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Precision positioning oriented modeling of electromagnetic actuator

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Abstract

This article presents a novel approach for accurate modeling of the relation between force, electric current and the actual air gap in the electromagnetic actuator used in active magnetic bearings (AMB) based precision positioning systems. The novelty of the presented modeling approach lies in a newly derived correlation for the relative permeability term in the electromagnetic actuator force model. This correlation, which can be easily obtained using the B-H curve for a given low-hysteresis actuator core material, describes the relative permeability term by means of electric current, actual air gap and a few parameters of the actuator. The resulting actuator model, which can be used either for more correct implementation of the well-known feedback-linearization technique or for obtaining more reliable results in simulations, was validated experimentally using a real single-degree-of-freedom (S-DoF) AMB plant.

Key words : Electromagnetic actuator, Modeling, Relative permeability, Precision positioning, B-H curve.

1. Introduction

Advantages of AMB technology continue to motivate research in this field for more than thirty years. Among these advantages are contactless actuation, which prevents mechanical wear and need of lubrication, and the ability to modify actively (by means of feedback control) system properties, such as stiffness and damping (Schweitzer and Maslen, 2009). Although the main application area of AMB is rotor machinery, this technology was also shown to be worthwhile in precise positioning related applications (Gutierrez and Ro, 2005; Comeaga and Alionte, 2010; Basovich et al., 2016). In particular, due to relatively large stroke and non-contaminating operation (Peijnenburg et al., 2006), magnetically levitated positioning systems can provide an excellent alternative to other solutions, such as piezoelectric actuators and air bearings.

The knowledge about AMB, which has been accumulated mainly in a context of rotor machinery, in general, can be applied to a filed of precise positioning. However, due to certain differences between these application areas, the subject of precise positioning based on AMB technology may require additional research efforts. In particular, among the aspects demanding a deeper investigation is the mathematical model of the AMB actuator.

Since in most cases AMB systems are operated by the model based feedback control, availability of a proper AMB model is crucial for achieving desired performance (Myburgh et al., 2010). In general, one can list three approaches for AMB modeling. The first approach assumes that the dependence of the AMB force in the air gap and the coil current is linear (Vischer and Bleuler, 1993). Although this approach can greatly facilitate the control system design, the linear representation of the force AMB is valid only in a close vicinity of a certain operation point. The second approach assumes that the AMB force is proportional to the square of the ratio between the coil current and the air gap (Grochmal and Lynch, 2007). This approaches is more precise than the first one, but it still provides a distorted description of AMB dynamics for relatively small air gap values, and that is due to the neglected relative permeability term. The third approach is also based on a nonlinear relation between the AMB force, the air gap and the coil current. However, in contrast to the second approach, it does not neglect the relative permeability (Li et al., 2003), which is usually assumed to be a constant. This approach provides a reasonable description of the AMB, and it is considered as the most precise approach for control purposes. It should be noted that there are other approaches for AMB modeling, such as experimental calibration (Chen and Knospe, 2005), or finite elements based method (Autila et al., 1998). However, since these approaches are not derived directly from a certain physical phenomena, they are left behind the scope of this article.

The vast majority of the available AMB models are oriented towards rotor machinery, where the main objective is to provide a contactless operation by keeping a rotating axis near the nominal working point, which is usually the geometrical center of the bearing. However, precise positioning involves tracking the reference position over the entire travel range, and not only in the vicinity of the nominal working point. In this regard, although the existing AMB models are sufficiently accurate for rotor machinery applications, they still don't provide a precision level required for precise positioning. In particular, the precision level of an AMB model can be deteriorated by neglecting or by improper evaluation of the relative permeability term.

This paper addresses the problem of accurate modeling of an AMB, which is intended to increase the tracking precision in positioning oriented applications. In order to meet this objective, within this study the precision level of a common AMB model is improved by a newly developed method for on-line evaluation of the relative permeability term. This method is based on a correlation between the relative permeability term, the electric current, the actual air gap and the standard AMB coefficients (such as the magnetic flux loop length and the number of coil turns). This correlation can be obtained using the B-H curve data for a given core material, that its hysteresis level is low. The resulting actuator model was validated experimentally using a single-degree-of-freedom (S-DoF) AMB levitating positioning system.

The remaining part of the paper is organized as follows. The method for evaluation of the relative permeability term is derived and compared to the existing methods in Section 3. The experimental validation of the model obtained in Section 3 is presented in Section 4. Section 5 concludes the paper.

2. Nomenclature

A	Cross section area of a single magnetic loop
B	Magnetic flux density
B_1	Magnetic flux density in the upper electromagnet
F	Electromagnetic force
H	Magnetic field
I	Electric current
N	Number of coil windings
a_0, a_1, a_2	Constant correlation coefficients
c	Electromagnetic coefficient
g	Gravity acceleration
i_1	Electric current in the upper electromagnet
i_2	Electric current in the lower electromagnet
l	Actual air gap length
l_0	Nominal air gap length
l_{Fe}	Magnetic loop length
m	Floating core equivalent inertia
u	Feedback component of the total control signal
u^*	Total control signal
x	Displacement of the floating core
μ_0	Permeability of vacuum
μ_r	Relative permeability of the medium (iron core)
$\bar{\mu}_r$	Constant estimate of relative permeability of the medium (iron core)

3. Electromagnetic actuator modeling

3.1. Equation of motion

Consider an S-DoF AMB positioning system (Fig. 1), which consists of a floating core made of ferromagnetic material and two static electromagnets. Each electromagnet consists of a coil wound around its ferromagnetic core. When electrical current is passed through the coil, magnetic field is produced and an attractive force is exerted on the floating core. Although the common model of the attractive force is well known (for instance, see (Schweitzer and Maslen, 2009)), for convenience we present here the main milestones of its development.

The constitutive law relates the magnetic field H and the magnetic induction (flux density) B as

$$B = \mu_0 \mu_r H \quad (1)$$

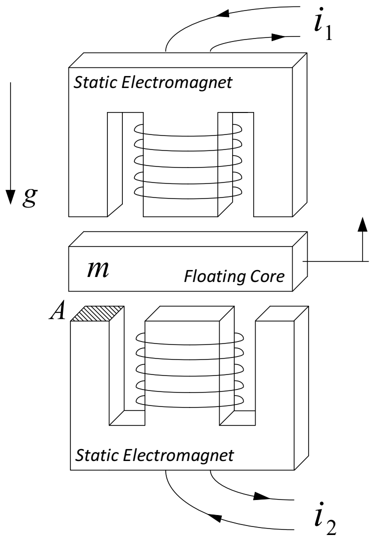


Fig. 1: S-DoF AMB positioning system scheme

where $\mu_0 = 4\pi \times 10^{-7}[Vs/Am]$ is the magnetic permeability of vacuum, and μ_r is the relative permeability of the medium (iron core in this study). Applying the above relation, in couple with Ampèr's circuital law, to the single electromagnet in Fig.1, one can obtain the following expression for the magnetic induction

$$B = \mu_0 \frac{NI}{\frac{l_{Fe}}{\mu_r} + 2l} \tag{2}$$

In (2) N is the number of coil turns, I is the electrical current passing through the coil, l_{Fe} is the magnetic loop length, and l is the air gap between the floating core and the core of the electromagnet. In the context of an AMB used for rotating axles, where most of the time the rotor is kept at the vicinity of the geometrical center of the bearing, it is very common (and reasonable) to neglect the l_{Fe}/μ_r term in (2), see (Schweitzer and Maslen, 2009). However, in a case of positioning systems based on AMB, the $2l$ and l_{Fe}/μ_r terms in (2) can be of the same order of magnitude, therefore neglecting the last term can lead to inexact AMB model which, in turn, can lead to undesirable performance.

The expression for the force applied by a single electromagnet in Fig.1 is obtained by differentiating the magnetic field energy with respect to the air gap. The result is given by

$$F = \frac{2AB^2}{\mu_0} \tag{3}$$

where A (see Fig.1) is the cross section area of a single magnetic loop (note that the total magnetic loop cross section for the entire electromagnet is given by $2A$). Substitution of (2) into (3) yields

$$F = c\frac{I^2}{\left(\frac{l_{Fe}}{2\mu_r} + l\right)^2} \tag{4}$$

where the constant term c is given by

$$c = \frac{1}{2}\mu_0 N^2 A \tag{5}$$

It is common to define a nominal air gap for both upper and lower electromagnets in Fig.1, which is denoted here by l_0. Thus, using (4)-(5), and the definition of l_0, one can obtain the dynamic model of the S-DoF AMB positioning system in Fig.1 through the Newton's second law. The result is given by

$$m\ddot{x} = c\frac{i_1^2}{\left(\frac{l_{Fe}}{2\mu_r} + l_0 - x\right)^2} - c\frac{i_2^2}{\left(\frac{l_{Fe}}{2\mu_r} + l_0 + x\right)^2} - mg \tag{6}$$

where x is the displacement of the floating core with respect to the nominal position (position where both air gaps equal to l_0), i_1 and i_2 are the control currents in the upper and the lower electromagnets respectively (see Fig.1), m is the inertia of the floating core, and g is the acceleration of gravity.

3.2. Influence of the relative permeability estimation on the closed loop performance

In order to assure desired performance of a real plant, the model of the AMB should be as accurate as possible. In addition to reliable simulations, a precise model is also crucial for implementation of feedback linearization technique, which is very popular in the context of AMB control.

An accurate model of AMB relies on a proper evaluation of its parameters. Considering (6), one can observe that the value of m can be obtained by weighing, and such parameters as c, l_0 and l_{Fe} can be easily evaluated, since they all depend on the AMB geometry. However, evaluation of μ_r is less trivial; therefore, the influence of its estimate on the system performance is discussed below.

Let a controller for (6) be based on the following feedback linearization transformation

$$i_1 = \begin{cases} \sqrt{u^*/c}\left(\frac{l_{Fe}}{2\hat{\mu}_{r1}} + l_0 - x\right), & u^* \geq 0 \\ 0, & 0 \end{cases}, \qquad i_2 = \begin{cases} \sqrt{-u^*/c}\left(\frac{l_{Fe}}{2\hat{\mu}_{r2}} + l_0 + x\right), & u^* < 0 \\ 0, & 0 \end{cases} \tag{7}$$

such that

$$u^* = u + mg \tag{8}$$

In (8) u is the feedback control signal, and $\hat{\mu}_{r1}, \hat{\mu}_{r2}$ are estimates of μ_r for the upper and lower electromagnets respectively. Defining

$$\rho(\mu_{r1}, \mu_{r2}, \hat{\mu}_{r1}, \hat{\mu}_{r2}, x) = \begin{cases} \rho_1(\mu_{r1}, \hat{\mu}_{r1}, x), & u \geq 0 \\ \rho_2(\mu_{r2}, \hat{\mu}_{r2}, x), & u < 0 \end{cases} \tag{9}$$

where

$$\rho_1(\mu_{r1}, \hat{\mu}_{r1}, x) = \frac{\left(\frac{l_{Fe}}{2\hat{\mu}_{r1}} + l_0 - x\right)^2}{\left(\frac{l_{Fe}}{2\mu_r} + l_0 - x\right)^2}, \qquad \rho_2(\mu_{r2}, \hat{\mu}_{r2}, x) = \frac{\left(\frac{l_{Fe}}{2\hat{\mu}_{r2}} + l_0 + x\right)^2}{\left(\frac{l_{Fe}}{2\mu_r} + l_0 + x\right)^2} \tag{10}$$

and substituting (7)-(8) into (6), one can obtain

$$m\ddot{x} = \rho(\mu_{r1}, \mu_{r2}, \hat{\mu}_{r1}, \hat{\mu}_{r2}, x)u + mg(\rho_1(\mu_{r1}, \hat{\mu}_{r1}, x) - 1) \tag{11}$$

As it can be observed in (10)-(11), for small values of $l_0 - x$ or $l_0 + x$ the $\rho(\mu_{r1}, \mu_{r2}, \hat{\mu}_{r1}, \hat{\mu}_{r2}, x)$ term deteriorates the feedback controller performance if the values of $\hat{\mu}_{r1}, \hat{\mu}_{r2}$ are not estimated properly. Moreover, even if the control signal u in (11) is computed by a controller with sufficiently large stability margins, inaccurate estimation of $\hat{\mu_r}$ will cause improper compensation of the gravity term.

3.3. Relative permeability modeling

3.3.1. Approach 1 Recalling (1) and assuming that during the operation of the AMB the magnetic force is never saturated, it is possible to estimate the value of μ_r through the slope of the B-H curve (Fig.2) over the (approximately) linear region (Fitzgerald, 1992). However, the B-H curve is nonlinear even over the so-called "linear" region, therefore, due to large differences between a real value of μ_r and its constant estimation $\bar{\mu}_r$ (see Fig.2), the above method is ineffective for the AMB positioning systems, which are designed to resist a constant disturbance. An example of such a system can be the plant in Fig.1, which counteracts the gravity term mg using a feedforward compensation of its value. In this case, as it was demonstrated in Subsection 3.2, for relatively small values of $l_0 - x$, the feedforward strongly depends on the accuracy of μ_r. For this reason, in the case described above, μ_r is better to be estimated using the following approach.

3.3.2. Approach 2 Observing (3) and (6), the following steady state equation can be obtained

$$\frac{2AB_1^2}{\mu_0} = mg \tag{12}$$

where B_1 is the flux density in the upper electromagnet. From (12) one can isolate B_1 and pick a corresponding value of H from the B-H curve. Then, given B_1 and H, the value of μ_r can be calculated using (1). Although, in a case of AMB used for levitation, the second method allows to obtain a more precise estimation of μ_r, it is limited to the set-point control task only. Also, the application of this method to a levitation system which is designed to carry uncertain payloads is not effective.

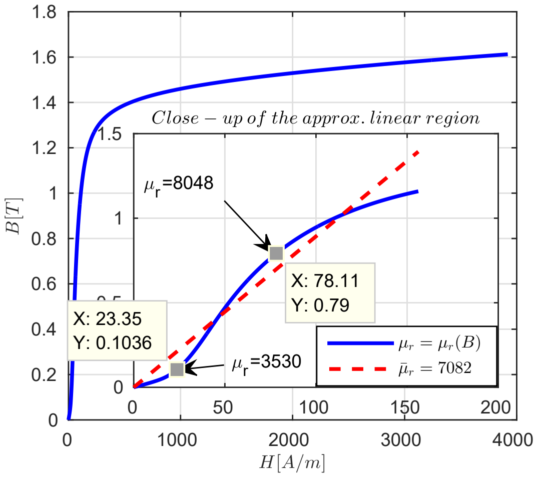


Fig. 2: B-H curve for M19 silicon steel

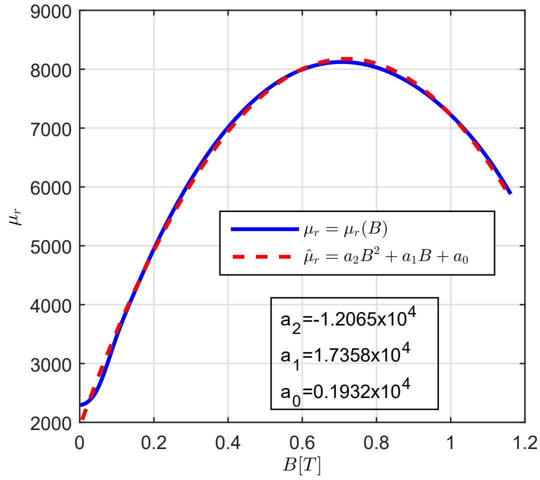


Fig. 3: Model of μ_r for the linear region of B-H curve

3.3.3. Proposed approach The drawbacks and restrictions of the above approaches are associated with the assumption of a constant μ_r. However, as one can see in Fig.2, μ_r is not a constant and its value depends on H (or B). Therefore, in order to achieve a better precision of μ_r within the unsaturated area of the B-H curve, we propose to use the following correlation

$$\hat{\mu}_r = a_2 B^2 + a_1 B + a_0 \tag{13}$$

The coefficients a_2, a_1, a_0 in (13) are obtained through least squares fitting (see Fig.3) of the B-H curve of silicon steel in Fig.2. The curve was extracted from a Finite Element Method Magnetics (FEMM) software (Meeker; Saraiva et al., 2008). Silicon steel is a common material used for AMB stators (Matsumura and Hatake, 1992; Mizuno and Higuchi, 1994), while one of its well known properties is its very small hysteresis. This fact allowed Gerami et al. to use an approximation of μ_r based on the B-H curve in (Gerami et al., 2014). However, since the main focus in (Gerami et al., 2014) was on the operation of AMB under conditions of magnetic saturation in order to achieve higher load capacity, and not on increasing the precision of a generic AMB model, their model of μ_r is linear in B, and it is valid only over a certain part within the saturated region of the B-H curve. The main strength of correlation (13) is that, for precisely known c, l_0 and l_{Fe}, it allows to obtain highly accurate mathematical description of the AMB, which can be used not only for achieving more reliable results in simulations, but also for an improved real time implementation of feedback linearization based control methods (Trumper et al., 2009).

In order to employ the proposed model of μ_r within a simulation, one can substitute (2) for B in (13) to obtain the following implicit expression

$$\hat{\mu}_r = a_2 \left(\frac{\mu_0 N I}{\frac{l_{Fe}}{\hat{\mu}_r} + 2l} \right)^2 + a_1 \frac{\mu_0 N I}{\frac{l_{Fe}}{\hat{\mu}_r} + 2l} + a_0 \tag{14}$$

The above expression can be either solved in real time numerically using the fixed-point iteration method, or be approximated (with small loss of precision) as follows

$$\hat{\mu}_r = a_2 \left(\frac{\mu_0 N I}{\frac{l_{Fe}}{\bar{\mu}_r} + 2l} \right)^2 + a_1 \frac{\mu_0 N I}{\frac{l_{Fe}}{\bar{\mu}_r} + 2l} + a_0 \tag{15}$$

In (15) $\bar{\mu}_r$ can be obtained using either Approaches 1 or 2 described earlier in this section. It should be noted that (15) exhibits low sensitivity to uncertainties in $\bar{\mu}_r$, and this claim is demonstrated experimentally (see Section 4).

Finally, in order to employ (15) for the feedback linearization control method, which is implemented by a digital computer in real time, all one has to do is to substitute the coil current and position measurements in I and x in (15), respectively. In a case when the coil current feedback is unavailable and the sampling rate of the discretized control algorithm is relatively high, I in (15) can be replaced by the current control signal of a previous sampling interval.

4. Experimental results

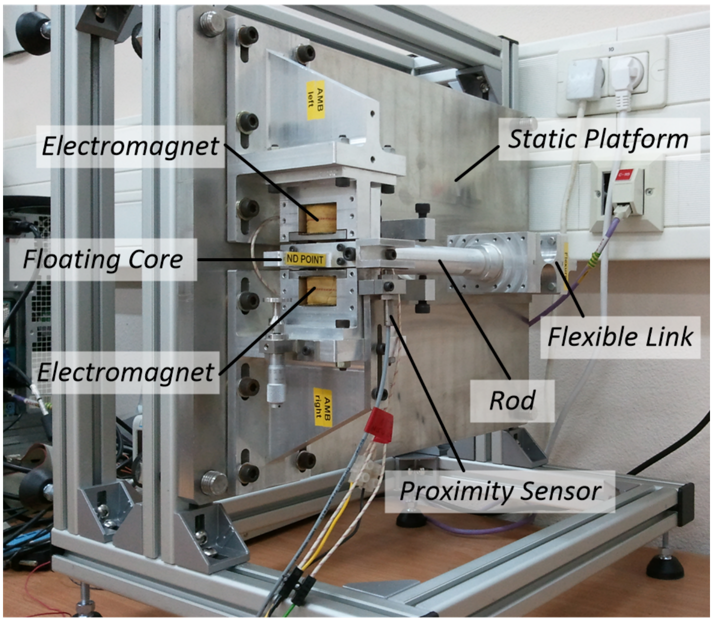
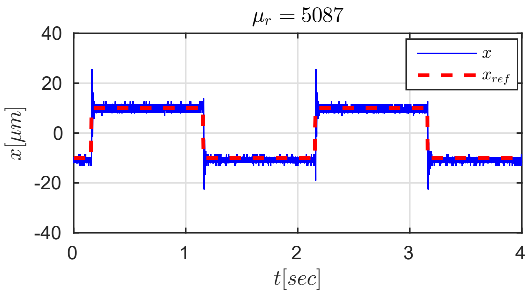
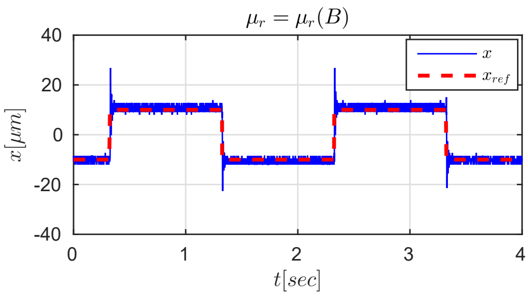


Fig. 4: S-DoF AMB test bed

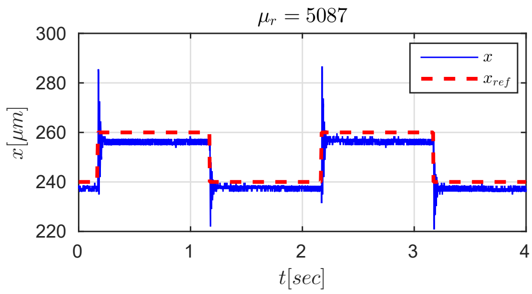


(a) μ_r = 5087

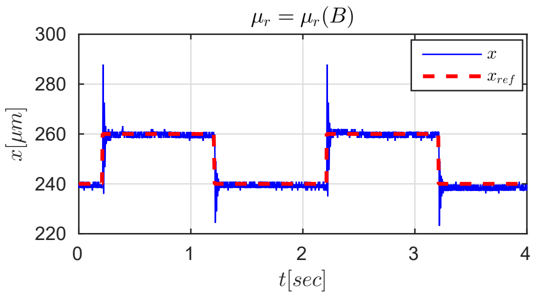


(b) $\mu_r = \mu_r(B)$

Fig. 5: Square wave tracking:
zero DC component,10[μm] amplitude



(a) μ_r = 5087



(b) $\mu_r = \mu_r(B)$

Fig. 6: Square wave tracking:
250[μm] DC component, 10[μm] amplitude

The improved AMB model was verified experimentally using the S-DoF AMB plant in Fig.4. The plant consists of a couple of electromagnets and a floating ferromagnetic core. The core is attached to the end of a long rod that its other end is connected to the static platform by a flexible link. The rod with the link restricts the floating core motion to a single (vertical) direction. At the same time (due to small displacements), the force produced by the flexible link is negligible with respect to the force due to gravity. The floating core displacement is measured by an eddy-current proximity sensor and these measurements are sampled by a data acquisition card. The sampled displacement of the floating core is used by the controller with sample time of $1 \times 10^{-4}[sec]$. The computed control signal is sent to two linear power amplifiers which produce control currents for the electromagnets.

Table 1: S-DoF AMB Parameters

Parameter	Meaning	Value	Units
m	Floating core equivalent mass	0.48	$[kg]$
N	Number of coil windings	275	$[-]$
A	Magnetic loop cross section	95×10^{-6}	$[m^2]$
l_{Fe}	Magnetic loop length	0.114	$[m]$
l_0	Nominal air gap	313×10^{-6}	$[m]$

In order to validate the AMB model proposed in this paper, the performance of the test bed in Fig.4 with the permeability estimation $\hat{\mu}_r$ (15), was compared to the performance with the perturbed constant value of the permeability $\bar{\mu}_r$. In particular, it was assumed that

$$\bar{\mu}_r = 5087 \tag{16}$$

while the constant estimate of μ_r computed from (12) for the plant parameters in Tab.1 is 4716. At the same time, the value of $\bar{\mu}_r$ in (16) was utilized to compute $\hat{\mu}_r$ for each electromagnet using (15).

The control law in the experiments is based on the feedback linearization transformation (7)-(8), while the control signal u in (8) is computed using a PD compensator. Experimental results that demonstrate tracking of a square wave reference with zero DC and 10$[\mu m]$ amplitude, for a case when the estimate of μ_r is given by (16) and for a case when μ_r is modeled by (15), are shown in Fig.5a and Fig.5b respectively. From Fig.5a and Fig.5b one can observe that there are no significant differences between (15) and (16) if the electromagnet air gap (i.e, the value of $l_0 - x$) is relatively large. However, when the DC component of the square wave is raised up to 250$[\mu m]$, a different result is achieved. In this case the air gap value is relatively small, and the differences between (15) and (16) are clear. From Fig.6a and Fig.6b it is obviuos that the perturbed estimation of μ_r (16) causes a steady state error. However, in a case when μ_r is modeled by (15), even with the perturbed value of $\bar{\mu}_r$ in (16), the tracking performance of the system remains almost unaffected. Thus, the performed experiments have shown that the model of μ_r proposed in this paper is effective and it can assist in achieving better performance of levitating precision positioning systems, based on AMB technology.

5. Conclusion

A novel approach for evaluation of relative permeability in AMB actuators was presented. The approach is based on a correlation between the relative permeability term, the electric current, the actual air gap and the AMB coefficients, which is obtained using the B-H curve data for a given core material. The presented modeling approach was validated by experiments using an S-DoF AMB based levitating positioning system.

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