Identification of Active Magnetic Bearing Systems Utilizing a Modulating Function Technique

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Abstract

An application of modulating function (MF) technique to the direct closed-loop identification for the estimation of a continuous-time model of multi-input-multi-output magnetic bearing system from sampled data are presented in this paper. The plant description given as a differential matrix polynomial is transformed into a matrix of difference equations using the modulating function approach, which allows to avoid numerical problems by aproximating time derivatives. Additionally, the initial conditions can be neglected as a characteristic of the MF technique. The system parameter estimation represents a solution of a least-squares problem derived from the difference system model with experimental data from an industrial magentic bearing system.

Key words : Active Magnetic Bearing, Closed-Loop Identification, Direct Identification, Modulating Function.

1. Introduction

Active magnetic bearings (AMB) for industrial applications are especially beneficial when a high rotational speed, an operation under special conditions (e.g. vacuum, gas, fluid) and the absence of lubrication are required. Moreover, the characteristics of the system (e.g. stiffness, damping) can be changed during its operation by a suitable control scheme. These advantages contribute to an application of magnetic bearings to such industrial products as high speed turbines, compressors, vacuum pumps, cryogenic circulators but also artificial hearts. However, instability and complex plant dynamics require thorough research and development, thus, increasing costs. This fact by others limits a wide industrial distribution of magnetic bearing systems to date.

The feedback control essential for a stable operation mode of an AMB is based on a sufficient knowledge of the system dynamics. This information can be available in form of an analytical model, extracted, for example, from finite element analysis or parametrized by an identification process. Magnetic bearing plants are unstable, complex, multi-input-multi-output (MIMO) mechatronic systems the accurate modelling of which is a demanding task. System identification provides a rewarding alternative solution in this case. Due to the open loop instabilities of the AMB, a closed-loop identification system identification is required.

Conventional methods to closed-loop identification can be classified into direct, indirect as well as joint input-output approaches and mainly use discrete-time (DT) system models (Isermann and Münchhof 2011), (Forssell 1999), (Ljung 1999), also with identification of rotor systems (Gähler and Herzog 1995), (Tiwari and Chougale 2014). Sun et al. (2002) suggested a two-stage algorithm for direct closed-loop identification of a magnetic suspension system by an inter-sampled model structure based on subspace methods. An initial SISO stucture of the identified system is transformed into a SIMO model due to intersampling. That proves the condition of an identification problem, but in the same time the computational effort increases. Besides, some authors focus on closed-loop identification concepts for continuous-time (CT) system models from sampled data (Garnier 2008). Identification of CT models was sidelined in the last years because of a rapid development of digital technologies and at the same time of DT identification. The control engineers still prefer to work with CT models by dynamical analysis of systems and control design because of their transparency (Rao and Unbehauen 2005). CT identification methods with application to AMB systems are proposed, for example, by Mohd-Mokhtar et al. (2004) and Aziz and Mohd-Mokhtar (2011). Both papers focus on the state-space model parameter estimation of an unstable MIMO plant. Mohd-Mokhtar used a Laguerre network and subspace methodology and Aziz and

Mohd-Mokhtar applied a two stage method, first identifying the order of the system followed by the identification of the corresponding parameters. Balini et al. (2010) use a predictor-based subspace identification algorithm for the estimation of a structure and parameters of an AMB system in closed-loop. The final CT model is obtained not immediately from a sampled data, but is subsequently reconstructed from a DT model. The problem of a correlation between the system input und the output noise is solved by an additional excitation signal.

The work presented in this article is focused on CT identification of a magnetic bearing system from sampled data and is based on the modulating function (MF) method (Shinbrot 1957). This method is suitable for identification of linear and nonlinear unstable systems in the time domain. Using this approach, the state-space model of single-input-single-output (SISO) and MIMO plants can be obtained (Pearson and Lee 1985), (Shen 1993), (Co and Ydstie 1990). The modulating function method, that is implemented to closed-loop identification of the magnetic bearing system in this work, is based on the procedure of Co and Ydstie (1990), which renders a partition of the problem in SISO or multi-input-single-output (MISO) subsystems unnecessary. The direct identification approach is illustrated and tested with an industrial AMB plant.

This paper is organized as follows: Section 2 illustrates the mathematical model of the magnetic bearing system with a flexible rotor. Section 3 gives a review of modulating function methodology and describes an implementation of this approach to the magnetic bearing system. The structure of the test bench and experimental results are addressed in Section 4. Finally, Section 5 concludes the paper.

2. AMB system model

The industrial magnetic bearing system available at the Technische Universität Ilmenau consists of the flexible shaft levitated by two electromagnetic actuators, which provide a support in five degrees of freedom (x, y, z, ϕ_x, ϕ_y). The rotor position is determined by the measurement of the displacements at five points on the shaft. Displacement sensors and actuators are collocated in the investigated plant. The stabilization of the system is realized by decentralized cascade control. A schematic of the system is shown in Fig. 1.
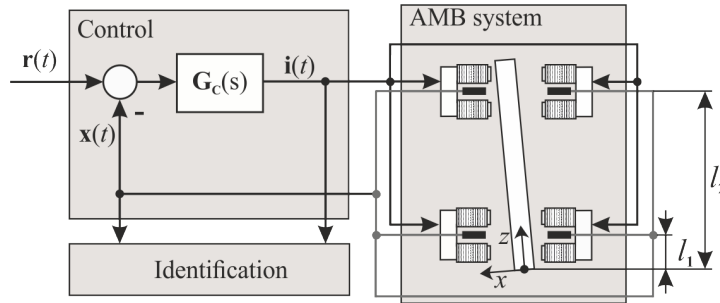

Fig. 1   Schematic of the magnetic bearing setup

Characteristics of electromagnetic actuators used in the setup allow a linearization of the system model around a working point. In this case, the longitudinal motion of the shaft (in z-direction) can be decoupled from transversal degrees of freedom (in x- and y-direction) and modeled separately, due to the small displacements and angles occurring within the operational range. Consider the equation of the rotor's radial motion in the time domain (Schweitzer and Maslen 2009)

$$[\mathbf{M}p^2 + (\mathbf{G} + \mathbf{D})p + \mathbf{K}]\mathbf{q}(t) = -\mathbf{K}_{sa}\mathbf{w}(t) + \mathbf{K}_{ia}\mathbf{i}(t), \quad \mathbf{G} = \mathbf{0} \tag{1}$$

$$\mathbf{w}(t) = \mathbf{C}\mathbf{q}(t), \tag{2}$$

where $p^i = d^i/dt^i$, $i = 0, \cdots, 2$ is a differential operator,

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_x & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_y \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} \mathbf{D}_x & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_y \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} \mathbf{K}_x & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_y \end{bmatrix}, \tag{3}$$

$$\mathbf{q}(t) = \begin{bmatrix} \mathbf{q}_x(t) & \mathbf{q}_y(t) \end{bmatrix}^T, \quad \mathbf{i}(t) = \begin{bmatrix} \mathbf{i}_x(t) & \mathbf{i}_y(t) \end{bmatrix}^T = \begin{bmatrix} i_{x1}(t) & i_{y1}(t) & i_{x2}(t) & i_{y2}(t) \end{bmatrix}^T, \tag{4}$$

$$\mathbf{w}(t) = \begin{bmatrix} \mathbf{w}_x(t) & \mathbf{w}_y(t) \end{bmatrix}^T = \begin{bmatrix} w_x(l_1, t) & w_x(l_2, t) & w_y(l_1, t) & w_y(l_2, t) \end{bmatrix}^T \tag{5}$$

are mass, damping and stiffness matrices, $\mathbf{q}(t)$ are modal coordinates, $\mathbf{i}(t)$ are actuator currents and $\mathbf{w}(t)$ are displacements of the shaft at points l_1 and l_2, where position sensors are placed.

Transformation of modal coordinates $\mathbf{q}(t)$ in the displacement coordinates $\mathbf{w}(t)$ is given through

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_x & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_y \end{bmatrix}. \tag{6}$$

Force-displacement factors and force-current factors of actuators are considered as follows

$$\mathbf{K}_{sa} = \mathrm{diag}[\mathbf{K}_{sax}, \mathbf{K}_{say}] = \mathrm{diag}[k_{sx1}, k_{sx2}, k_{sy1}, k_{sy2}], \tag{7}$$

$$\mathbf{K}_{ia} = \mathrm{diag}[\mathbf{K}_{iax}, \mathbf{K}_{iay}] = \mathrm{diag}[k_{ix1}, k_{ix2}, k_{iy1}, k_{iy2}] \tag{8}$$

It is assumed that the rotor is not spinning and the gyroscopic effects are omitted from the mathematical model (1). Thus, the $x-$ and $y-$ directions are decoupled (3). Furthermore the considered system is constructed rotationally symmetric. These assumptions and system properties allow an analysis and identification of one part of the AMB system, which describes the shaft motion, for example, in the $x - z$-plane, and a transfer of the obtained results to the $y - z$-plane. The equation of the rotor motion in $x - z$-plane can be extracted from Eq. (1) and transformed into a matrix polynomial model with two inputs $\mathbf{i}_x(t) = \begin{bmatrix} i_{x1}(t) & i_{x2}(t) \end{bmatrix}^\mathrm{T}$ and two outputs $\mathbf{w}_x(t) = \begin{bmatrix} w_x(l_1, t) & w_x(l_2, t) \end{bmatrix}^\mathrm{T}$

$$\underbrace{(p^2 + \mathbf{A}_1 p^1 + \mathbf{A}_2 p^0)}_{\mathbf{P}_{\mathrm{den}}} \mathbf{w}_x(t) = \underbrace{\mathbf{B}_2 p^0}_{\mathbf{P}_{\mathrm{num}}} \mathbf{i}_x(t) \tag{9}$$

wherein

$$\mathbf{A}_1 = \mathbf{C}_x \mathbf{M}_x^{-1} \mathbf{D}_x \mathbf{C}_x^{-1}, \quad \mathbf{A}_2 = \mathbf{C}_x \mathbf{M}_x^{-1}(\mathbf{K}_x \mathbf{C}_x^{-1} + \mathbf{K}_{sax}), \quad \mathbf{B}_2 = \mathbf{C}_x \mathbf{M}^{-1} \mathbf{K}_{iax} \tag{10}$$

are the parameter matrices with $\mathbf{A}_i = a_{ijk}$, $\mathbf{B}_i = b_{ijl}$, $i, j, k, l = 1, \cdots, 2$ to be identified.

The electric behavior of the actuators is described by the following relations

$$\mathbf{v}(t) = \mathbf{L} p^1 \mathbf{i}_x(t) + \mathbf{R} \mathbf{i}_x(t), \tag{11}$$

where $\mathbf{L} = \mathrm{diag}[L_1, L_2]$ are the inductivities and $\mathbf{R} = \mathrm{diag}[R_1, R_2]$ are the resistances of the coils.

The most intuitive control approach to stabilize an AMB is the decentralized control scheme, where each bearing axis has an individual controller (Schweitzer and Maslen 2009). The identical cascade controllers consisting of an inner current as well as an outer displacement control loop are implemented for every degree of freedom of magnetic bearing system. The outer control circuit is a PID control scheme

$$\mathbf{i}_{xref} = \mathbf{K}_p \mathbf{e}_x + \mathbf{K}_i \int \mathbf{e}_x dt + \mathbf{K}_d \dot{\mathbf{e}}_x, \tag{12}$$

$$\mathbf{e}_x = \mathbf{r} - \mathbf{w}_x(t), \quad \mathbf{r} = \begin{bmatrix} r_1(t) & r_2(t) \end{bmatrix}^\mathrm{T}, \quad \mathbf{K}_p = \mathrm{diag}[k_p, k_p], \quad \mathbf{K}_i = \mathrm{diag}[k_i, k_i], \quad \mathbf{K}_d = \mathrm{diag}[k_d, k_d] \tag{13}$$

The voltage for the actuators $\mathbf{v}$ is generated from the difference between the measured currents $\mathbf{i}_x$ and the reference currents $\mathbf{i}_{xref}$ in Eq. (12) by the control rule

$$\mathbf{v} = \mathbf{K}_{pst} \mathbf{e}_i, \quad \mathbf{e}_i = \mathbf{i}_x - \mathbf{i}_{xref}, \quad \mathbf{K}_{pst} = \mathrm{diag}[k_{pst}, k_{pst}] \tag{14}$$

The resulting transfer functions from the displacement control deviation $\mathbf{E}_x(s)$ to the actuator currents $\mathbf{I}_x(s)$ is denoted with $\mathbf{G}_\mathrm{C}(s)$ in Fig. 1.

3. Identification method

The modulating function technique was developed by Shinbrot (1957) for an analysis of linear and nonlinear SISO systems and allows to convert a differential equation into a set of parametrized algebraic equations on a finite time interval. The initial concept was refined by Pearson and Lee (1985), who proposed the trigonometric Fourier type modulating functions and a modification of the derivative calculation due to the implementation of the fast Fourier transform (FFT). An extension of this method for MIMO systems is proposed in (Shen 1993) and (Co and Ydstie 1990). The main idea of the modified closed-loop identification method used in this work is based on the concept of Co and Ydstie (1990).

Consider the polynomial description of the MIMO magnetic bearing system presented in Eq. (9)

$$\mathbf{P}_{\mathrm{den}} \mathbf{w}_x(t) = \mathbf{P}_{\mathrm{num}} \mathbf{i}_x(t) \tag{15}$$

with

$$\mathbf{P}_{\text{den}} = p^2 \mathbf{A}_0 + p^1 \mathbf{A}_1 + p^0 \mathbf{A}_2, \quad \mathbf{P}_{\text{num}} = p^0 \mathbf{B}_2, \quad \mathbf{A}_0 = \mathbf{I}, \quad p^i = d^i/dt^i, \quad i = 0, \cdots, 2 \tag{16}$$

$\mathbf{P}_{den}$ and $\mathbf{P}_{num}$ are the denominator and numerator polynomials with unknown system parameters $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{B}_2$.

The used algorithm operates with measured inputs and outputs of the analysed system. The approximation of the derivatives is based on the calculation of a convolution of the measured signal with a modulation function. There are many types of modulating functions implemented for system identification in the last years (Cieza Aguirre, Tafur, and Reger 2014). The following complex Fourier type of modulating function set (Pearson and Shen 1993) is used in this paper.

$$\phi_{m,n}(t) = 1/T e^{(-jm\omega_0 t)}(e^{(-j\omega_0 t)} - 1)^n = 1/T \sum_{k=m}^{n+m} c_{k-m} e^{-jk\omega_0 t} = 1/T \sum_{k=0}^{n} c_k e^{-j(k+m)\omega_0 t} \tag{17}$$

where

$$\omega_0 = 2\pi/T, \quad m = 0, 1, 2, \ldots, M \tag{18}$$

and n is the system order, ω_0 is a resolving frequency for the fixed time interval $[0, T]$ and m is the modulating frequency index, which defines the frequency range for the identification. The derivative of $\phi_{m,n}(t)$ is determined as follows

$$p^i \phi_{m,n}(t) = \frac{1}{T} \sum_{k=0}^{n} c_k(-j(k+m)\omega_0)^i e^{-j(k+m)\omega_0 t} . \tag{19}$$

Furthermore, the coefficients c_k are chosen so that the modulating function $\phi_{m,n}(t)$ satisfys

$$p^i \phi_m(0) = p^i \phi_m(T) = 0. \tag{20}$$

For example, factors c_k sind equal the binomial coefficients $(-1)^{n-k}\binom{n}{k}$. The use of Eq. (20) ensures that the initial conditions of the experiment are irrelevant to the identification process. The derivatives of the modulating function $\phi_{m,n}(t)$ are shown in Fig. 2
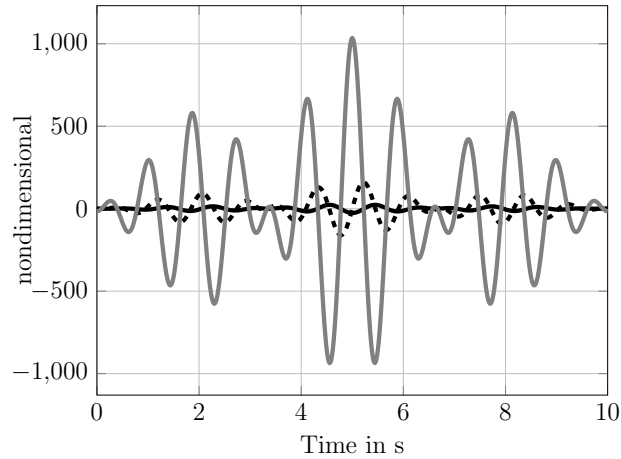

Fig. 2   An example of the modulating function derivatives $p^0\phi(t)$ (black solid line), $p^1\phi(t)$ (black dashed line) and $p^2\phi(t)$ (gray line) with $T = 10$ s, $w_0 = 0.1$ Hz, $m = 10$.

Multiplying both sides of Eq. (15) with $\phi_{(m,n)}(t)$ and integrating with respect to time t over the interval $[0, T]$ leads to the complex-valued vector regression form

$$\mathbf{Y} = \mathbf{\Gamma}\mathbf{\Theta} \tag{21}$$

with a parameter matrix $\mathbf{\Theta} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 & \mathbf{B}_2 \end{bmatrix}^{\mathrm{T}}$ and matrices of the modulated input and output signals

$$\mathbf{Y} = \begin{bmatrix} \gamma_{21}^{\mathbf{w}_x}(0) & \gamma_{22}^{\mathbf{w}_x}(0) \\ \gamma_{21}^{\mathbf{w}_x}(1) & \gamma_{22}^{\mathbf{w}_x}(1) \\ \vdots & \vdots \\ \gamma_{21}^{\mathbf{w}_x}(M) & \gamma_{22}^{\mathbf{w}_x}(M) \end{bmatrix}, \quad \mathbf{\Gamma} = \begin{bmatrix} -\gamma_{01}^{\mathbf{w}_x}(0) & -\gamma_{02}^{\mathbf{w}_x}(0) & -\gamma_{11}^{\mathbf{w}_x}(0) & -\gamma_{12}^{\mathbf{w}_x}(0) & \gamma_{01}^{\mathbf{i}_x}(0) & \gamma_{02}^{\mathbf{i}_x}(0) \\ -\gamma_{01}^{\mathbf{w}_x}(1) & -\gamma_{02}^{\mathbf{w}_x}(1) & -\gamma_{11}^{\mathbf{w}_x}(1) & -\gamma_{12}^{\mathbf{w}_x}(1) & \gamma_{01}^{\mathbf{i}_x}(1) & \gamma_{02}^{\mathbf{i}_x}(1) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -\gamma_{01}^{\mathbf{w}_x}(M) & -\gamma_{02}^{\mathbf{w}_x}(M) & -\gamma_{11}^{\mathbf{w}_x}(M) & -\gamma_{12}^{\mathbf{w}_x}(M) & \gamma_{01}^{\mathbf{i}_x}(M) & \gamma_{02}^{\mathbf{i}_x}(M) \end{bmatrix}. \tag{22}$$

Every matrix entry is a convolution of the signal with the derivative of the modulating function of corresponding order

$$\gamma_{ij}^{\mathbf{w}_x}(m) = (-1)^i \int_0^T w_{xj}(t) p^i \phi_{(m,n)}(t) dt, \quad i = 1, 2, \tag{23}$$

$$\gamma_{kj}^{\mathbf{i}_x}(m) = (-1)^k \int_0^T i_{xj}(t) p^k \phi_{(m,n)}(t) dt, \quad k = 2, \quad j = 1, 2. \tag{24}$$

The regression in Eq. (21) can be devided into real and complex parts

$$\mathbf{Y} = \mathbf{Y}_R + j\mathbf{Y}_I, \quad \mathbf{\Gamma} = \mathbf{\Gamma}_R + j\mathbf{\Gamma}_I, \quad \boldsymbol{\epsilon} = \boldsymbol{\epsilon}_R + j\boldsymbol{\epsilon}_I. \tag{25}$$

The parameter matrices can than be estimated by using the maximum likelihood method

$$\hat{\mathbf{\Theta}}_{LS} = (\mathbf{\Gamma}_R^T\mathbf{\Gamma}_R + \mathbf{\Gamma}_I^T\mathbf{\Gamma}_I)^{-1}(\mathbf{\Gamma}_R\mathbf{Y}_R^T + \mathbf{\Gamma}_I\mathbf{Y}_I^T). \tag{26}$$

The direct closed-loop identification can be applied without any information regarding the controller and works with the inputs and outputs of the plant. Thus, the method can be applied without any alterations of the equations. The actuator currents are taken as system inputs and the displacements of a shaft as outputs for the identification. The open loop state-space model of the AMB is obtained by applying the described algorithm to the experimental data.

4. Experimental results

The magnetic bearing test bench consists of a flexilble rotor levitated by two radial and one axial magnetic bearings (as illustrated in Fig. 1). The reference signals are generated by an ADWin-Gold II system. The processor cycle frequency of this real-time platform is 300 MHz, with a closed-loop sample time is 50 μs. Displacement measurements are done using eddy current sensors, where the currents are measured with shunts. Information from sensors is provided to ADWin-Gold II, where the control input for the actuators is produced according to the cascade control scheme. Parameters of the displacement und current controllers are listed in Table 1.

Table 1　Parameters of the position und the current controller

Symbol	Value	Units
k_p	1×10^4	A/m
k_i	8×10^4	A/ms
k_d	15	As/m
k_{pst}	15	V/A

The rotor system is excited simultaneously at both reference channels r_1 and r_2. The characteristics of the different excitation signals such as a chirp, white noise and a Schroeder-phased multisine (Pintelon and Schoukens 2012) are investigated. The multisine signals have been recognized to be the best suited excitation for the MIMO AMB system and the proposed identification algorithm. A Schroeder-phased multisine is obtained according to following formula (Pintelon and Schoukens 2012)

$$r_i(t) = \sum_{k=1}^F A \cos(2\pi f_k t + \phi_k), \quad \phi_k = -k(k-1)\pi/F, \quad f_k = l_k f_0, \quad l_k \in \mathbb{N}. \tag{27}$$

All identification results are obtained based on the application of multisine reference signals and the measurements of currents $i_{x1}(t)$, $i_{x2}(t)$ and displacements $w_x(l_1, t)$, $w_x(l_2, t)$. The time interval of every single measurement is 10 s and consequentially the resolving frequency $\omega_0 = 0, 1$ Hz. Collected data is processed by an identification algorithm proposed in Section 3. The rigid body eigenfrequencies and flexible eigenfrequencies are identified separately. Increasing the model order in Eq. (9) allows a modeling of all four frequencies. Eigenfrequencies of the resulted model are shown in Fig. 3, which show a typical distribution for a AMB system (Schweitzer and Maslen 2009). Eigenfrequencies of rigid body modes cannot be obtained from a modal analysis of a shaft themselves and strongly depend on stiffness of magnetic bearings, unlike the eigenfrequencies of flexible modes. Parameters of actuators such as force-displacement-factors $\mathbf{K}_{sax}$ and force-current-factors $\mathbf{K}_{iax}$ vary during operation. Therefore, a quantitative comparison of identified and theoretical obtained eigenfrequencies is not reliable.

There is also no way to measure a frequency response of an open loop AMB plant directly because of the unstable system behaviour in a frequency range of rigid body modes. Thus, a validation of the identified model consist of the comparison of measured and simulated responses of the closed-loop system, see Fig. 4. The experimental curve is obtained with Newtons4th Frequency Response Analyzer by exciting the plant with a sine chirp signal. The input is a
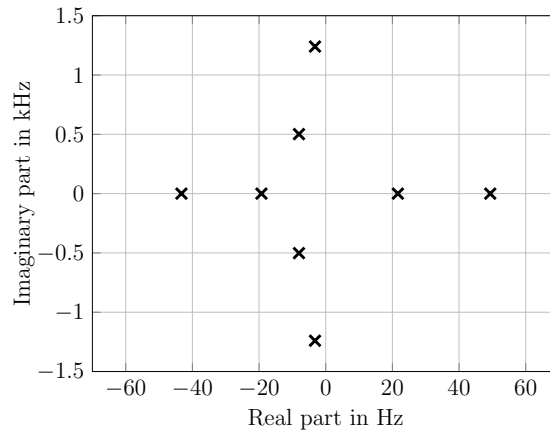
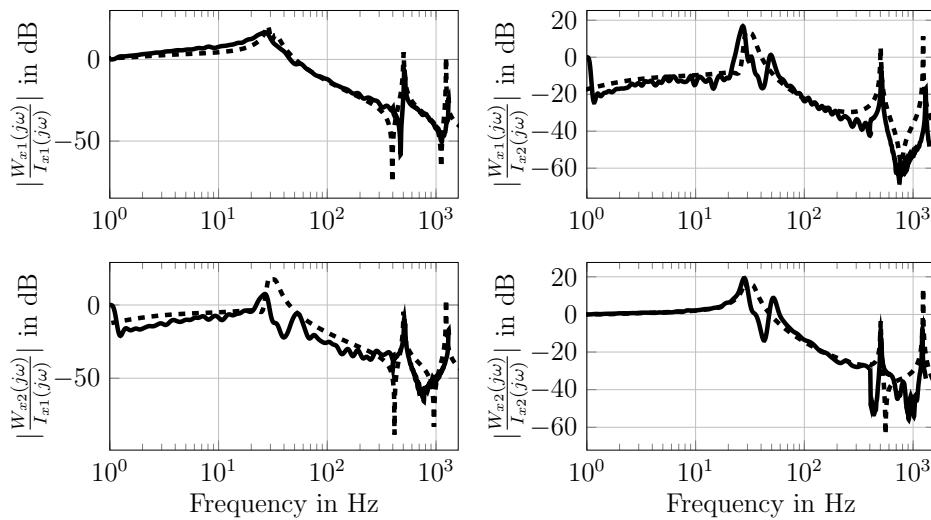Fig. 3   The first four estimated eigenfrequencies of the magnetic bearing system

Fig. 4   Measured (solid lines) and identified (dashed lines) amplitude responses of the magnetic bearing system in closed-loop.

desired position generated by the measurement device and the output is an actual position of the rotor. The identified response is designed based on the identified open loop model by implementing a controller in MATLAB.

Figures 3 and 4 show that the proposed identification algorithm copes with a problem of estimation both unstable poles of rigid body modes and slightly-damped poles of flexible modes. Resonance and antiresonance frequencies match in diagonal and off-diagonal transfer functions. Small differences in estimated and measured values of the second flexible mode could be explained by high signal-to-noise ratio in this frequency range.

5. Conclusions

An application of the identification algorithm based on MF technique for direct closed-loop identification of an AMB plant is proposed in this work. The methodology processes a raw input and output data and needs no information about the control scheme used to stabilize the system. To the best of the authors knowledge, this approach has not been applied to such an AMB system. The separation of the complete MIMO model into SISO or SIMO subsystems is not neccessery. Thus, reducing the computational effort and allows an estimation of the parameter matrix in a single step. An identified continuous-time polynomial model can be directly transformed into state space, which is more universal for control design. The efficiency of the algorithm is demonstrated on the real magnetic bearing system. Presented results of identification show good performance and can be successfully used for control design. Future work will concentrate on

the expansion of the developed identification methodology to AMB systems with spinning flexible rotor.

6. Acknowledgements

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