

# Using a pseudorandom binary sequence for rotor-bearing system identification in active magnetic bearing rotor systems

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## Abstract

This paper examines usage of a pseudorandom binary sequence (PRBS) in system identification of the rotor-bearing system in active magnetic bearings (AMBs). In the work, PRBS system identification is compared with adaptive amplitude stepped sine identification. PRBS is a deterministic, periodic signal varying between two levels. A stepped sine wave is a type of sine wave that has all the energy on one frequency only. The paper compares the running time, memory consumption and accuracy of the identification methods studied. A five degree-of-freedom (DOF) AMB testrig is used to test the accuracy of the stepped sine and PRBS system identification approaches. Both methods are implemented in a real-time open automation system. Required running time of the PRBS identification was significantly shorter than that of the stepped sine identification, but more memory was needed for data points. The accuracy of both identification methods was similar at lower frequencies, but at higher frequencies there was greater fluctuation with PRBS identification than when using the stepped sine method. The results show the suitability of using a PRBS in AMB rotor-bearing system identification. The PRBS based approach would be the choice for initial fast identifications of the system, while the stepped sine method would remain as the choice when focusing on identification at specific and higher frequencies.

**Keywords:** System identification, Frequency response function (FRF), Active magnetic bearing (AMB), Pseudorandom binary sequence (PRBS), Stepped sine

## 1. Introduction

System identification is the construction of a mathematical model of a dynamical system based on observed and measured data from the system. Input-output data of the system are recorded during specific identification experiments to make the input-output data maximally informative (Ljung, 1987). Different types of identification experiments are used depending on the system to be analyzed. One approach is to use step and impulse signals to obtain identification data from the step and impulse responses of the system. Sine waves can also be used in identification experiments, for example, multisines or stepped sines. A multisine is a sine wave where power is divided into different frequencies, and a stepped sine is a sine wave that has all the power on one frequency only. Random signals such as random Gaussian signals and pseudorandom binary sequence signals have also been used in the identification experiments, for example, by (Shariff, et al., 2013), (Fairweather, et al., 2011). Multi-level pseudorandom signals have been applied for nonlinear system identification, for example, by (Braun, et al., 1999).

For active magnetic bearing (AMB) rotor-bearing system identification, which is the application studied in this work, sine wave based identification experiments have been performed by a number of researchers: stepped sine identification experiments were used by (Gähler, 1998), (Lösch, 2002) and (Vuojolainen, 2015), and multisine identification by (Hynnen, 2011).

This paper applies a pseudorandom binary sequence (PRBS) in system identification of a five degrees-of-freedom (DOF) AMB rotor-bearing system. The suitability of a PRBS approach for AMB rotor-bearing system identification is investigated, and the performance of the PRBS system identification is evaluated and compared with that of an adaptive amplitude stepped sine identification algorithm presented by (Vuojolainen, 2015). Accuracy, memory consumption and

running time are the evaluation criteria used. To the best of the authors' knowledge, no results for the application of PRBS in AMB rotor-bearing system identification and diagnostics have been presented in the literature.

The paper starts by describing the pseudorandom binary sequence and adaptive amplitude stepped sine signal approaches. Experimental results from utilization of the methods in rotor-bearing system identification are then given. The identification results are evaluated and compared with a nominal finite element method (FEM) model of the rotor. In addition, the inner current control loop from the reference control current to the measured control current from the electromagnets is identified. Nonlinearity in the system is investigated with a constant amplitude stepped sine by analyzing the harmonics in the frequency response of the rotor displacement. Finally, conclusions are drawn about the suitability of the PRBS based approach for AMB rotor-bearing system identification.

## Nomenclature

$A$	amplitude
$d$	order of the PRBS
DX	drive end $x$ -axis
DY	drive end $y$ -axis
$f$	excitation frequency
$f_r$	frequency resolution
$f_s$	sampling frequency
$G$	transfer function
$L$	number of data points
$N$	length of the PRBS
NX	non-drive end $x$ -axis
NY	non-drive end $y$ -axis
$U$	plant input(reference control current)
$V_1$	plant output(rotor displacement)
$V_2$	plant output(measured control current)
$Z$	axial $z$ -axis

## 2. Methods

Pseudorandom binary sequence is a type of binary signal which can be used in the system identification. Adaptive amplitude stepped sine is a type of signal where the sine wave amplitude is adjusted to get an acceptable response and is used in the system identification. Nonlinearities in the system can be analyzed with looking at the harmonics present in the frequency response of the rotor displacement.

### 2.1 Pseudorandom binary sequence

A pseudorandom binary sequence is a periodic, deterministic signal that varies between two levels, typically between amplitudes  $+A$  and  $-A$ . The pseudorandomness of the signal means that although it is deterministic, it seems to behave like a real random sequence and is hard to predict.

Binary signals such as pseudorandom binary sequence have an optimal spectrum for the excitation signal. They are easy to generate, have controllable spectral energy and high spectral energy over a wide band range.

The PRBS is determined by the selected excitation frequency  $f$  and the order  $d$ . Length  $N$  of the PRBS is calculated with:

$$N = 2^d - 1. \quad (1)$$

An example of a third order PRBS signal is shown in Fig. 1a. Figure 1b shows a higher, seventh order PRBS signal. The amplitude of both signals is 1. Using Eq. (1), the length of the third order PRBS signal is 7 and the length of the seventh order PRBS signal is 127. PRBS excitation frequency is 3.33 kHz.

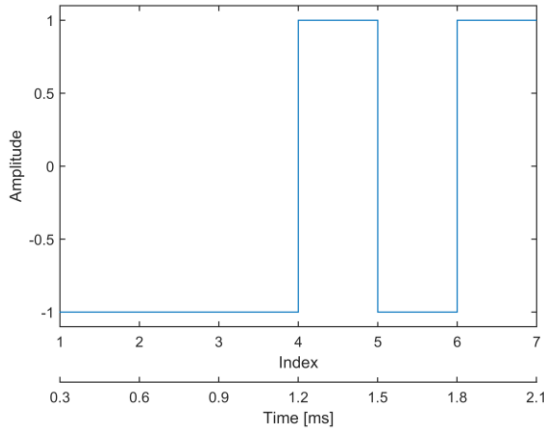


Fig. 1a Example of a third order PRBS.

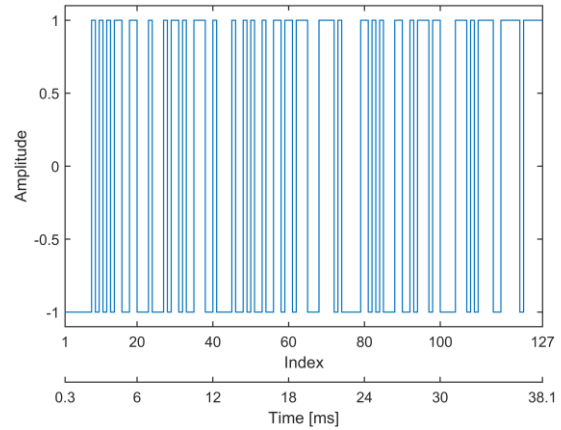


Fig. 1b Example of a seventh order PRBS.

Frequency resolution  $f_r$  of the pseudorandom binary sequence is:

$$f_r = \frac{f}{N}, \quad (2)$$

where  $f$  is the selected excitation (PRBS generation) frequency and  $N$  is the length of the PRBS. The number of data points  $L$  needed to save the PRBS identification data is:

$$L = \frac{Nf_s}{f}, \quad (3)$$

where  $f_s$  is the sampling frequency used, which is 20 kHz in this case. A maximum of one second of the PRBS identification can be saved based on memory consumption considerations. The maximum number of data points in this case is thus 20 000. Now Eq. (3) can be reformulated as the following inequality:

$$\frac{Nf_s}{f} \leq 20\,000. \quad (4)$$

Substituting the value of sampling frequency  $f_s$  in Eq. (4) yields:

$$\frac{N}{f} \leq 1. \quad (5)$$

Equation (5) shows that the length  $N$  of the PRBS and the excitation frequency  $f$  have to be chosen based on the maximum number of data points to be saved and the sampling frequency  $f_s$ . Multiplying Eq. (5) with excitation frequency  $f$  and dividing by the length  $N$  gives:

$$1 \leq \frac{f}{N}. \quad (6)$$

From Eq. (2), which defines the frequency resolution  $f_r$  of the PRBS signal, and Eq. (6) it can be noted that the frequency resolution of the PRBS is constrained to being equal or greater than one. Equation (2) shows that by decreasing the excitation frequency  $f$  and increasing the length  $N$ , a higher frequency resolution  $f_r$  is achieved. The maximum value of the length  $N$  is limited by the maximum number of data points  $L$  and the ratio of  $f/f_s$ , as seen in Eq. (3). Thus, higher frequency resolution can be achieved by increasing the sampling frequency  $f_s$  and the maximum number of data points  $L$  and decreasing the excitation frequency  $f$ .

PRBS identification was implemented in a real-time open automation system. In this implementation, the order of the PRBS can be chosen from 3 to 13 (where 14 is a limit from Eq. (5)). The amplitude and the number of PRBS measurement periods can be chosen freely. A minimum of two periods of the PRBS signal is used. The first period is a transient period, and data points related to the transient period are discarded because they lead to incorrect results. Data points related to the second period (measurement period) are saved and used for the identification. If two or more measurement periods are used, an average over the measurement periods is calculated and saved. The use of two or more measurement periods enables a better signal-to-noise ratio (SNR) to be achieved and the effect of possible outliers (measurement errors) to be minimized.

An advantage of the PRBS identification is the short runtime required, with ten measurement periods a maximum

of 11 s per axis is needed. A disadvantage is the large amount of memory needed to save the data, which in this case comprise 20 000 data points.

## 2.2 Adaptive amplitude stepped sine

A stepped sine is a type of sine wave where all the power is on one frequency only. Consequently, the SNR of the stepped sine is very high. In this paper, an adaptive amplitude stepped sine algorithm presented by (Vuojolainen, 2015) is used. Adaptive amplitude means that the sine wave amplitude is adjusted to get an acceptable response. An advantage of the adaptive amplitude stepped sine identification algorithm is that memory is needed only for 250 data points. However, the runtime is long, up to 12 minutes per axis, because every frequency has to be excited separately, and a waiting time is used between frequencies and if the amplitude is adjusted.

## 2.3 Harmonic analysis of nonlinear systems

Nonlinear systems produce additional harmonics in the frequency response. These harmonics degrade the quality of the measured frequency response functions (FRFs). Quadratic systems generate harmonics that are on the second multiple of the excited frequency. If a system contains harmonics that are on the odd multiple of the excited frequency, these harmonics add to the signal power of the excited frequency and are unwanted (Hynynen, 2011). In the case under study in this work, the rotor-bearing system is quadratic and the odd harmonics are analyzed. Harmonics are analyzed from the spectrum of the rotor displacement for the excited axis. In this study they are analyzed for the drive end  $x$ -axis (DX) using a constant amplitude stepped sine.

## 3. Experiments and results of the system identification

A five degree-of-freedom AMB testrig was used to test the PRBS and adaptive amplitude stepped sine system identification approaches. The testrig consists of two radial and one axial active magnetic bearing. For the PRBS identification, the excitation frequency was 3.33 kHz, the order was 11, the frequency resolution was 1.63 Hz, the amplitude was 2.5 A, and ten measurement periods were used. For the adaptive amplitude stepped sine identification, the frequency range was from 1 Hz to 750 Hz, the frequency resolution was 3.01 Hz, and the maximum sine wave amplitude was 2.5 A. A simple FRF, an empirical transfer function, was used to form the experimental transfer functions based on the identifications.

In the harmonic analysis of the rotor-bearing system, two different frequency bands were used, both of which used a sine wave amplitude of 2.5 A and step size of 4 Hz. The first frequency band was from 2 Hz to 146 Hz and the second frequency band was from 150 Hz to 302 Hz.

### 3.1 Rotor-bearing system identification

For the rotor-bearing system identification, excitation was applied at the reference control current. The plant input  $U$  and output  $V_1$  were measured for the corresponding axis to form the open loop plant transfer function from the reference control current to the rotor displacement. The open loop plant transfer function  $G$  is written as:

$$G = \frac{V_1}{U}. \quad (7)$$

After forming the open loop plant transfer functions for the PRBS and adaptive amplitude stepped sine system identification, respectively, the transfer functions and the FEM-model rotor transfer function were compared. The comparison is shown in Fig. 2.

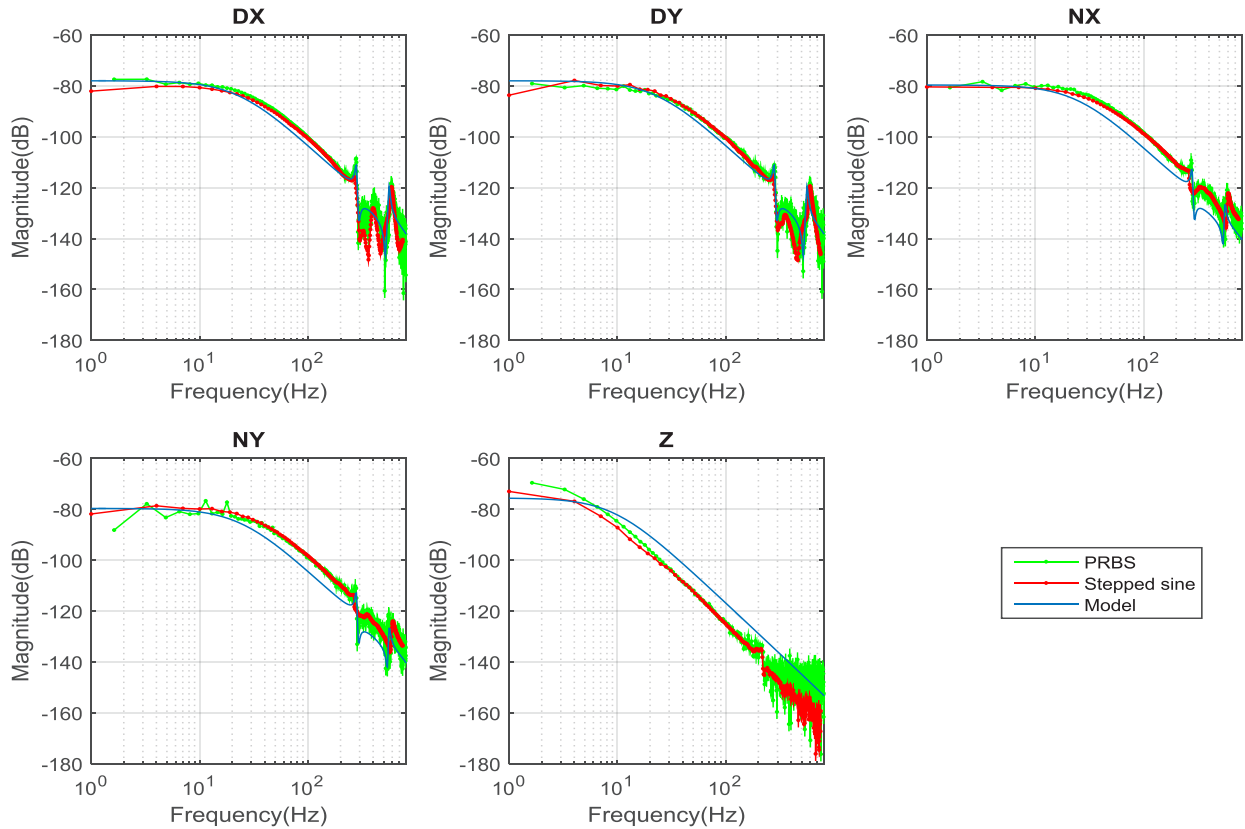


Fig. 2 Comparison of the open loop plant transfer functions, from the reference control current  $U$  to the rotor displacement  $V_1$  for the corresponding axis. D is the drive end, N is the non-drive end and Z is the axial direction.

From Fig. 2, it can be seen that when compared to the FEM-model of the rotor, both the adaptive amplitude stepped sine and PRBS identification yield similar accuracy. The stepped sine identification has less fluctuation than the PRBS identification, particularly in the frequency range after the first resonance/anti-resonance frequency pair located at around 280 Hz and 290 Hz on the radial axes. On the Z-axis PRBS identification starts to have more fluctuation at around 220 Hz. PRBS identification on the other hand matches the model more accurately on the first resonance/anti-resonance frequency pair on the radial axes. PRBS has higher resolution 1.63 Hz compared to the stepped sine with 3.01 Hz resolution.

### 3.2 Inner current control loop identification

For the inner current control loop identification, excitation was applied at the reference control current. The plant input  $U$  and output  $V_2$  were measured for the corresponding axis to form the transfer function from the reference control current to the measured control current. Transfer function  $G$  from the reference control current to the measured control current is written as:

$$G = \frac{V_2}{U}. \quad (8)$$

Comparison between the stepped sine and the PRBS identification of the inner current control loop is shown in Fig. 3.

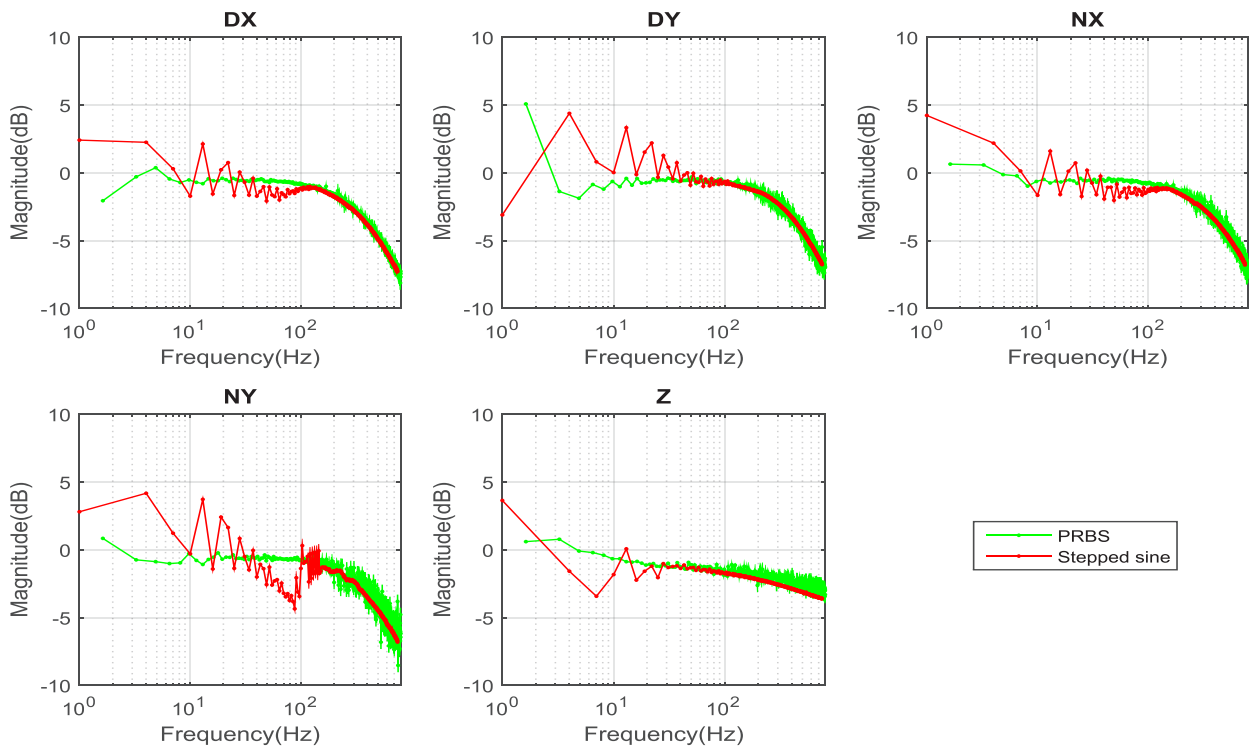


Fig. 3 Comparison of the inner current control loop transfer functions, from the reference control current  $U$  to the measured control current  $V_2$  for the corresponding axis. D is the drive end, N is the non-drive end and Z is the axial direction.

From Fig. 3, it can be noted that in the frequency range below 100 Hz on the radial axes, the PRBS identification has less fluctuation between the identified points. On the Z-axis, PRBS identification has less fluctuation up to 30 Hz. On the radial axes, inner current control loop identification results start to converge at around 100 Hz, except on the NY-axis where convergence begins at around 150 Hz. On the Z-axis convergence occurs at around 30 Hz. After the convergence point, fluctuation in the PRBS identification starts to increase on all axes.

### 3.3 Harmonic analysis of the rotor-bearing system

In the harmonic analysis of the rotor-bearing system, excitation was applied at the reference control current on the DX-axis and the spectrum of the output  $V_1$  (rotor displacement) was measured. Figure 4 shows the harmonic analysis of the frequency band from 2 Hz to 146 Hz in 4 Hz steps. Figure 5 shows the harmonic analysis of the frequency band from 150 Hz to 302 Hz in 4 Hz steps.

From Fig. 4, it can be seen that there are harmonics in the rotor-bearing system. A second harmonic, implying a quadratic system, is visible. A third harmonic, which is unwanted, is clearly visible. Other harmonics such as the fourth, sixth and unwanted fifth harmonic can be noted, especially at lower frequencies.

In Fig. 5, the second harmonic is clearly visible. The unwanted third harmonic is also visible but has less amplitude than in the harmonic analysis presented in Fig. 4. A fourth harmonic can also be seen. Other harmonics are not visible.

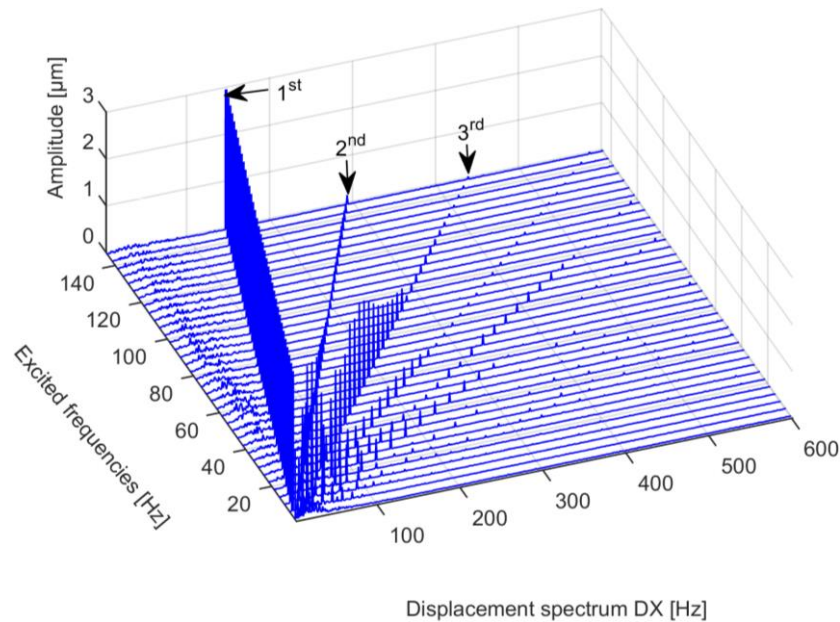


Fig. 4 Harmonic analysis of the rotor-bearing system on the drive end  $x$ -axis (DX). Frequencies from 2 Hz to 146 Hz in 4 Hz steps were excited. The second and third harmonics show clearly.

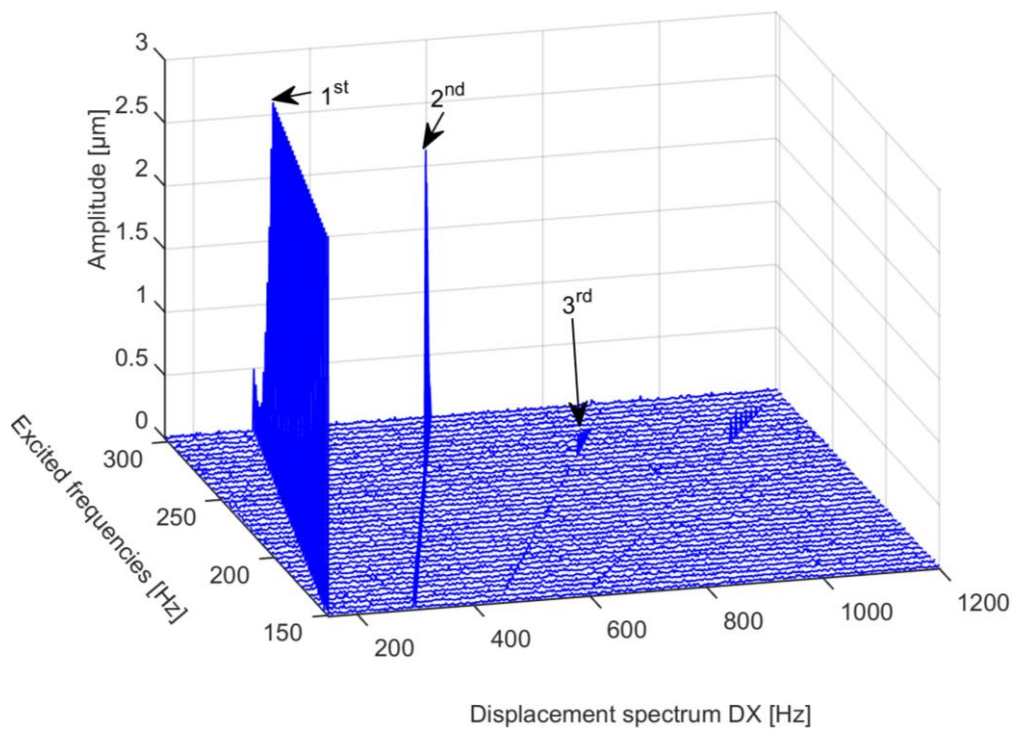


Fig. 5 Harmonic analysis of the rotor-bearing system on the drive end  $x$ -axis (DX). Frequencies from 150 Hz to 302 Hz in 4 Hz steps were excited. The second harmonic is clearly seen and a small third harmonic is present.

#### 4. Discussion

Stepped sine and multisine identification experiments have typically been used in AMB rotor-bearing system identification. The results in this paper show the suitability of using a PRBS based approach in AMB rotor-bearing system identification. Comparison between the PRBS system identification and the adaptive amplitude stepped sine system identification showed that the required runtime of the stepped sine system identification is longer than that of the PRBS system identification. However, less memory is needed. The accuracy of the stepped sine system identification of the rotor-bearing system was greater at higher frequencies and the approach also produced more accurate results for inner

current control loop system identification at higher frequencies. The PRBS system identification had better accuracy for inner current control loop system identification at lower frequencies. Both identification methods had similar accuracy for rotor-bearing system identification at lower frequencies, but the PRBS system identification matched better the first resonance/anti-resonance frequency pair on the radial axes.

The harmonic analysis demonstrated that the second harmonic is clearly visible, indicating that the rotor-bearing system is quadratic. An unwanted third harmonic is also visible, especially at lower frequencies in the frequency band from 2 Hz to 146 Hz.

## 5. Conclusion

Use of a PRBS based approach was found to be feasible for AMB rotor-bearing system identification. The required running time is shorter than with stepped sine identification, but more memory is needed to store data points. The accuracy of both identification methods was similar for rotor-bearing system identification at lower frequencies. For inner current control loop identification, PRBS identification was more accurate at lower frequencies. Stepped sine identification was more accurate for rotor-bearing system identification at higher frequencies and gave better results for inner current control loop identification.

PRBS would be the first choice for fast and safe identification and diagnostics of the system. Other identification methods such as the stepped sine approach used in this work and multisine approaches could then be used to obtain a more accurate correspondence between the experimentally identified model and the FEM-model of the rotor, especially at higher frequencies.

Future work could include the testing of multi-level PRBS identification in rotor-bearing system identification. Additionally, the effect of adapting the amplitude of the PRBS signal based on noise and the maximum response of previous experiments could be investigated.

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