiffness in middle frequency band are bathtub-shaped. The dynamic stiffness of the system can flexibly designed by changing the controller parameters in the stable region of the system.

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