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On system identification for active magnetic bearings at nonzero speeds

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Abstract

In this paper we consider the problem of obtaining a dynamic model of rotating machinery equipped with magnetic bearings for a given speed range. Theoretical models are typically not accurate enough and cannot be trusted to predict system behaviour for the whole operating range of the machine. In this work we first obtain a dynamic model of the system at several speed levels; second, an extrapolation/interpolation of these models is performed to produce an estimate of the behaviour for the whole speed range. This procedure overcomes two common issues in system identification for magnetic bearings: first, it is costly and time-consuming to perform system identification at many speed levels to cover the whole operating range. Second, it is typically not safe to reach certain speeds unless the stability and unbalance response at high speeds has been verified. This can be only done once a reliable model is available. From a technical perspective, our approach relies on local system identification techniques aimed at linear parameter varying (LPV) systems. Compared to existing work, we believe the main contributions of this work are: we do not rely on global approaches that might lead into nonconvex problems, our approach is amenable to automation, and it allows considering the presence of casing modes in the system.

Key words : System Identification, Gyroscopic Effects, Prediction, LPV Systems, Coordinate transformation, Extrapolation, Parametrisation

1. Introduction

Active magnetic bearings for rotating machinery represent many benefits compared with conventional bearings, such as vibration control and removal of lubrication circuits, among others. Nonetheless, these systems are open loop unstable and therefore require feedback control for normal operation. In order to design a feedback control loop that renders the system stable, it is necessary to first have a fairly accurate model of the system. Since theoretical models are typically not accurate enough, it is common practice to resort to system identification techniques. System identification is not only needed in terms of control, but it also enables remote monitoring and diagnostics of the machine. The problem of system identification at standstill or at a fixed, particular speed is a challenging task, as we are dealing with a nonlinear, multiple-input multiple-output, open-loop unstable system. It has been widely studied in the literature (Gähler et al (1997), Herzog and Siegwart (1993), Vázquez et al (1993)). However, it is necessary to obtain a dynamic model of the system not just for a few speed values but for the whole speed range of the machine, both for control purposes and to verify international norms requirements (namely, ISO 14839 and API 617). In this paper we propose a method to obtain a dynamic model of a system that represents the behaviour of the system for the whole operating speed range.

Most results available in the literature on system identification for speed-dependent models consider a gray box approach, and attempt to identify the gyroscopic effects (either the whole gyroscopic matrix or just certain parameters, such as moments of inertia). Most techniques formulate the task as an optimisation problem, that happens to be nonconvex, with many local minima and too many variables (Senn (1997), Lösch (2002)). Some authors exploit a priori knowledge from finite element models to ease the task, also known as model updating (Wroblewski et al, 2012). In our experience, none of these approaches provide accurate and numerically robust results for flexible shafts, where the number of parameters to identify might be relatively large. Moreover, a gray box approach might lead to inaccurate results, as speed-dependent behaviour does not always match the expected theoretical equations.

We propose a new technique by leveraging existing work on LPV system identification. Indeed, rotating equipment levitated by magnetic bearings can be mathematically modelled as linear parameter varying (LPV) systems, since for a fixed speed the system can be modelled using a linear description. There are two main approaches to identify linear-parameter varying systems. A local approach uses identified models at different speeds to obtain a model describing the behaviour for the whole speed range, either via interpolation or extrapolation. The other possibility (known as global approach) would be to identify the system via a single experiment where the speed is continuously changing. This global approach takes advantage of the fact that the varying parameter (speed) can be measured. Standard global approaches (*i.e.*, Tóth et al (2009)) are computationally very demanding and extremely complex. Given the limited resources available on our product platform, we focus in this work on local approaches to develop a dynamical model of the system - especially as it is possible to run the machine at a constant speed. Moreover, there are already well-known, efficient, system identification techniques for linear time invariant systems. Our method relies on obtaining several models at different speeds and interpolates the dynamics to describe the behaviour at intermediate steps. From a business perspective, the main advantages of this method are a reduction of commissioning time and avoiding unexpected landings during commissioning(as we can predict the behaviour of the machine at high speeds from low speed models).

The structure of the document goes as follows: in Section 2 the main steps of the proposed algorithm are described. Section 3 shows the experimental results in a GE product, and we conclude this document with a discussion of possible extensions of this work.

2. An LPV-based approach to system identification for magnetic bearings

The dynamics of a flexible structured supported by active magnetic bearings can be mathematically represented by the following state space description:

$$\dot{x} = Ax + A_G(\Omega)x + Bu \tag{1}$$

$$y = Cx + Du \tag{2}$$

where x represents the state (typically, positions and velocities at certain points along the shaft), y contains all measured signals (namely displacements), u is the input forces, A, B, C, D are the state space matrices, A_G represents the gyroscopic matrix, and Ω is the speed of the system. While in reality there are many nonlinearities (especially in the magnetic bearing dynamics) in the system, our experience shows us that a linear model is sufficient for the range of applications we envision.

It is clear that for a fixed value of Ω, the dynamics becomes linear. Given that Eq. (1) corresponds to a linear-parameter-varying structure, we leverage existing work on identification of LPV systems (Caigny et al (2008), Adegas et al (2013)). As our work aims at flexible shafts, the complexity of the dynamic models involved is relatively high and constrains the set of techniques that can be employed; in our experience, most of the techniques suffer from numerical issues when used for high order systems or for rotor structures, where poles and zeros are nearly collocated. Our proposed method consists of the following steps:

A) Perform system identification experiments at different speeds (at least 2), in order to obtain a nonparametric model (e.g., frequency response of the system) at those speeds.

B) Parametrize all available models, constructing state space representations.

C) Obtain a common basis for all available models.

D) Specify a speed dependence for the dynamics and identify terms in this dependence using models from (C).

As mentioned previously, step (A) has been deeply studied, and it is considered out of the scope of this paper. We refer the interested reader to the references provided in the introduction of this article. While any 2 (or more) models at different speeds can be used for our procedure, normally a model at standstill and another one at low speeds are used at a first stage.

2.1. Parametrisation

Once two or more non-parametric models of the system have been obtained, the next step is to construct a mathematical description for each model. This is known as parametrisation. Among all existing parametrisation algorithms, the two most popular ones are subspace identification (SID) and maximum likelihood/prediction error methods (MLE/PEM). The SID methods can be seen as a least squares estimation approach of the state matrices based on measured values of the output, input and estimated values of the state (Katayama, 2005). PEM is based on an iterative minimisation of a weighted norm of the prediction error. This iterative procedure is typically started from parameter values that were obtained at a first stage via the subspace identification method. Subspace methods are not iterative and therefore require much less computation time. While subspace methods are useful to get a first idea of order models, PEM typically provides the most precise parametric models. Moreover, subspace methods can derive two similar models with similar I/O behaviour but very different poles and zeros, making step (C) of our method much harder (more on this later). In this work we will use PEM as it allows for more control of the final result, via specification of initial values and limiting the search space.

Among all the design parameters in PEM, there are two factors that play a big role in obtaining a good parametric model: initial model and imposed structure.

- Initial model: as the minimisation problem is nonconvex, starting at a good initial model is vital to obtain good results. Given that a rough theoretical model of the system is typically available, it can be used for the parametrisation of the first model. To parametrize the other models, we could use as initial condition either the theoretical model (if this is accurate enough), or the first model. In the presence of casing modes, the theoretical model cannot be used directly as the initial guess (or at least it could cause very poor results), and it would have to be augmented with the appropriate number of states to model the system accurately.
- Imposed structure: PEM allows the user to define constraints in the minimisation formulation, such as maximum value admissible for a certain coefficient, or minimum modal frequency expected in the model. For instance, one could resort to the mathematical model to define an expected range for certain values in the state matrices. Based on our experience, it is beneficial to fix the elements that are expected to be 1 or 0 in the A, B and C matrices of the state space representation (the D matrix is typically 0 for these kind of systems). A typical structure of a theoretical model of a flexible shaft supported by magnetic bearings is as follows:

$$\dot{x} = \begin{pmatrix} \multicolumn{2}{c}{0} & \multicolumn{2}{c}{I} \\ A_A & 0 & A_C & 0 \\ 0 & A_B & 0 & A_D \end{pmatrix} x + \begin{pmatrix} \multicolumn{2}{c}{0} \\ B_A & 0 \\ 0 & B_B \end{pmatrix} u \tag{3}$$

$$y = \begin{pmatrix} C_A & 0 & \\ 0 & C_B & \\ C_C & 0 & 0 \\ 0 & C_D & \end{pmatrix} x \tag{4}$$

where $A_A, A_B, A_C, A_D, B_A, B_B, C_A, C_B, C_C$ and C_D are matrices to be determined (see for instance (Schweitzer and Maslen, 2010) for details of how to obtain such a structure). Fixing the elements that are 0 or 1 implies that the number of parameters to be determined is reduced. Moreover, this simplifies the next steps in our procedure, as it will be shown in the next section. It is important to keep in mind that in some cases overconstraining the problem might produce worse results. Hence, it could be the case that for some systems it is better to set all values in the A, B and C matrices as free (i.e., fully identifiable).

Remark. It is important to realize that the parametrisation of the radial coupling terms at low speed is a hard task, as it will mostly be affected by noise (since the coupling at low speeds is low, the signal-to-noise ratio of the identification experiment is low). Nonetheless, even if one may think that these terms are negligible, it is important at least to obtain reasonable values on this parametrisation, as these values are used during the extrapolation to obtain models at higher speeds. This step can be partially controlled by constraining the PEM algorithm.

2.2. Obtaining a common basis

In order to draw parallelisms and establish connections between the two (or more) available models, it is first necessary to ensure that the coordinates of both systems coincide, that is, state vector x in the state space description corresponds to the same set of variables in both systems. We then need to obtain a common basis or coordinates set for all available systems. Since the models at speeds Ω_i were obtained from experimental data, there is no guarantee that the two state space representations use the same set of coordinates for each state. The inputs and outputs are clearly defined since they correspond to physical signals, but the same cannot be guaranteed for the states. Let the first identified system be given by:

$$\dot{w}_{sysid} = A_w(\Omega_1) w_{sysid} + B_w u \tag{5}$$

$$y \quad = C_w w_{sysid} \tag{6}$$

If we run the identification algorithm again at a different speed, the state space model would be:

$$\dot{z}_{sysid} = A_z(\Omega_2) z_{sysid} + B_z u \tag{7}$$

$$y \quad = C_z z_{sysid} \tag{8}$$

The signals y and u represent the same physical signal in both models (displacements measured by the sensors and currents at the bearings), while it is not known if the states w and z correspond to the same signals. Hence, it is necessary to convert all available models to a common, unique basis where all the states represent the same signal. To perform this step, a set of similarity transformations T_{ij} need to be found for each system. That is, if we consider state space realisations in Eqs. (5) and (7) with states w and z, we need to find T_{wx} and T_{zx} such that $w = T_{wx}x$, $z = T_{zx}x$ and the following identities hold:

$$\dot{w}_{sysid} = A_w(\Omega_1)w_{sysid} + B_w u \quad \Leftrightarrow \dot{x}_{sysid} = A_x(\Omega_1)x + B_x u \tag{9}$$
$$y \quad = C_w w_{sysid} \quad \Leftrightarrow y = C_x x \tag{10}$$

$$\dot{z}_{sysid} = A_z(\Omega_2)z_{sysid} + B_z u \quad \Leftrightarrow \dot{x}_{sysid} = A_x(\Omega_2)x + B_x u \tag{11}$$
$$y \quad = C_z z_{sysid} \quad \Leftrightarrow y = C_x x \tag{12}$$

There exist many techniques for such a task, but most of them are not numerically robust (especially for high order systems or rotor structures), or do not apply to multiple-input multiple-output (MIMO) and/or unstable systems. Our approach has been inspired by the work in (Caigny et al, 2008). There are 3 steps to be performed for each identified system to obtain a common basis:

- Choose an input/output pair of the MIMO system (any pair).
- Compute the poles and zeros of this single-input single-output (SISO) system. Sort these poles and zeros in the *same* order for all systems (more on this later).
- Divide this SISO system into a combination of first order and second order subsystems and apply a canonical transformation to each subsystem. This representation is unique provided the sorting in step 2 is done *properly*.

The most complicated task in these steps corresponds to the sorting of poles and zeros. For this task we need to compare the poles and zeros of all available models, and assign the *same* order to all models. This task can be fairly straightforward or complicated. The idea is to guess how poles and zeros move as we vary the speed. Figure 2.2 depicts a simple case where one can guess the root locus as a function of speed.

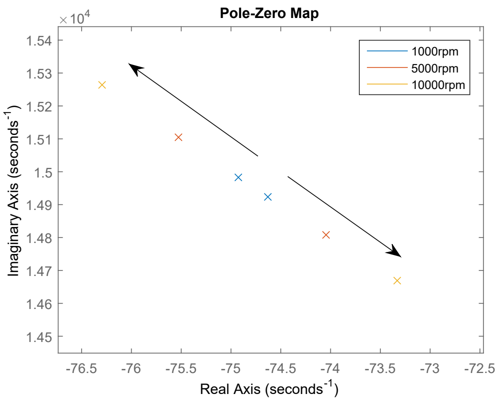

Fig. 1 Evolution of poles and zeros of a typical rotor as a function of speed. The arrows show the root locus that can be used to sort out poles and zeros

While this task is fairly trivial in theoretical models, it is quite complicated for state space representations that have been obtained via parametrisation of identified frequency responses, as the parametrisation step might place zeros in very different places based on a particular noise realisation. To avoid these issues, one can specify an initial model, a fixed structure and a limited range for each parameter, as described before. If several models are available, one could even look for root locus patterns. Based on our experience, the poles are sorted according to the real part; for the zeros, they are first split into very undamped zeros (imaginary part much larger than the real part) and damped zeros. The undamped are then ordered according their magnitude, and the damped according to the real part.

In the third step, this SISO system is converted into a combination of first order and second order subsystems and rendered in a canonical form. This step allows us to reach a unique state-space basis and to avoid numerical issues. We represent each system $\mathcal{F}^A$ as a series connection of first and second order submodels:

$$\mathcal{F}^A = K \prod_{i=1}^{N} \mathcal{F}_i^A$$

with K being a scalar gain and $\mathcal{F}_i^A$ the subsystems. For each $\mathcal{F}_i^A$, a canonical representation is obtained. Notice that, since this representation is computed for small order systems, no numerical issues can arise in this step. This representation is unique provided the sorting in step 2 is done properly. Since we want to obtain this unique representation for all subsystems, we look for a similarity transformation T_{12} that generates system $\mathcal{F}^B$ from system $\mathcal{F}^A$ and apply it to the original MIMO system:

$$A_1 = T_{12}^{-1} A_2 T_{12} \qquad T_{12} B_1 = B_2 \qquad C_1 = C_2 T_{12}$$

Hence, the problem of finding a matrix T_{12} can be reformulated as:

$$\min_{T_{12}} \ \alpha |T_{12} A_1 - A_2 T_{12}|^2 + \beta |T_{12} B_1 - B_2|^2 + \gamma |C_1 - C_2 T_{12}|^2 \tag{13}$$

where α, β and γ are design parameters to give equal weight to each term in the optimisation process. They are typically selected as:

$$\alpha = \frac{|B_2||C_2|}{|A_2|} \qquad \beta = \frac{|A_2||C_2|}{|B_2|} \qquad \gamma = \frac{|B_2||A_2|}{|C_2|} \tag{14}$$

Using the *vec* operator and the Kronecker identity we end up with a linear least squares problem that can be easily solved.

$$\begin{pmatrix} \alpha(A_2^T \otimes I - I \otimes A_1) \\ \beta(B_2^T \otimes I) \\ \gamma(I \otimes C_1) \end{pmatrix} vec(T_{12}) = \begin{pmatrix} 0 \\ B_1 \\ C_2 \end{pmatrix} \tag{15}$$

In general, there are many similarity transformations that satisfy this set of equalities. Among all these possible solutions, we are interested in those with a low condition number; otherwise precision will be lost during the computations. In that case, an extra term $cond(T_{12})$ can be added to the minimisation problem in Eq. (13) to improve the conditioning number of the solution. The minimisation problem then becomes nonconvex.

2.3. Merging models

Once all available systems have been transformed, an interpolation or extrapolation can be done directly on the state space matrices, since the states for all models represent the same signals. To proceed, a certain speed dependence needs to be considered for the state space matrices, for instance:

$$\dot{x} = Ax + A_G f_A(\Omega))x + B f_B(\Omega) u \tag{16}$$

$$y = C f_C(\Omega) x + D f_D(\Omega) u \tag{17}$$

From our theoretical models of rigid and flexible shafts supported by magnetic bearings (Schweitzer and Maslen, 2010), we expect the A matrix to have an affine dependence on the speed, while the B and C matrices should be constant:

$$\dot{x} = Ax + A_G \Omega x + Bu \tag{18}$$

$$y = Cx + Du \tag{19}$$

Once this dependence is specified, a set of equations can be posed and solved via linear least squares. For instance, in the case of the linear dependence we have:

$(A_0 + A_G \Omega_0) = A_1$ (A matrix of identified system at Ω_0, transformed to a common basis)

$(A_0 + A_G \Omega_1) = A_2$ (A matrix of identified system at Ω_1, transformed to a common basis)

We are looking for matrices A_0,A_G that satisfy these equations in the least squares sense. Likewise, we can consider dependences for the B and C matrices:

$$\min_{A_0,A_G} \sum_i^{all\ models} |A(\Omega) - (A_0 + A_G \Omega_i)|^2 \qquad \min_{B_0,B_G} \sum_i^{all\ models} |B(\Omega) - (B_0 + B_G \Omega_i)|^2 \qquad \min_{C_0,C_G} \sum_i^{all\ models} |C(\Omega) - (C_0 + C_G \Omega_i)|^2$$

This problem can be solved again using the Kronecker identity. We can consider nonlinear dependencies as well (i.e., $A_i = A_0 + A_{G1}\Omega_0 + A_{G2}\Omega_0^2$). It is possible as well to relate these identified A_0 and A_G with the expected theoretical values, since we can use this coordinate transformation procedure to the theoretical model as well. The whole procedure is summarized in the following chart:

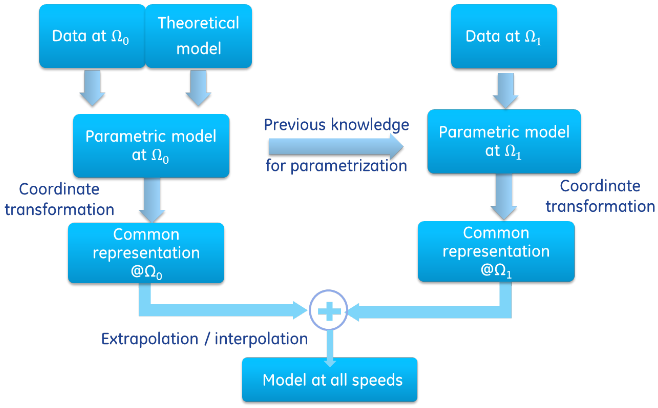

Fig. 2 Steps for the system identification procedure.

3. Experimental results

In this section we share results of the proposed procedure as implemented in a GE product. Due to confidentiality issues we only disclose the relevant information that is needed to understand the system identification problem. The system consists of 4 inputs (currents going through the magnets) and 4 outputs (measured displacements, in the radial direction at two different axial positions along the shaft); gyroscopic effects are not negligible, as it will be seen later on. The operating speed range of the machine is [0, 12000]rpm. Frequency responses being depicted in this paper have been scaled for confidentiality issues. First, two models of the system were obtained at 2000 and 4000rpm, following the procedure in (Blom, 2010). As an example, Figure 3 shows the frequency response of the system at 2000rpm and its parametric equivalents using PEM (fixing the structure as in Eq. (3)). Based on these two models, we compute a model valid for the whole operating range, following the steps described above. Figure 4 shows the expected behaviour at 8000rpm, compared against an identified model. Figure 5 zooms into the block (1,1), where we can see how the splitting of the modes is properly estimated.

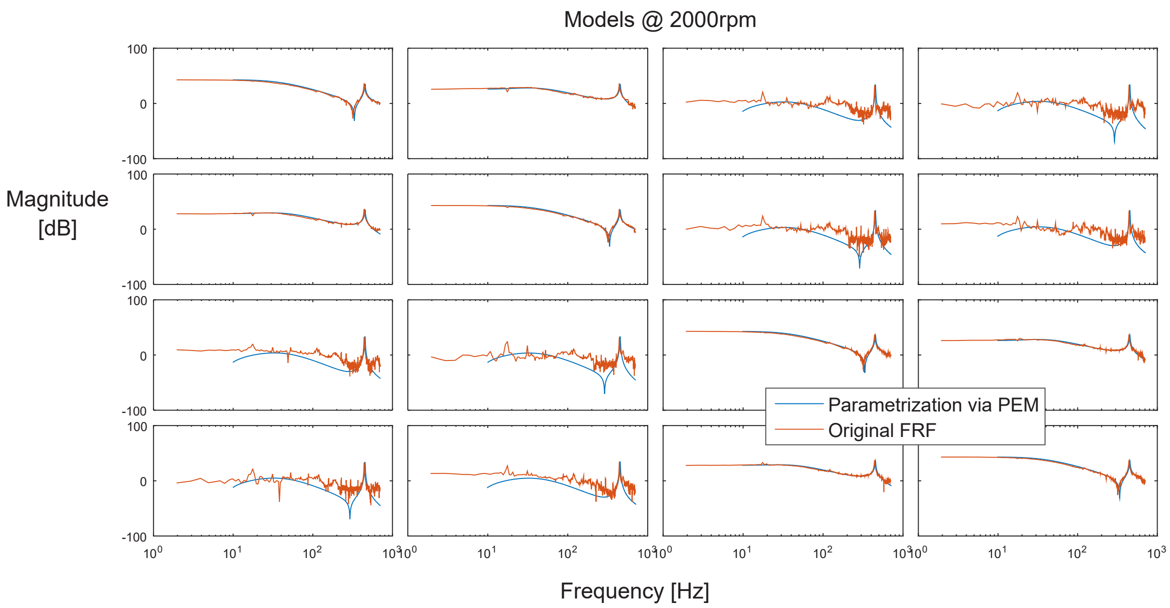

Fig. 3 Comparison of parametric and non parametric model at 2000rpm. The off-diagonal terms exhibit the largest discrepancy.

It is expected that using models at low speed might not give very good accuracy at high speeds. In this case, our algorithm can be used in an iterative way as follows:

- Obtain two models of the system at low speed, Ω_0 and Ω_1.
- Use such models to predict the behaviour of the system at a higher speed Ω_2.
- Use this prediction to analyse the stability and performance of the machine at Ω_2.
- Once it is safe to accelerate the machine, run experiments at Ω_2, and now use this new model (in conjunction with other models) to improve the overall predicted model.

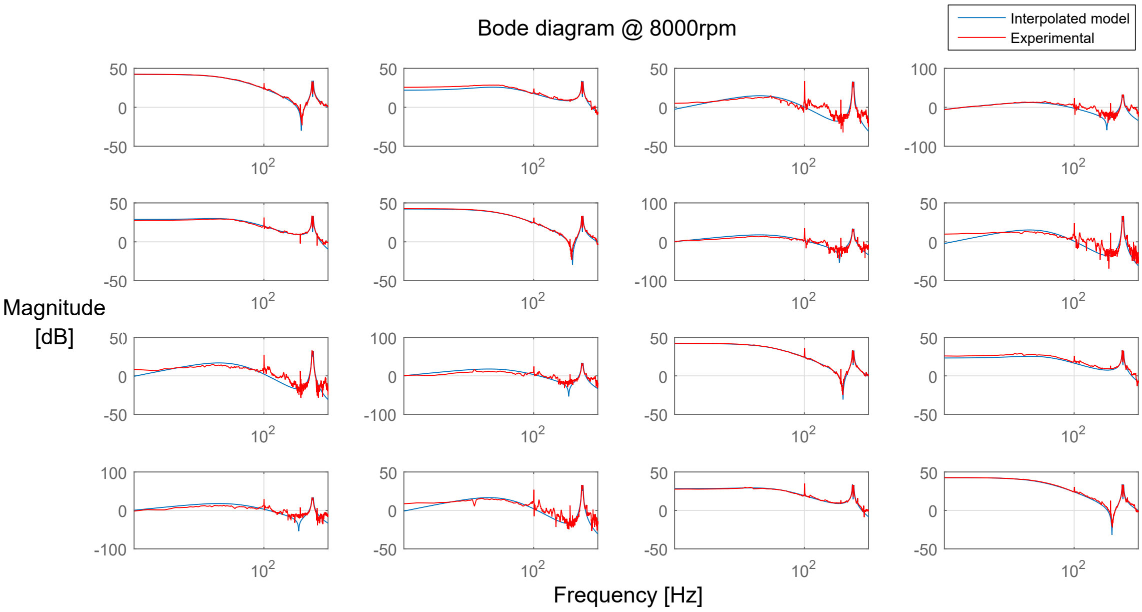

Fig. 4 Comparison of predicted and identified behaviour of the system at 8000rpm

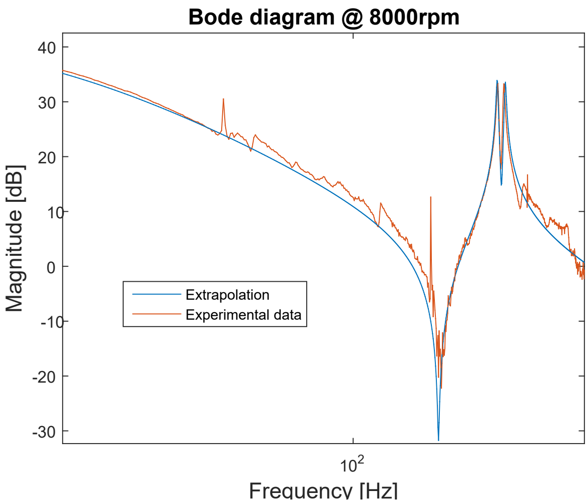

Fig. 5 Comparison of predicted and identified behaviour of the system at 8000rpm - block (1,1), i.e., frequency response between the first input and the first output. We can observe how the splitting of the modes is properly captured in our prediction.

Finally, we would like to point out that our approach is general enough that we can consider other speed dependences that go beyond the expected theoretical ones. In fact, in some of our machines, we observe a quadratic dependence rather than a linear one. Likewise, experimental results show that, in some cases, the B matrix also depends on the speed.

4. Conclusions and discussion

We have presented a method to obtain a dynamic model for the whole speed range. The advantages of such a procedure in comparison with existing techniques are clear: avoiding landings during the commissioning phase (as we can predict the behaviour at high speeds in advance) and faster system identification (as fewer experiments are needed to validate the machine). Of course, such a procedure implies a leap of faith that one can infer the behaviour of the system at high speeds from low speeds. To reduce this assumption, in reality one could implement this procedure in an iterative way, as shown in the previous section. This approach is amenable for automatisation, reducing the need for an expert during commissioning phase. There are still open problems in this regard: for instance, automatic order selection for the parametric model or a robust procedure to sort out poles and zeros.

One of the advantages of this approach is that we do not rely on the theoretical model or any knowledge of modal frequencies: this is beneficial when unexpected modes (i.e., casing modes) appear during the commissioning phase. Notice that theoretical models for the casing are in many cases not even available, or they are not very reliable whenever they exist.

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